Waiting Line Management

7

Unit Introduction

The phenomena of waiting are common in business and industry. In most business situations, the customers are the machines, not people, like trucks waiting to be unloaded, ships waiting to dock, orders waiting to be filled, jobs waiting to be processed. Waiting is annoying, but we do wait. Given time most of the facilities can process more than they are called upon to process. The waiting line theory or queue theory is commonly used in planning and analyzing service capacity of a system. Based on type of the structure a specific queue model, from innumerous models, is applied. The main characteristics under consideration are, the arrival pattern of customers, queue discipline, length of the queue, queue behavior etc. Because there are large number of possible models, a notation set has been developed by D.G. Kendall that makes it easy to identify the model applicable for a particular system. After finding out the values of different measures of performance, the next is to determine the efficiency of the service system. At the very beginning it should be remembered that all the measures of performances are not required to determine the efficiency of the system. Therefore this unit will include a discussion on introduction of the concept of waiting line management, characteristics of waiting line, and the methodologies of waiting line management.

Lesson One: Introduction To Waiting Line Management

Lesson Objectives
After completing this lesson you will be able to:

- Explain the theories of satisfaction
- Discuss the quality factors affecting satisfaction
- Justify the customer's reaction to delay
- Describe the methods of delay management

Many things in life are worth waiting for. We would not mind waiting for the course completion to earn an MBA. We love to wait for good news from our dear ones. Waiting in line is common in our day to day life. We wait in line to avail buses, to deposit our money in the bank, to pay bills, to buy goods or to use lift. The phenomena of waiting are also common in business and industry. We find mechanics waiting for their tool cribs or standing idle for mechanics. Students wait for photocopy of books. In most business situations, the customers are the machines, not people, like trucks waiting to be unloaded, ships waiting to dock, orders waiting to be filled, jobs waiting to be processed.

Waiting is annoying, but we do wait. It is estimated that we spend at least 10% of our working moments waiting. Given time most of the facilities can process more than they are called upon to process. But still, because of the random nature of arrival of customers, waiting is bound to occur. The waiting line theory or queue theory is commonly used in planning and analyzing service capacity of a system. It is a mathematical approach not only to analyze waiting times, but also used in large number of situations for example,

> The waiting line theory is a mathematical approach not only to analyze waiting times, but also used in large number of situations

- to determine the number of beds in a hospital or required size of a restaurant
- to determine the number of runways in an airport
- to determine the number of lifts in a building or the number of ATM machines in a bank branch
- to determine the number of switches in a telephone box
- to schedule delivery of jobs.

Delay in service has a direct bearing on customer's satisfaction. To reduce delays, management needs to expand capacity. But it is expensive to increase capacity. Managers have to weigh the cost of providing additional capacity against potential decrease in delays in service. Therefore, management is interested in finding the *appropriate level of service* that will ensure the customer satisfaction.

Theories on Customer Satisfaction
During and after consumption of goods and services, consumers develop a feeling about the quality of the commodity. The feeling may be positive or negative. If the consumer develops a positive feeling, we say that the consumer is satisfied with the service or product. If the consumer develops a negative feeling we say that the consumer is dissatisfied. Consumer satisfaction or dissatisfaction is a post-acquisition and post consumption judgement of the quality of service or product. Consumer satisfaction has been defined as "the overall attitude consumers have towards a good or service after they have acquired or used it"[1].

> Consumer satisfaction or dissatisfaction is a post-acquisition and post consumption judgement of the quality of service or product.

[1] Robert A Westbrook and Richard L. Oliver, The Dimensionality of consumption Emotion Patterns and Consumer Satisfaction, Journal of Consumer Research, Vol 18 (June 1991).

From a managerial perspective, maintaining and enhancing customer satisfaction is critical. Satisfied customers are important to companies because, on average, approximately 70% of all sales are derived from repeated purchases. Firms can no longer maintain volume or profits by seeking new customers. Attracting new customers are expensive. It is much cheaper to keep the current customers happy and satisfied. After the purchase and consumption of products or services, customers compare their pre-purchase perception of the quality of the product/service and their post-consumption perception of the quality of the service/product. Depending how the actual performance measures against expected performance, the customer will experience positive, negative or neutral emotions. To understand how consumers develop satisfaction or dissatisfaction regarding a service or a product, we will look into

1. Confirmation model
2. Disconfirmation model, and
3. Equity theory of satisfaction

1. **The confirmation model:** Early thinkers about consumer satisfaction assumed it as meeting expectation of consumers. The confirmation model describes the successful outcome as contentment. We are content when our computer is working without problem or the photocopy machine does not get jammed. This low arousal state is matched by discontent when negative expectations are met. But poor satisfactions are ignored because of habit. People put up with many troubles because they are used to them and no longer notice them as a problem. For example, we are used to slow responses of the salesmen, late arrival of buses, or congestion in roads. The tolerance of deficiencies in services or products is explained by the "adaptation theory". This theory makes expectation relative to experience. As long as gap between expectation and reality is small, the customers tend to be accommodative. Only when the gap is too large and does affect the adaptability of the customers it leads to discontentment.

Disconfirmation model explain satisfaction or dissatisfaction of consumers by explaining the gap between expectation and reality.

2. **The disconfirmation model:** Unlike the Confirmation Model, the Disconfirmation Model focuses on high arousal conditions. The model attempts to explain satisfaction or dissatisfaction of consumers by explaining the gap between expectation and reality. If the product or service has more features than expected, the customer is surprised and is satisfied. On the other hand, if the product or service contains fewer features than expected, again the customer is surprised and is dissatisfied. The model recognizes that people use standards of assessment to judge a product or service. According to this model satisfaction is a cognitive state of a consumer of being adequately or inadequately rewarded for the sacrifices he/she has made to acquire and consume the service or product. In the disconfirmation model, the degree of satisfaction or dissatisfaction depends on,
   (i) the size of discrepancy between expectation and experience,
   (ii) value of the product, and
   (iii) the perception of consumer regarding the performance of the service or product.

   The larger the discrepancy between expectation and experience the more aroused (positive or negative) we get. The more money we spend for a service or product the higher the expectation. Finally, the higher the expectation and the higher the resulting experience the higher is the satisfaction.

3. **The equity theory:** Another approach to explain consumer satisfaction is the Equity Theory. The Equity Theory holds that the customer will analyze the outcome of the transaction by comparing the value of the input he has put in the effort and the value of input provided by the other party. If the consumer feels that his input contribution is more than that of the other party he will develop a feeling of inequity. According to this theory, the norm is that each party to the exchange should be treated equitably. The ratios of outcomes and inputs of both the parties should be equal. Equity theory holds that satisfaction results from comparing one's outcomes and inputs with that of others. From the consumers perspective inputs are information, money, effort and time spent to acquire the good or service. Outcome consists of receiving the goods or services, performance of the product, and the feeling associated with the use of the product or service.

Equity theory analyze the outcome of the transaction by comparing the value of the input in effort and the value of input provided by other party.

Activity: Choose a service product of your choice and think why you as a customer need better service for that product? Exchange your views with any of your friend(s).

Factors Affecting Customer Satisfaction

Consumer's expectation of a product or service is derived from the perceived quality of the commodity. Quality has multiple dimensions in the mind of the customer, and applying them one or more at a time at any one time. Quality is defined as *customer's overall evaluation of the excellence of the performance of a good or service*. Because of the difference between a product and a service, they have different quality dimensions. Table 7.2.1 provides a summary of the quality dimensions of products and services. They are discussed in detail below:

Table 7.2.1: Dimensions of quality

Dimensions	Description
Quality of Products	
Confirmation to Specification	Degree to which the product meets industrial standards
Fitness of Use	Chances of failure or malfunction
Value	Worth of the product in relation to similar products
After Sales Services	Ease of repair, and timeliness of personnel
Psychological Impression	The look, feel and sound of the product
Quality of Services	
Tangibles	Physical facilities, appearance and equipment
Reliability	Dependability of employees
Responsiveness	Prompt response
Assurance	Ability and knowledge of employees
Empathy	Individualized attention

Customer's Response to Delay

Customers do not like to wait for services. But delay in delivery of service is a perennial feature of service outlets. Customers have to wait to deposit or withdraw money from bank; they have to join queues to post their letters; they

From the economic point of view *waiting* is wasteful nor is it productive.

have to wait to buy tickets for bus; they also have to wait for transport and get held up in traffic jams. For the individuals it is frustrating to wait. For the economy it is wasteful because people waiting for service is neither consuming nor they are producing. But the service providers are unable to remove delays in their system. Even if they plan for peak demand they still end up with queues in their system. Waiting for service is a result of the random arrival of the customers. Customers arrive for service at random, creating localized peaks and valleys in demand pattern. In most service outlets arrival of customers cannot be programmed or scheduled. Researches show that, on average, people wait for more than half an hour each day for different services and appointments.

a) *Evaluation of service:* Organizations that create delay by poor design of their service facilities do so at their own peril. The customers consider waiting a punishment, and they associate it with the whole service experience. The longer the delay the lower is the evaluation of the whole system. Many of our local organizations, specially the nationalized institutions like banks, do not notice the irritations caused by delay.

