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COURSE MANUAL

C7: Quantitative Techniques

Module 5

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Module 5

Decision-making, risk and challenges

Introduction

In this final module, you will consider some recent ideas and challenges in the business and management world. There are new theories based on applications drawn from mathematics, decision theory and psychology.

Decision making often takes place in uncertain conditions and criteria have been developed to make decisions in these situations. Decision trees and utility functions can also be used to optimise decisions.

Managers have to face situations where their "opponent" is not just fate or uncertainty, but another human being who can strategise and plan against them. Tools like game theory, described later in this module, provide a framework for analysing these situations.

Now, more than ever before, decision-makers are being swamped with "information" that needs to be evaluated and sifted. This module's philosophy is to make you aware of the 21st century developments and tools you will need to know about when others confront you with technical terms (for example, minimax criteria, utility or "black swans"). You want to be able to point colleagues in the right direction, refer them to research tools and delegate some decision making.

Upon completion of this module you will be able to:

Outcomes

- **evaluate** risk and risk aversion;
- **apply** utility theory to risk management;
- **solve** decision-making problems under uncertain conditions;
- **construct** and **use** decision trees for risky decision making;
- **analyse** and **evaluate** strategies using game theory;
- **appreciate** new challenges in management and economics (black swans and fat tails); and
- **critique** the philosophies of economists and the new theory of behavioural economics.

Unit 17

Risk and decision-making

When you read this, the world will still be trying to claw its way out of a serious recession. Fortunes were made in the 1990s and were still being made in the early 2000s. Then came the crash. What went wrong?

Upon completion of this unit you will be able to:

Outcomes

- **be aware** of the changes and challenges in business and management in the 21st century**;**
- **understand** and **apply** the criteria of decision making under uncertainty (Laplace, maximin, minimax and maximax); and
- **model** risky decision-making processes using decision trees.

Decision-making under certainty, risk and uncertainty

Changes in economies

Over the last 300 years, the economy has changed from agricultural to industrial, then from manufacturing to service, and now to a knowledge economy.

The advantages of high technology and almost instantaneous communication were supposed to end our exposure to business cycles and fluctuations. Inventory control (a big source of fluctuations and instability after World War II) was modernised and, as the economy moved from manufacturing to service industries, the role of inventory decreased even more. The so-called “New Economy” promised much innovation and more productivity; at the same time, it gave us the boom and bust cycle. It exposed us to more risk, but with less ability to manage the risk.

As demand decreases, unemployment increases, with firm and worker loyalties at a low. Income from pensions is risky, unemployment insurance and social security uncertain. Scandals rock the world and income inequalities seem to be on the increase. Risk seems to be increasing, making informed decision making all the more important.

Risk

A risk describes the possibility of something happening that will have an adverse impact on business or a business project.

There are four stages to risk management:

1. Risk identification
2. Risk quantification
3. Risk decisions and actions
4. Risk monitoring

There are different types of risks: financial risk, operational risk (including model risk), credit risk, and so on. Various risks and ways of measuring them have been dealt with in previous modules, for example by calculating standard deviations or mean square errors.

This unit looks at decision making and management actions in the face of risk and uncertainty.

Decision making

Decisions are made after considering the possible outcomes, probabilities and risk, and are then implemented in order to make a difference to the future of an institution. (The implementation should include monitoring of people, processes and procedures. Recent cases of fraud, such as the USD 7 billion loss at Société Générale caused by Jerome Kerviel, have been blamed on poor supervision and control, or management just "turning a blind eye".)

The aim of decision making is to choose the best of a number of alternatives. Some decisions are made under conditions of certainty, others under conditions of risk, and many under uncertain conditions. These conditions are determined by the type of information you have and, in turn, determine the models you will apply.

In this discussion, risk and uncertainty are not the same thing. Risk assumes there are various possible known outcomes with probabilities attached to them. Uncertainty means there are various possible outcomes, but probabilities are not available and practically nothing useful is known about them.

Information lies at the heart of decision making. The decision-making model should encapsulate all the knowledge that enables the manager to make an informed and responsible decision. But even under conditions of certainty and with a wealth of information, you should be aware that information and knowledge are not the same thing!

Reflection

Information and knowledge

The world's biggest electronic network for sharing data and information is the global computer network between banks. (The United States military communications network is the second largest.) Thousands of trillions of dollars or bits of information fly around the "global village" every year. Real goods and services have become subsumed by endless cycles of information (money) being passed around.

However, the global computer network is subject to instabilities and crashes. Who takes responsibility for these? Where and when does information become more than symbols and structures?