Customers perceive to waited long to get service where as employees feel that customers did not wait long.

b) *Difference in Perception:* There is a perpetual difference in opinion regarding the length of delays between the provider of service and the customer. Customers always perceive to have waited long where as on the other side the employees feel that the customers did not have to wait long. Research found that when a customer at a bank thought the average delay was 5.6 minutes the staff perception was 3.2 minutes and the actual delay was 4.7 minutes.

c) *Type of Dissatisfaction associated with delay:* There are two types of dissatisfaction associated with delay. A low involvement discontents when delay is predicted and a high involvement affects when delay is not predictable. As delays become more common and impose a more frequent time-cost, people find it easier to put up with it.

d) *Reasons for delay:* If people are informed in advance as to the reason for delay, they are less irritated and are patient in their waiting. The provision of delay information is now standard among the transport operator, such as airlines. Such information seems to be much appreciated despite the fact that it does nothing to reduce waiting time.

e) *Entertainment during Delay:* People are less dissatisfied by delays if they can fill in their waiting time. For example, ANZ Grindlays Bank at their Dhanmondi branch provides TV entertainment to their customers waiting in queue; another is the provision of mirror in places where people have to wait, e.g., at lifts in buildings. Additional examples are places where waiting is very frequent, to provide the latest newspapers and magazines for the customers to read. Research has shown that many of those waiting do not even realize that they had been held up.

f) *Control on delay:* People become more irritated when they believe that the service provider has control over the situation. If delays are beyond the control of the providers, the customers are more tolerant to delays. For example, airplane delayed due to inclemental weather does not create negative impression about the provider. But on the other hand if the customer believe that the delay is a result of slow check-in at the counter it is bound to create negative feeling about the provider.

g) *Delay at the beginning or end of the process:* When delay occurs at the beginning or at the end of the process it is evaluated more negatively than when it occurs at the mid-process. It is better to impose delays in the middle of the process, rather than at the end or beginning. In a restaurant it is more appropriate to impose delay in providing food, but not to keep the customers waiting before taking orders.

h) *Value of exchange:* People expect that their cost of waiting to be compensated by appropriate rewards. This implies that those receiving benefits in an exchange would be more tolerant to delays than those incurring costs. For this reason, in a bank, those are in queue to pay bills complain more than those waiting to encash their cheques.

Delay Management

Among the many quality dimensions that contribute to satisfaction or dissatisfaction of customers, delay is one of the major and immediate contributors to dissatisfaction. This is amply clear from the discussion in the previous section. Management of service organizations should attempt to reduce delay in providing services. There are three approaches that managers can adopt to reduce delays:

> Management of service organizations should attempt to reduce delay in providing services.

1. Operations management
2. Influencing demand
3. Perception management

1. **Operations management:** Wherever feasible, management should try to avoid delays by increasing service supply. This they can do by increasing number of counter at peak hour and reduce it at off peak hours. Or they can use experienced and quick workers during peak hours. This is easy to say but very difficult to maintain. It is very expensive to provide additional facilities. To plan for peak demand situation will result in under utilization at non-peak hours. Because of the random nature of customer arrival, even if management decides to provide for peak demand, they still will face delays in their services. Some make use of part-time workers, others train their non-service workers to assist the service workers during peak periods, and still others use fast automatic machines.

2. **Influencing demand:** The second approach is to regulate demand or influence demand to occur at specific time. Scheduling appointments, as done by doctors for non-emergency patients, is one good example of regulating demand for service. Providing incentive is another method of regulating demand. For example, using differential price to shift demand to particular period, e.g., Mondays are cheap days at the cinema in Britain. Some even segment their customers according to the nature of service required. If a group of customers need something that can be done very quickly, they are given special line so they do not have to wait for the slower customers. For example, at some departmental stores the customers with less than five items have a fast channel to check out, whereas, customers with more than five items have to pass through a slow line.

> Providing incentive is another method of regulating demand.

3. **Perception management:** The third approach is to try to ensure that the customers see the delay in a favorable way. By supplying delay information customers can be made more tolerant to delays. By providing diversions can the customers be made less dissatisfied to delays, like providing

> Queuing ticket system allows the customer to conduct other business while waiting.

entertainment as they wait for service. Another way is the queuing ticket system that allows the customer to conduct other business while waiting. For example, in banks, the customers can sit down and read instead of standing in line. But this approach implies that delay is normal and management is only interested in alleviating the discomfort of waiting, not in eliminating it.

Activity: Think you as a bank manager in Bangladesh. Now why and how you will manage your customers when you find that due to some unavoidable circumstances they are waiting long for a service. Justify your answer.

Employees that are not part of service should be kept out of sight. Nothing is more frustrating to someone waiting in line to see employees, who potentially could be serving those in line, doing nothing or working on other activities. Greeting the customers by name, or providing some other special attention, can go a long way toward overcoming the negative feeling of a long wait. Psychologists suggest that the workers should invoke friendly actions such as smiling when greeting or serving. Test result shows significant increase in perceived friendliness of the servers in the eyes of the customers when the servers smile while dealing with customers.

Discussion questions

1. Describe the different theories of consumer satisfaction.
2. Describe the factors that influence consumers' satisfaction?
3. How do consumers respond to delay?
4. What should management do to manage delays?

Lesson Two: Waiting Line Queuing System

Lesson Objectives

After completing this lesson you will be able to:

- Describe the major characteristics of waiting line
- Explain the arrival pattern
- Discuss the service pattern

Characteristics of the Queuing system

Analysis of waiting line problem starts with a description of the structure of the service system. Based on type of the structure a specific queue model, of innumerous models, is applied. The main characteristics under consideration are:

a) The calling population or population source
b) The arrival pattern of customers
c) The distribution of customer arrival
d) The service pattern or number of servers
e) The service time distribution
f) The queue discipline
g) The length of the queue
h) The queue behavior
i) The exit of customer from the system

In structuring a waiting line model the inputs are called arrival. The arrival times and service times are controlled by different probability processes. The output rate of the system is dependent on the interplay of arrival pattern and service pattern. Based on the probability distribution of arrival and services the basic values of the parameters under study can be determined. The Figure 7.2.1 shows a typical queuing system with its different components.

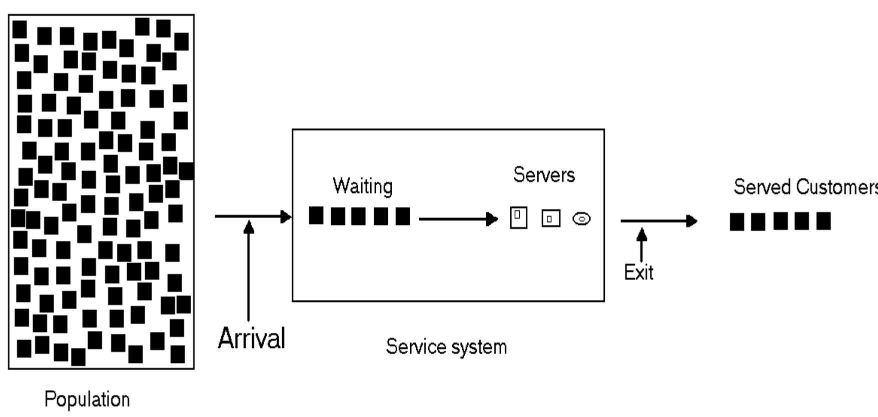


Figure 7.2.1: Typical queuing system

Population source of the system can be of two types: finite and infinite.

a) The calling population or population source: The population is the source of input to the service system. In the Figure 7.2.1 it is the first parameter of the system. Each system has a definite source called its population from which demand for its service is created. The population source is unique for each type of system. Source of the system can be of two types: *finite or infinite*. It is a major concern for the analyst to determine whether the source for potential customer is finite or infinite. A *finite population* source refers to the limited size customer pool that will use the service. If the source is finite, and a customer leaves the population and joins the system for service, the population gets reduced which, in turn, reduces the probability of creation of next demand for service. For example, a photocopy machine in an office has a finite population source. Demand for its use can arise from among that office employees, no one outside the office can use the machine. So if an employee uses the machine, the chance of another using it is very small. If the population source is too small it may even affect the probability of an arrival.

Alternatively, an *infinite population* is one in which the number of customers in the system doesn't affect the rate at which the population generates new arrivals. For example, the photocopy machines at Nilket, New Market at Dhaka have an infinite population source. Its customers are all living in and around Nilket area. Because the customer population is large and only a small fraction is at any one time demanding its service, the number of new arrivals it generates is not affected by the number who is using or has already used its service. Such population is called infinite population.

An infinite population is one in which the probability of future arrival is not affected by the number of customer already in avenue.

b) The arrival pattern of customers: Arrival pattern is also an important issue to the analysts. The arrival into a service system can be *scheduled or unscheduled*. When customers arrive at service centers by appointment it is called scheduled arrival. For example, general patients can consult specialist doctors only by appointment. Unscheduled arrivals are common in many service centers like retail outlets or restaurants. Unscheduled arrivals are random in nature and are also called *random arrivals*. Random arrivals are more common than scheduled arrivals.

Arrival pattern of customer can be classified as scheduled and unscheduled, or controlled and uncontrolled or even single and in batches.

Arrivals can also be *controlled or uncontrolled*. The arrivals at a system are more controlled than is generally recognized. Barbers may decrease their Friday arrivals by charging more than other weekdays, or a doctor, in a private clinic, not seeing more than a fixed number of patients a day. The simplest of all arrival-control devices is having fixed business hours. But some service demands are clearly uncontrollable, like hospital emergencies. Controlled arrivals are more common than uncontrolled.

Arrivals can also be *single* or *in batches***.** Customers can arrive in batches, such as the arrival of family to a restaurant or garment factories receive orders for a particular item in batches. When customers arrive individually or singly it is called single arrival. For example, a housewife going for shopping at 1-Stop Mall; a student going for class; or a person waiting to use the lift in a building.

c) The distribution of customer arrival: The arrival of customers for services can be described as either *average arrival rate* or as *average inter-arrival time*. Average arrival rate means the average number of arrivals per a given time and average inter-arrival time means the average time between arrivals, that is, the time between one arrival and the next. In case of scheduled arrivals, the arrival rate is relatively predetermined, while that in unscheduled it is a random variable and therefore we have to find its average time and also need to find out its frequency distribution.

Arrival of customers for services can be described by either average arrival rate or by the average inter-arrival time.

In scheduled arrival the arrival distribution is generally *constant* with exactly the same time period between successive arrivals, as shown in the Figure: 7.2.1. It would have same inter-arrival time (t) and with variance zero (0). But in case of random arrivals, the time between arrivals would vary and would have a positive variance. When arrivals at a service outlet occur on a purely random fashion, a plot of the inter-arrival times yield an *exponential* distribution (Figure 7.2.2).

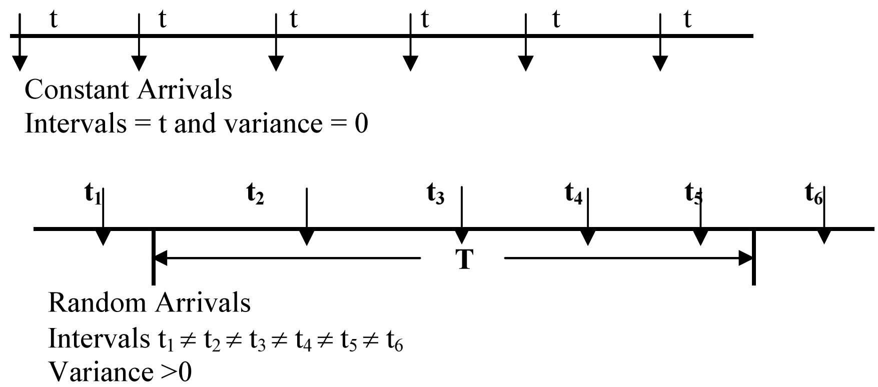


Figure: 7.2.1 Constant Arrivals and Random Arrivals

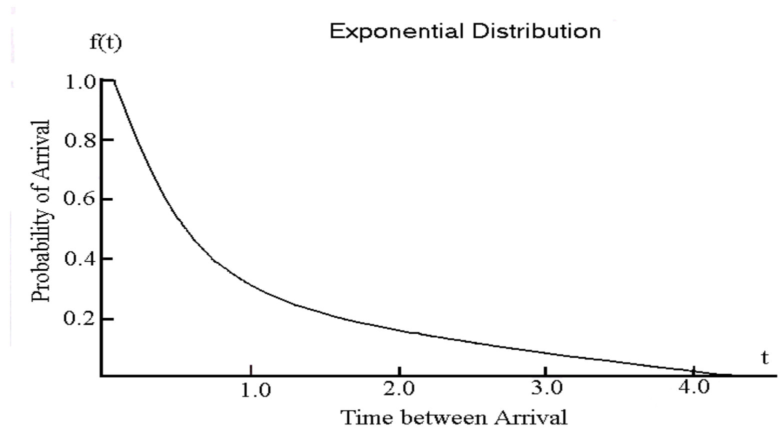


Figure 7.2.2: Inter-arrival times yields an exponential distribution

But when one is interested in the number of arrivals during a given period of time (T), the distribution appears as shown in Figure 7.2.3. If the arrival process is random, the distribution is Poisson. Random arrival means that even if the mean number of arrivals in a time period is known, the exact moment of arrival cannot be predicted. Thus each moment in the time span has the same chance of having an arrival. Such behavior is observed when arrival is independent of each other and one arrival does not affect the chance of other future arrivals.

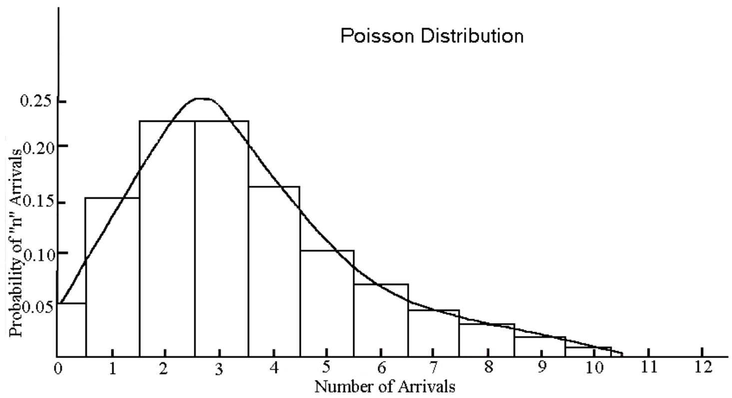


Figure 7.2.3: Interested number of arrivals in a given period

Poisson distribution is commonly observed in average arrival rate and exponential distribution for inter-arrival time. Other distributions observed in queuing situation are hyperexponential, hyperpoisson, and erlang distribution with k parameter. The average arrival rate is usually designated by the Greek letter *lambda*, λ and interarrival time, which is reciprocal of average rate, is 1/λ.

d) The service pattern or number of servers: The capacity of a service system is a function of the capacity of each server and the number of servers being used. The terms *server* and *channel* are synonymous, and it is generally assumed that each channel can handle one customer at a time. The physical flow of customers through the facility can be in single line or in multiple line, or mixture of both. The format of flow depends on the nature of service demanded and the pattern of service offered.

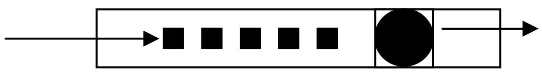
The physical flow of customers through the facility can be in single line or in multiple line, or mixture of both.

A *Single-channel-single-phase* flow is the simplest of all waiting line formats. When all the services demanded by the customer can be performed by a single server the facility is arranged such that customers form a single line and go through the service facility one at a time. Examples are checkout counter at retail stores and machine that must process several batches of parts.

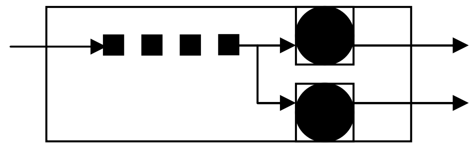
A *Multiple-channel-single-phase* (identical service) approach is used where the demand for services are identical but is very high to warrant use of more than one server for the same type of service. Depending on the design of the system, customers may form separate lines in front of each server or they may form a single line but separate into different lines at the point of service. This approach is also known as snake-line queue. It is commonly seen in service centers with high service demand but low in waiting space, like Standard Chartered Grindlays Bank office at Dhanmondi, Dhaka.

A *Multiple-channel-single-phase* (non-identical service) is a format used when different set of customers has different type of service requirement. Service desired by customers are not identical. Each demand set is passed through the system in separate lines, but for each line a single server performs the desired services. For example, we observe in bank separate counters for deposits and encashment of cheques.