Information is not knowledge. The first step towards knowledge and informed decision making comes from organising all the information you are acquiring here about modelling: understanding its limitations and assumptions, understanding its power and the insight that models can provide, and in this way turning information into knowledge.

Find and discuss some philosophical viewpoints on information, knowledge and reality. The Internet's Google search engine can provide many examples. Here are three:

1. Immanuel Kant (1724–1804) wrote that the human mind organises information. It processes (filters) information into perceptions of the outside world through our conceptual apparatus. These perceptions then function as knowledge.
2. The theory of post-modernism holds that what is seen as reality depends on the observer.
3. George Soros (see Module 4, Unit 13) believes that knowledge (statements) and facts (information) interfere with each other, and that knowledge will always be biased and incomplete.

Decision making under conditions of certainty

With complete certainty, there is no risk.

Example 1

A risk-free investment for one year can be made with a number of banks or by buying government bonds. The annual rates of return are:

Trust Bank	4.5%
Fidelity Bank	3.9%
Bond AAA	5.1%
Bond AA	4.8%

What decision will you make?

This is decision making under certainty, as we know what the returns are and we are assured that all four investments are riskless. You should choose Bond AAA.

Decision making under conditions of uncertainty

Conditions of uncertainty exist in situations where there is almost no information – probabilities cannot be assigned, the risk cannot be measured and there are too many uncontrollable factors, which may include social and political factors. Decision making here is little more than guessing.

It becomes difficult to choose optimum values under these conditions of strict uncertainty. A number of decision criteria have been suggested and the Laplace, Wald (minimax) and maximin and maximax criteria are briefly explored below.

Laplace decision criterion

The mathematician Laplace (born 1749) suggested that if no probabilities are available, **all outcomes should be treated as equally likely**. The various decisions or alternative strategies and their monetary or satisfaction values are set out in a pay-off table. The rows in the table represent strategies, and the columns show possible outcomes. The mean value of outcomes is then calculated for each strategy – that is, the mean of the values in each row in the table. The strategy with the best average should be chosen.

Example 2

An event's organiser must decide how many food hampers to order today to sell at a festival in two weeks' time. She can order a large, medium or small quantity. The risk is the weather – it could be warm, cool or extremely cold, and no probabilities are available. Her profits (in pounds) are set out in the pay-off table (matrix) of profits in Figure 1. (Note that a negative profit is a loss.) How should she make her decision?

Pay-off table of profits

Profit pay-offs	Weather outcomes			
		Warm	Cool	Cold
Strategies	1. Buy large quantity	1200	600	−560
	2. Buy medium quantity	750	650	−120
	3. Buy small quantity	590	300	120

Figure 1

The three different strategies form the three rows of the table or matrix.

For example, if the event organiser decides to follow Strategy 1, "Buy large quantity", then she will make a profit of 1,200 pounds if the weather is warm, a profit of 600 pounds if the weather is cool and a loss of 560 pounds if the weather is cold.

Laplace's method can now be applied. Since we assume all three outcomes for weather conditions are equally likely, we attach a probability of ⅓ to each. The mean values of profits for each strategy are (in pounds):

Strategy 1: $\frac{1}{3}(1200) + \frac{1}{3}(600) + \frac{1}{3}(-560) = 413.33$ (rounded off to 2 decimals)

Strategy 2: $\frac{1}{3}(750) + \frac{1}{3}(650) + \frac{1}{3}(-120) = 426.67$ (rounded off to 2 decimals)

Strategy 3: $\frac{1}{3}(590) + \frac{1}{3}(300) + \frac{1}{3}(120) = 336.67$ (rounded off to 2 decimals)

Decision: Follow Strategy 2.

Note: The Laplace criterion is optimistic and assigns equal probabilities to outcomes. Outcome values are expressed in terms of profit or satisfaction and the best average is then chosen.

The Wald (minimax) decision criterion

This criterion concentrates on the maximum losses. **The pay-off table must now be given in terms of losses or regrets**. Remember that a positive profit is a negative loss and a negative profit is a positive loss. For the last example, you should use the negative of the given table if you want to apply the Wald criterion.

The idea here is to calculate the worst outcome (maximum loss) for each strategy. Then choose the strategy with the best worst outcome (minimum of maximum losses). Minimax stands for minimum of maxima.

Maximax and maximin criteria

The maximax rule is for risk-seeking people. It ignores probabilities of loss and looks purely for the **maximum profit** from each strategy and then the overall maximum.

The maximin criterion is more pessimistic. It considers the **minimum profit** or satisfaction from each strategy and then takes the maximum of these minima.

Activity 5.1

Activity

Apply maximax and minimax

What will you do?