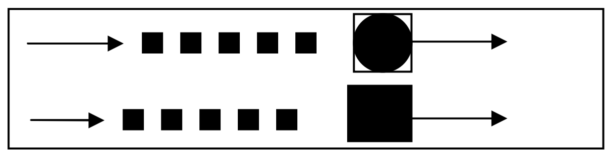
A *Single-channel-multiple-phase* arrangement is used when the demand for service can be performed best by a series of servers in sequence, but the flow of the customers is in one line. Customers form a single line and proceed from one service facility to the next. This approach is very common in University Cafeteria where different food items are arranged in sequence and the students have to pass through the system in sequence.



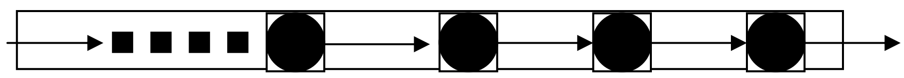
Single Channel Single Phase



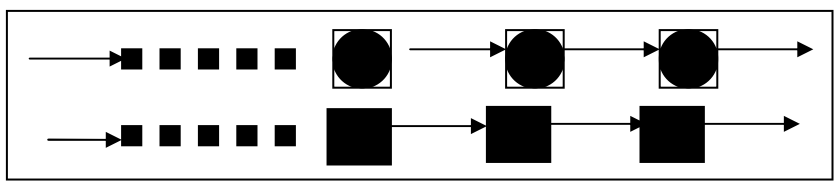
Multiple Channel Single Phase
(identical facilities, single line)



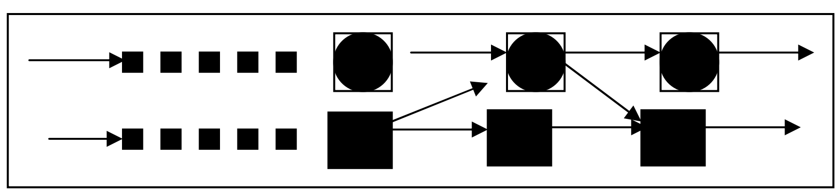
Multiple Channel Single Phase (non-identical facilities)



Single Channel Multi Phase



Multi Channel Multi Phase



Mixed

Figure 7.2.4: Single line, Multiple line, and Mixture physical flow of customer

A *Multiple-channel-multiple-phase* arrangement is appropriate where the demand for services is high; there are more than one set of demand type, and where the demand can be best served by sequence of servers. The facility is arranged such that each set of service type has a separate line and the customers move from one server to the next. In most cases, once the service has started the customers cannot switch channel.

A *mixed-channel* is used in situations where the demand patterns of the customers is varied that it is not possible to make special arrangement for each set of demand type. Some customers may use part of the facility and leave, whereas, others may pass through each phase. Arrangements are flexible and customers have the option to choose the sequence of services to avail.

e) **The service time distribution:** The service given at a facility consumes time. The service time may be constant, say exactly 12 minutes for each service, or it may fluctuate, for some it may be 4 minutes for another it may be 4.5 minutes, etc. Like random arrival, if a service time fluctuates then a service time probability is required to describe the length of time that a customer would spend in getting the service. There are two ways of describing the fluctuation of service time. One is to describe it by the *average length of service* and the other is *average rate of service*. Average length of service deals with the length of time required to give each service on average, for example, 25 minutes per service. Average rate of service means the number of customers served within a given time, for example, 10 customers per hour on average.

> If a service time fluctuates then a service time probability is required to describe the length of time that a customer would spend in getting the service.

f) The queue discipline: A queue discipline is a priority rule or set of rules for determining the order of selection of customer to receive services. The rules selected have effect on the length and size of the line and also the time spent in the line. The most common priority rule is *first come first service* (FCFS) where the first to enter a line receives the first service. Another is *last in first served* (LCFS), where the last person in line gets the first service. Other customer selection methods are *emergency* (preemptive) *priority*, *ladies first priority*, *highest profit customer first*, *largest order first*, *regular customer first*, *service in random order* (SIRO), *earliest promised due date* (EDD), *shortest expected processing time* (SPT), etc. Whatever selection rule is used, its effectiveness depends on the customer being aware of the rule and the server can apply it without discrimination.

> A queue discipline effects the length and size of the line and the time spent in the line.

g) The length of the queue: As in calling population, a distinction has to be made as to the number of customers allowed to wait in line for service. In theory, an infinite line can form to receive services, but in practice, because of limitation of space for waiting, an infinite queue is never encouraged. But it is also true that no store manager would discourage customer to enter his store because there are too many customers already inside the store. Good example of infinite queue length is the lines of vehicles backed up for miles, recent days phenomenon in the streets of Dhaka or the eager cricket fans waiting in lines to buy tickets for the Pepsi Asia Cup Cricket Tournament.

> An infinite line with limited space for waiting is never encouraged.

Systems that have capacity or time limitations generally restrict the queue size. If the queue length is finite or limited, the customer cannot enter the system when it is full. Typical examples of finite queues are full lifts in busy buildings or full parking lots or private practicing doctors refusing to see more than a fixed number of patients a day. Note that, if the queue size is finite, then the system size is also finite. Restricted queue size influences the actual arrival distribution, because the customer denied entry may rejoin the population, or may try again or may seek service elsewhere. These actions have impact on the population size specially when the population is finite.

h) The queue behavior: Customers behavior also has effect on the parameters of queue system. Customers refuse to join a line when they see a very long line and chances of getting quick service are very low. This is called *balking*. But customer also *reneges*. Reneging mean leaving the queue before

Customers refuse to join a line when they see a very long line and chances of getting quick service are very low.

getting served. Customers waiting in line are either impatient or patient. Those who renege or bulk are called impatient customers. Patient customers are those who enter the system and do not leave till served. But patient customer, while waiting for service, seeing a shorter line may switch between lines. This is called *jockeying*. Customers also *recycle* by returning to the queue immediately after obtaining service. This phenomenon is commonly observed when the authority restricts the number of tickets that an individual can buy as in Pepsi Asia Cup Tournament.

i) The exit of customer from the system: Once the customer is served, two exit fates are possible: (a) the customer may return to the population source and immediately become candidate for re-arrival, or (b) the customer exits the system but does not return to the calling population. The first case can be illustrated by the customer who patronage the same restaurant for every meal or the machine in a factory that has been repaired and has joined production line. In the second case, the served customer has a zero probability of returning for re-service. For example, a patient who had an appendectomy operation will never require a second similar operation. If the population is finite the non-return of served customer to the population will modify the population structure and will also modify the arrival rate for service.

Discussion questions

1. Explain how the source population can affect arrival rate.
2. What do you mean by arrival pattern?
3. Describe the common service system structure in use.
4. What do you mean by single-channel-multi-phase queue?
5. Describe, in brief, the different components of a service system.

Lesson Three: Waiting Line Methodology (i)

Lesson Objectives
After completing this lesson you will be able to:

- Explain the managerial problems in waiting line situation
- Describe the methodology of queue analysis

The Managerial Problems in Waiting Line
The central problem in every waiting line situation is a trade-off decision. There are two basic categories of cost in queuing situation. The first is the cost of providing service. The greater the service level, the higher the cost of providing service. The second cost is related to customer waiting. Waiting cost decreases as the capacity to provide service increases.

The manager must weigh the added cost of providing more rapid service against the inherent cost of waiting. Unfortunately, the two costs are in direct opposition to each other, as shown in Figure 7.3.1. That is, the cost of providing service increases with the increase in service level, while the waiting cost declines with the increase in service level. For simplicity, in the figure, the cost of providing service is shown to have linear relationship. Although a step function is often more appropriate, use of a straight line does not distort the picture. As capacity increases, the number of customer waiting decreases. As is typical in a trade-off relationship, total cost is U-shaped. The goal of analysis is to find the service level where the total cost is minimum.

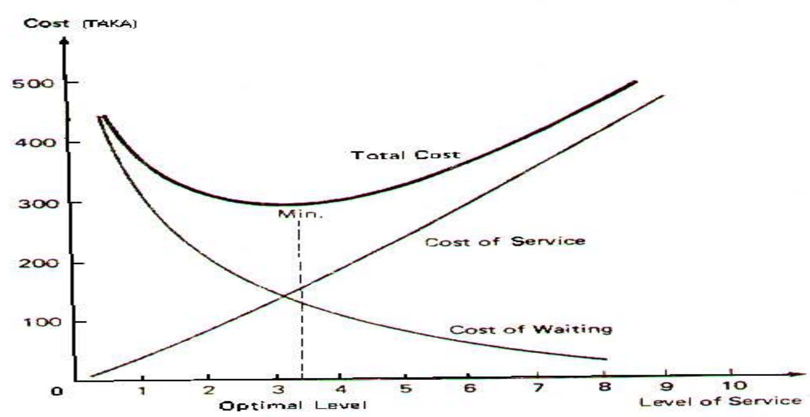


Figure 7.3.1: Two types of waiting cost

Thus the costs involved in a queuing situation are:

i. Service facility cost, and
ii. Customers' waiting cost.

i. **Service Facility Cost:** All costs associated with providing services are facility cost. It includes capital investment cost, operating cost, and maintenance cost. Adding facility to the system would require more space, furniture and fixtures. To provide for it the organization would have to borrow more capital or spend their money on the facility. Capital investment is expressed in term of interest, amortization, and opportunity costs. Thus, cost of capital increases with the increase in facility or service level. Operating cost includes cost of labor, energy, and materials required for the additional facility. Additional facility would require additional maintenance, repair, insurance, taxes, rental of space, and other fixed costs.

ii. **Waiting Cost of Customers:** Customers waiting for service also incur cost. A customer who is waiting is not producing. The longer they wait the longer the time they waste in non-productive activity. In some cases, it is easy to assess the cost of waiting. When the customer belongs to the same organization as those providing services, the cost of waiting is the wage wasted while waiting. For example, if a worker has to wait in line to use a computer, his waiting cost is the wage for the period he has to wait for the computer.

Longer the people wait the longer the time they waste in non-productive activity.

In other situations, the cost of waiting cannot be determined so easily. For example, if the customer were external to the organization providing service, then what would be the cost of waiting? When the same organization have different levels of customers how do we assess their lost wages? Other questions that arise are, is the cost directly proportional to the waiting time? What is the cost of ill will resulting from long waiting? What is the cost of lost sales, when customers get impatient and leave? etc. Answers to such questions are not simple, because they raise questions on personal values, social priorities, and other qualitative factors.

Activity: As a manager of a nationalized bank branch, how would you identify the cost of waiting of your customers? Justify.

Therefore, the total cost (TC) of a queue system are equal to the sum of cost of providing service (C_F) and the cost of waiting (C_W). That is,

$$TC = C_F + C_W$$

Where,

TC	=	Total cost of the system
C_F	=	Cost of providing service
C_W	=	Customers' cost of waiting for service

The cost of waiting (C_W) per unit time for the queuing system as a whole is given by:

$$C_W = W\lambda C = LC$$

Where,

C_W	=	Total cost of waiting
W	=	Average time waiting in the system
λ	=	Customer rate of arrival
C	=	Waiting cost per customer
L	=	Average number of customer in the system

The management may have the objective of minimizing total cost of services (both facility and waiting cost) or achieving a specific level of services or both.

i. **Cost minimization:** Where both the facility cost and waiting cost can be assessed easily, management would try to provide a service level that would minimize total cost of the system.
ii. **Specific service level:** As a policy matter management may try to achieve a specific level of service, disregarding the cost of service. For example, a fast food restaurant advertise that customer does not have to wait for more than three minutes for a burger, or a private mobile telephone company promise to provide new connection within 24 hours, or a service facility should be in use at least 75 percent of the time. Determining the best level of service is a matter of organization policy and is influenced by external factors as competition and consumer pressures, etc.

Methodology of queuing analysis

The major objective of queuing method is to predict the behavior of the system to reflect its operating characteristics or measures of performance.

Queuing can be analyzed by either mathematical process or by simulation. Here we will describe the mathematical procedure. The method is basically a descriptive tool of analysis. Unlike most other mathematical procedure it does not provide any optimum solution rather it only describes the parameter of the queue system. The major objective of this method is to predict the behavior of the system to reflect its operating characteristics or measures of performance. Management needs this set of information to determine the appropriate service level of the system. Although queuing theory is basically descriptive, it can be used to determine the optimum number of servers or optimum speed of service, but such applications are very limited. The entire process of analyzing a queuing system involves the following three steps:

1. Establish the performance criteria to measure.
2. Compute the measures of performance.
3. Analyze the situation and take action, if required.

1. **Establish the performance criteria to measure:** An operations manager's main aim is to improve the efficiency of the system and also balance the cost involved. Typically the operations manager looks at the following measures to evaluate the existing or proposed service system:

A long queue may indicate poor service or the need to increase capacity.

   i. *Queue Length:* The queue length or the number of customer in the line reflects one of two conditions. Short line means either a very good customer service or it means the system has too much service capacity. Similarly, a long queue may indicate poor service or the need to increase capacity. For these reasons, the first parameter that any operations manager looks for is the line size.

   ii. *Number of customers in the system:* There is a major difference between number of customer in line and number of customer in the system. Number of customer in line implies those customer who are waiting in line but are not being served, but number in the system means not only those waiting in line but also those who are being served. This measure reflects service efficiency and capacity. A large number of customers in the system can create congestion and overcrowding resulting in dissatisfaction, unless capacity for waiting is increased. This criterion mainly indicates the need to increase space where customers can wait. As for example, after checking in at the airport but before availing flight out of the city the passengers need space where they can wait.

iii. *Waiting time in line:* Short lines do not always mean a good service level neither a long line implies a bad service level. Efficient system can service large number of customers in short period whereas inefficient system may take long to do so. So it is important for the operations manager to know, on average, how long do the customers have to wait in line. If they have to wait too long the customers would perceive the service as poor.

iv. *Total time in the system:* Total elapsed time in the system is also important from operations point of view. If customers spend too long a time in the system it either indicates the need to change customers' behavior or the capacity needs to be increased.

v. *Utilization rate of the facility:* On the surface, it might seem that the operations manager would want to seek 100 percent utilization. However, too little idle time indicates under capacity. Increase in system's utilization can be achieved only at the expense of long waiting and long queues for the customers. Management's goal is to maintain high utilization rate without adversely affecting other performance criteria.

Too little idle time indicate under capacity.

vi. *Probability that an arriving customer must wait for service and number of customers in the system:* This criteria gives indication of the chance of a particular number of customer in the system. If the probability is high then the operations manager has to ensure facilities for the expected number of customers. But if it low he may have to reduce waiting space capacity.

vii. *Implied cost related to the level of service:* The manager also would like to know the cost associated with different service levels and the justification behind such cost. If the return against investment is not right, even if the capacity is under staffed, the manager may not opt for high investment in the facility. But placing taka value to service level is not easy. In such situation, the operations manager must weigh the cost of alternative arrangements and use subjective assessment to select the best alternative.

Activity: Do you think for every purpose the manager should establish the performance criteria to measure the quality of every individual or material performance? Why or why not? Discuss.

2. **Compute the measures of performance:** Once the performance criteria have been selected the next step is to apply appropriate queue model to define the parameters of the system. Based on the characteristics of the system, different mathematical models are available. Determining the appropriate model would require a thorough understanding about the basic characteristics of the system under study. How to compute the performance parameters is described, in detail, in the next lesson.

Once the alternatives have been identified, the analyst has to measure performance parameters for each alternatives.

3. **Analyze the situation and take action, if required:** Operations manager first of all identifies a set of alternatives and then selects the one that best suits his requirement, either in term of service level or in term of cost. In queue analysis, there are only a few alternatives to evaluate. For example, an MBA computer course teacher has to set up computer in a classroom of 10 by 10 feet. Assuming each student needs ten square feet of space, including space for movement, 10 computers would be a realistic consideration, but not 100 computers. The number of feasible alternatives is usually small because of technical and physical constraints. In this particular example the teacher may have additional options like type of computers, whether to network them or not, whether to have server, etc. But still, the total alternatives cannot be too large. Once the alternatives have been identified, the analyst has to measure performance parameters for each of the alternatives. Based on these measures the analyst would select the best alternative that either meets his desired service effectiveness or meets the overall cost-benefit consideration.

Discussion Questions

1. What are the main objectives of analyzing a queue system?
2. Describe the different costs associated with a queue system.
3. Describe the procedure of analyzing a queue system.
4. Describe the relation, with the help of a graph, between different components of a queue system.
5. Describe the important factors taken into consideration in measuring performance of a system.

Lesson Four: Waiting Line Methodology (ii)

Lesson Objectives

After completing this lesson you will be able to:

- Read Kendall's notations
- Describe the parameters of a single server model
- Make comparative analysis of a single server system

Kendall's notation contains six symbols divided into two sets.

The Kendall's Notations Used in Queue Analysis

A queue or waiting line forms because the short-run demand rate for service exceeds the short-run rate of providing service; that is, a queue forms whenever an arrival occurs and finds the server busy. Many analytical waiting-line models exist; each is based on certain unique assumptions about the nature of arrivals, service times, and other characteristics of the service system. Because there are large number of possible models, a notation set has been developed by D.G. Kendall that makes it easy to identify the model applicable for a particular system. Kendall's notation contains six symbols, divided into two sets, representing different characteristics of the system. They are:

(a/b/c) : (d/e/f)

The symbols represent the following features of the service system:

a. Arrival distribution
b. Service distribution
c. Number of servers or service configuration
d. Service discipline or service priority
e. Permissible number of customers in a queue
f. Calling population size

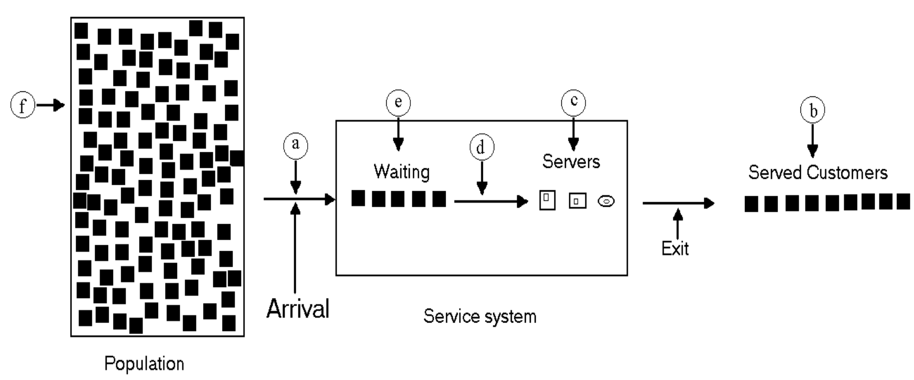


Figure 7.4.1: Service System with Kendall's Notations

All of these features of a service system have already been discussed in the previous lesson. In addition, to represent different characteristics of each of these features more standard symbols are commonly used. They are described in the Table 7.4.1.

Table 7.4.1: Symbols of Kendall's queue analysis

Item	Symbol	Distribution
Arrival Process	M	Poisson
	D	Constant or Deterministic
	E_K	Erlang with parameter k
	G	General independent - with only mean and variance known
	U	Uniform
	N	Normal
Service Process	M	Exponential
	D	Deterministic or Constant
	E_K	Erlang with parameter k
	G	General independent - with only mean and variance known
	U	Uniform
	N	Normal
Number of Servers	Finite	Actual number (1, 2, 3, etc)
Queue Discipline	FCFS	First come first served
	LCFS	Last come first served
	SIRO	Service in random order
	GD	General service
	PR	Priority
Queue length	α	No limit
	Finite	Actual number (1, 2, 3, etc)
Population	α	No limit
	Finite	Actual number (1, 2, 3, etc)

The notation for the simple one server single-phase queue is,

$$(M/M/1) : (FCFS/\alpha/\alpha)$$

It stands for Poisson arrival rate (M), exponential service time (M), one server (1), a first-come-first served discipline (FCFS), and infinite queue length (α) and infinite calling population (α). If there are two servers in the system, and a maximum of 25 customers allowed in the system, and all other parameters remaining the same, then the notation would be:

$$(M/M/2) : (FCFS/25/\alpha)$$

There are various queue situations, for example, queue with priority, cyclic queue, truncated queue, and many others. In theory there are innumerable number of queue models. With difference in any one of the features of the system and difference even in the distribution of each of the features a unique model, for that particular situation, is applicable. Before analyzing any queue system, it is very important to identify the right model otherwise the resulting system parameters would not be the correct parameters of the system under study. Some commonly observed waiting line models are shown in the Table 7.4.2.

Table 7.4.2: Some commonly observed waiting line models

No of Servers	Service Phase	Population Size	Arrival Pattern	Queue Discipline	Service Pattern	Queue Size	Example
Single	Single	Infinite	Poisson	FCFS	Exponential	Infinite	One lane toll bridge
Single	Single	Infinite	Poisson	FCFS	Constant	Infinite	Train ride in an amusement park
Single	Single	Finite	Poisson	FCFS	Exponential	Infinite	Machine breakdown and repair in a factory
Multiple	Single	Infinite	Poisson	FCFS	Exponential	Infinite	Toll counter at Jamuna Bridge
Multiple	Multiple	Infinite	Poisson	FCFS	Exponential	Infinite	Bank counters with multiple of verification points

In this lesson we would present a sample waiting line model. There are many more types of queue models than the one shown here. While using the formula, it is important to keep in mind that it is a steady-state formula derived on the assumption that the process under study is an ongoing concern with no chance of change in arrival and service rates. Steady state of a system occurs when the system is independent of the initial conditions and elapsed time. In other words, the system is not in a transient state when its measures of performance are still dependent on the initial conditions of changing arrival and service pattern.

Activity: Make a list of the importance of Kendall's Notations Queue analysis for your own?

The Basic Poisson-Exponential Model (M/M/1 FCFS/α/α)

M/M/1 FCFS/α/α is the most basic and the most common of the queue models. It has the following characteristics or assumptions:

a) *Arrival Rate:* In this system the arrival rate is random and is described by Poisson distribution. The average arrival rate is λ.
b) *Service Rate:* The time required for service is assumed to follow negative exponential distribution. The average rate of service is μ.
c) *Number of Servers:* The service system has only one server.
d) *Number of Phases and Channels:* Because the system has single server, it is a single channel system with a single service phase. Question of multiple number of channels and phases arises when there are more than one server.
e) *Service Priority:* First-come-first-served policy is assumed.
f) *Queue Length:* It is assumed that the waiting space is unlimited to allow infinite queue size.
g) *Population Size:* The population is also assumed to be infinite.
h) *Steady-State:* The system is assumed to be in a steady-state.
i) *Service Rate Larger than Arrival Rate:* It is assumed that the service rate is faster than the arrival rate. Otherwise the system would end up with an exploding queue, a never-ending waiting line.

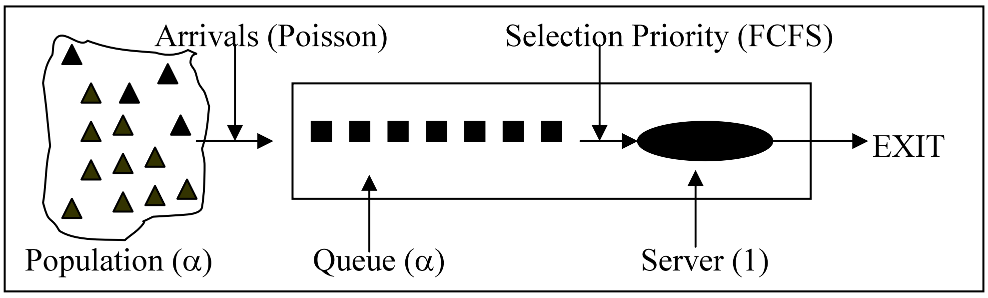


Figure 7.4.2: The service system under consideration

In analyzing a waiting line the analyst generally evaluates one or all of the following measures of performance of the system.

i. Total time in the system or average waiting time (W)
ii. Average waiting time in line (W_q)
iii. Average number of customers in the system (L)
iv. Queue length or average number of customer in line (L_q)
v. Utilization rate of the facility or the probability of the system being busy (P_b)
vi. Probability of an empty system or the system not busy (P_e)
vii. Probability of a customer being in the system (waiting and being served) longer than time t (P{T>t})
viii. Probability of a customer being in the system (only waiting but not yet being served) longer than time t' (P{T'>t'})
ix. Probability of finding exactly N customers in the system {P(N)}
x. Probability of finding more than N customers in the system {P(N>n)}
xi. Cost related to the level of service

For definition and explanation of these performance measures please refer back to the previous lesson. The analyst needs only two variables to analyze all of the performance measures of the system. They are λ and μ, the arrival and service rate. The arrival rate should be smaller than the service rate, and they should be expressed in the same time dimension. The formulas for the respective performance measures are given below:

i. *Total time in the system or average waiting time* (W): The total time waiting implies both the times spent waiting in line and also the time required for the service. The formula is,

$$W = \frac{1}{\mu - \lambda}$$

ii. *Average waiting time in line* (W_q): This measure of performance deals with only the time that the customer spends waiting in line but does not includes the time required for service. This is an important measurement parameter because customers get annoyed or dissatisfied if they have to wait too long for the desired service or services. They are inclined to be less dissatisfied if the service takes long, because they know something is being done for them.

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$$

iii. *Average number of customers in the system* (L): This parameter included both the numbers of customers waiting in line and those being served.

$$L = \frac{\lambda}{\mu - \lambda}$$

iv. *Queue length or average number of customer in line* (L_q): Average number of customer in queue measures the average number of customers waiting in line and their service is yet to start.

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

v. *Utilization rate of the facility or the probability of the system being busy* (P_b): The probability that the system is busy, is the same as not finding an empty system. It measures the proportion of time that the server is busy serving the customers.

$$P_b = \frac{\lambda}{\mu} = 1 - P_e = \rho$$

vi. *Probabilities of an empty system or the system not busy* (P_e): The probability that the system is empty means the proportion of time there is no customer in the system and the server is ideal.

$$P_e = 1 - \frac{\lambda}{\mu} = 1 - P_b$$

vii. *Probability of a customer being in the system (waiting and being served) longer than time t* (P{T>t}): The probability that a customer spends a total time more than time "t" includes the time spend waiting and the time required for service.

$$P\{T > t\} = e^{(\lambda - \mu)t}$$

Where,

e	=	2.718 (a constant)
T	=	Time in the system, and
t	=	A specific time, say 8 minutes, etc.

viii. *Probability of a customer being in the system (only waiting but not yet being served) longer than time t'* (P{T'>t'}): The probability a customer spends a time more than t' time waiting in line before the start of his/her service, is

$$P\{T' > t'\} = \left(\frac{\lambda}{\mu}\right)(e)^{-(\mu - \lambda)t'}$$

ix. *Probability of finding exactly N customers in the system* (P(N)): The probability of finding exactly a specific (N) number of customers in the system is given by:

$$P(N) = \left(\frac{\lambda}{\mu}\right)^N \left(1 - \frac{\lambda}{\mu}\right)$$

x. *Probability of finding more than N customers in the system* (P(N>n)): The formula of the probability of finding more than a specific number (n) of customer in the system is:

$$P\{N > n\} = \left(\frac{\lambda}{\mu}\right)^{n+1}$$

Activity: What are the different parameters of a system under study in a queue model? Explain why.

Example of Poisson-Exponential Queue (M/M/1 FCFS/α/α) Model

In a charity clinic, for the underprivileged, patients arrive randomly for treatment, at an average rate of 7 per hour. Doctor Tahsin estimates that an average consultation lasts 8 minutes. However, he is worried about the possibility of a long waiting and wonders whether he should ask his fellow doctor to help in consultation. The arrival process is Poisson and the service rate can be assumed to be negative exponential.

Solution

In this problem the arrival is Poisson and service is negative exponential distributed with only one server, therefore,

The arrival rate	$= \lambda$	=	7 per hour	
Inter service rate	$= 1/\mu$	=	8 minutes	
Therefore, the service rate	$= \mu$	=	60/8	=7.5 patients per hour

i. *Total time in the system or average waiting time* (W): On average each patient spends 2.0 hours (waiting and being treated) in the clinic.

$$W = \frac{1}{\mu - \lambda} = \frac{1}{7.5 - 7} = \frac{1}{0.5} = 2.0 \text{ hours}$$

It is the time that elapses between the moment the patient entered the clinic and left the clinic.

ii. *Average waiting time in line* (W_q): The first parameter indicates that the patient spends a long time in the clinic. It includes both waiting and getting treatment. But the performance measure does not tell us anything about the proportion of time that the patient spends getting treated. If the proportion of time getting treated is longer than the proportion of time waiting for treatment, then the long time spent in the clinic is justified, otherwise not. By finding W_q we would be able to determine, out of the total time, the time the patient waited for treatment.

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{7.0}{7.5\,(7.5 - 7.0)} = \frac{7.0}{7.5\,(0.5)} = \frac{7.0}{3.75} = 1.87 \text{ hours}$$

The performance measure produces a very gloomy picture. Out of two hours spent in the clinic, each patient on average spends 1.87 hours in waiting. In other words, for a 7.8 (2.0-1.87 hours) minutes of treatment, each patient has to wait for 1 hour and 52.2 minutes.

iii. *Average number of customers in the system* (L): This performance measure shows both the number of customers waiting in line and those being served. This parameter is essential to know whether sufficient space is available for patients to wait and also treated.

$$L = \frac{\lambda}{\mu - \lambda} = \frac{7.0}{7.5 - 7.0} = \frac{7.0}{0.5} = 14 \text{ patients}$$

On average the clinic will always have 14 patients, either waiting for treatment or being treated. The operations manager has to ensure that sufficient space is there to accommodate the patients.

iv. *Queue length or average number of customer in line* (L_q)*:* This parameter measures the average number of customers waiting in line waiting for their turn to consult the doctor.

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{(7.0)^2}{7.5(7.5 - 7.0)} = \frac{49}{3.75} = 13.07 \text{ patients}$$

On average there would be 13 patients waiting for treatment. This measure of performance is not very important in a single-server-single-phase system. Out of the total customer one would be getting served and the rest would be waiting (14-1 = 13).

v. *Utilization rate of the facility or the probability of the system being busy* (P_b): To measures the proportion of time that the server is busy serving the customers we would use the following formula.

$$P_b = \frac{\lambda}{\mu} = \frac{7.0}{7.5} = 0.9333 = 93.33 \%$$

This parameter indicates that the doctor is busy 93.33% of the time. This is too high a utilization rate of resources. The doctor is too busy he does not have time for rest, which may lead to inefficiency in the long run.

vi. *Probability of an empty system or the system not busy* (P_e)*:* The probability that the clinic is empty, or in other words, the doctor is not busy is,

$$P_e = 1 - \frac{\lambda}{\mu} = 1 - \frac{7.0}{7.5} = 1 - 0.933 = 0.06$$

As indicated in the immediate previous measure the doctor is not busy is only 6.7% of the total time.

v. *Probability of a customer being in the system (waiting and being served) longer than time* t (P{T>t}): The probability that a customer spends a total time more than a specified time "t" (both waiting and being served) is given by the formula below.

For our example, let us assume
t = 1.5 hours

That is, the probability of a patient being in the clinic more than one and a half hour, that includes the time for waiting and the time for treatment inclusive.

$$P\{T > t\} = e^{(\lambda - \mu)t} = 2.718^{(7.0 - 7.5)1.5} = 2.718^{-0.75}$$
$$= \frac{1}{2.718^{0.75}} = 0.4724 = 47.24\% \text{ of the time}$$

On average each patient has a probability of 47.24% of being in the clinic for more than 1.5 hours.

vi. *Probability of a customer being in the system (only waiting but not yet being served) longer than time* t' (P{T'>t'}): The probability a customer spends a time more than t' time waiting in line before the start of his/her service is,

$$P\{T' > t'\} = \left(\frac{\lambda}{\mu}\right)(e)^{-(\mu - \lambda)t'}$$

For our example we would like to know the probability of waiting for service for more than *one hour* is,

$$P\{T' > 1.0\} = \left(\frac{7.0}{7.5}\right)(2.718)^{-(7.5 - 7.0)1.0}$$
$$= (0.93333)(2.718)^{-0.5}$$
$$= \frac{0.93333}{2.718^{0.5}}$$
$$= 0.5661 = 56.61\% \text{ of the time}$$

There is 56.61% chance that a patient has to wait more than 1 hour before he/she gets to see the doctor.

vii. *Probability of finding exactly N customers in the system* (P(N)): The probability of finding exactly 5 patients in the clinic would be,

$$P(5) = \left(\frac{\lambda}{\mu}\right)^N \left(1 - \frac{\lambda}{\mu}\right)$$
$$= \left(\frac{7.0}{7.5}\right)^5 \left(1 - \frac{7.0}{7.5}\right)$$
$$= (0.9333)^5 (1 - 0.933)$$
$$= 0.0472$$

There is only 4.72% chance of finding exactly 5 patients in the clinic.

viii. *Probability of finding more than N customers in the system* (P(N>n)): The formula of the probability of finding more than a specific number, let us assume 8, of patients in the clinic is:

$$P\{N > 8\} = \left(\frac{\lambda}{\mu}\right)^{n+1}$$

$$= \left(\frac{7.0}{7.5}\right)^{8+1}$$

$$= (0.933)^9$$

$$= 0.5374$$

The chances of finding more than 8 patients are 53.74 percent.

Management Decision

All the measures of performances are not required to determine the efficiency of the system.

After finding out the values of different measures of performance, the next step is to determine the efficiency of the service system. At the very beginning it should be remembered that all the measures of performances are not required to determine the efficiency of the system. Depending on the nature and desire of the management different sets of parameters are evaluated for determining the efficiency.

There are two ways to determine the efficiency of the system. One method is to have a set of values for each parameter, previously determined as desirable, and compare the result against them. If the computed results matches or are within acceptable range of the predetermined values the system it is assumed to be performing satisfactorily, otherwise not. The second method is to apply cost analysis on multiple of alternative systems, and select the alternative that gives a satisfactory cost analysis.

Let us assume that the management already has a set of performance criteria available for each of the measures of performance. Let us also assume that they would prefer their system to meet the standards. The Table 7.4.3 given below shows a comparative study of the desired results and the computed results.

Table 7.4.3 Comparative study of desired & computed results

Performance Parameters	**Desired Performance Values**	**Computed Performance Values**	**Remarks**
W	30 minutes	2.0 hours	Excessive time in the system not desired at all. If possible add more doctors.
W_q	20 minutes	1.87 hours	Too long waiting, to reduce anxiety arrange preliminary consultation by paramedics.
L	20 patients	14 patients	Sufficient arrangement for patients.
L_q	19 patients	13.07 patients	Sufficient arrangement for patients.
P_b	85.00%	93.33%	The doctor works too much, need relief.
P_e	15.00%	6.67%	Same as above.
P(5)	10.50%	4.72%	Lower than expected.
P(N>8)	< 20.00%	53.74%	Too high, find solution.

The present arrangement is not satisfactory. The management has to find solution to the problem. They may try to make the doctor work faster, but the doctor is already working fast. He takes only 7.8 minutes for each patient. Alternative is to increase the number of doctor, or try to change the arrival pattern of the patients, or even change the service system itself. In queue analysis, the evaluations do not give any solution but only describes the parameter of the system, so that the management can evaluate different alternatives to find the best that suits their purpose.

Discussion Questions

1. Describe the different symbols of Kendall's notation.
2. During the height of the construction season in Rajshahi, trucks arrive at a brick-field according to Poisson distribution at the rate of 26 trucks per hour. The present capacity of the brick-field permits loading of 32 trucks per hour. The service rate has exponential density function. Find
   a) The average length of time a truck would be in the brick-field.
   b) The proportion of time the brick-field is empty of trucks.
   c) The probability of waiting more than 15 minutes.
   d) The probability of finding more than 6 trucks in the brick-field.
   e) If the service capacity decreases to 30 trucks per hour what would be its effect on the average waiting time for each truck.
3. The manager of a neighborhood store is interested in providing good service to the customers of his store. Presently, the store has a checkout counter for the customers. On average, 30 customers arrive at the counter every hour, according to Poisson distribution, and are served at an average rate of 35 customers per hour, with exponential service time. Find the following averages:
   a) Utilization of the checkout clerk.
   b) Number of customers in the system.
   c) Number of customers in line.
   d) Time spent by the customer in store.
   e) Waiting time in line.
4. A plant distributes its products by trucks. The average loading time is 20 minutes per truck. Trucks arrive at an average rate of two each hour. Management feels that the existing loading facility is more than adequate. However, the drivers complain that they have to wait too long. Analyze the situation and find how much money the company can save by speeding up loading if the waiting time of a truck is figured at Taka 250 per hour and the plant operates eight hour each day.

Case Analysis

Conquering Those Killer Queues

By: *N. R. Kleinfeld*

Lines are one of Richard Larson's odd fascinations. He is a steadfastly gleeful professor of electrical engineering and computer science at the Massachusetts Institute of Technology, and something of an expert on waiting. Thus the Zayre Corporation, whose discount prices make it something of an expert on making people wait, has hired him to come up with fresh ideas to combat that immemorial bagaboo – customer lines.

Eugene Fram, a professor of marketing and management at the Rochester Institute of Technology, feels that businesses are recognizing that by keeping customers waiting they become "time bandits." They are finding that people will pick one establishment over another because of shorter lines. All this means more pressure on companies – from bank to restaurants, supermarkets to airlines – to solve the waiting problem. There are quick cures: spend more money and provide more service. But most businesses cannot afford to – or do not want to – and so they have been trying harder to find imaginative methods to curtail waiting or at least make it less repugnant.

Those who wrestle with waiting often contact someone like Richard Larson. More and more researchers and consultants are studying lines and, in their efforts to demystify and quantify them, have produced a mathematical discipline. Queuing theory makes heavy use of probability theory and gets into abstruse mathematical equations that gauge such things as how many people will arrive at a drugstore to buy dental floss during the noon hour.

The father of queuing theory is A. K. Erlang, a Danish telephone engineer who, in 1908, began to study congestion in the telephone service of the Copenhagen Telephone Company. A few years later, he arrived at a mathematical approach to assist in designing telephone switches. Queuing theory has grown far more sophisticated. While it continues to have its chief application in telecommunications and computer design, it has seeped elsewhere.

Ideas about lines matter to the people at Zayre, because they have tried to differentiate their store from competitors I part by how swiftly they take customers' money. For years, Zayre has had a policy that if more than three people are lined up at a checkout register, another register will open.

The system, Zayre admits, has its flaws. Lines do occasionally swell to four or five customers when no one is free to handle another register. At times, every register is open and yet the lines are four deep. "We found the system was difficult to manage, because it was hard to predict when customers would arrive,: said Frank Capek, Zayre's manager of operations planning.

Years of listening to howling customers have also taught the airlines some baggage-retrieval lessons. When American designed its baggage-claim area in Dallas-Fort Worth Airport, it put it close to the gates, so disembarking passengers would not have to trudge too far. But even though passengers reach the area quickly, they must wait for their luggage. At Los Angeles International Airport, passengers have to walk quite some distance to the claim area, but when they arrive, their suitcases are usually there. Even though the Los Angeles travelers spend more total time picking up their baggage, American has found they do not grouse as much about baggage delays as do the Dallas passengers.

Few businesses have taken more hard shots about bad management of lines than banks. “Our research indicates that people don’t like to spend time in bank branches,” said David Mooney, senior vice president of Chemical Bank. “It’s probably second in their disdain to waiting to see the dentist.”

To reduce lines, most banks have installed automatic teller machines and tried to persuade employers to alter their check distribution or to directly deposit payroll checks in the bank. The lines, however, persist.

One way to take some of the stings out of waiting is to entertain customers. Since 1959, the Manhattan Savings Bank has offered live entertainment during the frenzied noontime hours. In 13 branches, a pianist performs and one branch has an organ player (Willard Denton, the former Chemical chairman who dreamed up the idea, liked organs, though present management thinks they are a trifle loud for a bank). Occasionally, to make line-waiting even more wonderful, Manhattan Savings has scheduled events such as a fancy-cat exhibit, a purebred dog show and a boat show.

Because of all this, Manhattan Savings believes customers endure long waits better than those who go to banks where the only music is the person in front of you grinding his teeth. “At very hectic times, we get very few complaints,” said Jean Madsen, a senior vice president.

At hotels and office buildings, mirrors affixed to elevator doors make people less maniacal during waits. Instead of deciding whom to kill, they can comb their hair. A study done by Russel Ackoff showed that hotels that had mirrors received far less grumbling about elevator delays than ones without mirrors.

Just telling people how long they have to wait often cheers them up. Disneyland is sensitive to waiting, since the line for a hot attraction like Star Tours can run to 1800 people. Like many amusement parks, Disney employs entertainment for waiters, but it is also big on feedback. At various spots along lines, signs give estimated delays from those points. Queuing experts say nothing is worse than the blind waiting familiar to people at bus stops, who don’t know if the next bus is one minute or 15 minutes away. Disney’s feedback permits parents to weigh odd options: Is it wiser to wait 25 minutes fo rMr. Toad’s Wild Ride or 30 minutes for Dumbo?

Peter Kolesar, a professor of operations research at the Columbia University Business School, thinks there ought to be more efforts to shift demand by altering pricing. Some rail lines, for example, charge less for off-peak trains and restaurants offer early-bird discounts.

During a whimsical moment, Dick Larson speculated that if the average American waited hald an hour a day in one line or another, then the population expended 37 billion hours a year in lines. It strikes him, he said, that businesses ought to consider merchandising products to idle waiters to take their minds off pulling out their hair. “Like those flower peddlers outside tunnels and bridges,” he said. “They are very shrewd.”

Case questions

1. What options might a chain store such as Zayre employ to increase capacity? To influence demand?
2. What techniques can be employed to make a wait seem more pleasant (besides the ones mentioned here)?
3. What products could be realistically merchandised?

Source: Excerpted from New York Times (Sunday, September 25, 1988)
Mentioned in the book “Service Operations Management” by Robert, G. Murdick, Barry Render, and Roberta S. Russell, Prentice Hall, pages 344-345.