Consider the situation of the events manager ordering food hampers. Apply the maximax and maximin decision criteria and give the chosen decision in each case. Discuss your results.

What do you think of the three criteria? Which suits your personality or business style?

Activity 5.2

Activity

Apply the minimax

What will you do?

Consider the situation of the events manager ordering food hampers. Apply the minimax decision criterion to decide which strategy to follow.

Here's our feedback

The pay-off table of losses in Figure 2 is a table of losses and is the negative of the table in Figure 1. Remember that positive losses are real losses and negative losses are actually profits.

Pay-off table of losses

Loss pay-offs	Weather outcomes			
Strategies		Warm	Cool	Cold
	1. Buy large quantity	−1200	−600	+560
	2. Buy medium quantity	−750	−650	+120
	3. Buy small quantity	−590	−300	−120

Figure 2

Strategy 1: The maximum loss is 560.
Strategy 2: The maximum loss is 120.
Strategy 3: The maximum loss is −120.

Remember that −120 > −300 > −590. All numbers are negative in row 3, which means that there are only profits. The maximum loss is now the same as the minimum profit.

The minimum of the maxima (least of the biggest regrets) is −120.

Decision: Follow Strategy 3.

Note: Different criteria or attitudes towards risk lead to different decisions. The Laplace criterion (reasonably optimistic) led to a decision for Strategy 2 and the Wald criterion (pessimistic or cautious) led to a decision for Strategy 3. Neither of the two criteria led the manager to choose Strategy 1. The risk of the 560 pound loss is simply too great.

Decision making under conditions of risk

Here, there is risk associated with the decision, but some information is available about the probabilities of various outcomes.

Example 3

You can invest in one only of shares X, Y or Z. The outcomes in terms of rates of return R over the next year are risky, but have specific probabilities:

	Rate of return R	Probability $p(R)$
Share X	8%	0.6
	10%	0.2
	12%	0.2
Share Y	9.6%	0.5
	7%	0.4
	13%	0.1
Share Z	7.5%	0.3
	11%	0.6
	14%	0.1

Figure 3

So Share X can have a rate of return of 8 per cent with probability 0.6 or a rate of return of 10 per cent with probability 0.2 or a rate of return of 12 per cent with probability 0.2. You do not know which outcome will be realised at the end of the year, but you must make a decision now.

What decision will you make?

This is decision making under risk. Although you don't know the outcomes with certainty, you do have probability distributions of the random variables R. This allows you to use statistical and probabilistic tools to make decisions.

Under conditions of risk, you can analyse the situation by calculating expected (mean) values of returns and a measure of risk. Risk for financial products is usually measured by the volatility of the product's value – that is, the standard deviation of returns.

Expected values of returns

Share X: Expected return = $\overline{R} = \sum R\, p(R)$ summed over returns R

$$= 0.08(0.6) + 0.10(0.2) + 0.12(0.2)$$

$$= 0.092 = 9.2\%$$

Share Y: Expected return = $\overline{R} = \sum R\, p(R)$ summed over returns R

$$= 0.096(0.5) + 0.07(0.4) + 0.13(0.1)$$

$$= 0.089 = 8.9\%$$

Share Z: Expected return = $\overline{R} = \sum R\, p(R)$ summed over returns R

$$= 0.075(0.3) + 0.11(0.6) + 0.14(0.1)$$

$$= 0.1025 = 10.25\%$$

You may decide to invest in share Z because it has the highest mean return. However, it would be wise to calculate the respective volatilities (risks) before finalising the decision. Do this in the following activity.

Activity 5.3

Activity

Final share decision

What will you do?

1. Calculate the volatilities of shares X, Y and Z. (Refer to Modules 2 and 4 if you have forgotten the definition of volatility.)
2. Repeat the calculations using Excel or Open Office.
3. What is your final decision about investing in one of the shares? Justify your answer.
4. Research the concept of diversification. Discuss the benefits of investing in more than one share.

Decision trees

Now consider situations where there is not only one decision between risky alternatives, but a sequence of decision making, with one decision leading to a new set of alternatives and another decision. This can be displayed by a **decision tree**, rather similar to the probability trees discussed in Module 2, Unit 7.

Decision trees help to display information and can present an overall picture of the risks and rewards associated with different courses of action. They are valuable tools for deciding between different projects or strategies under conditions of risk.

Building a decision tree

Decision trees consist of nodes (squares and circles) joined by lines in a tree-like structure. Square nodes represent decisions and circles represent risky outcomes. Lines emanating from squares are different options (courses of action), and lines emanating from circles are possible outcomes.

You are faced with a decision problem in management. Represent this with a square at the left midpoint of a sheet of paper. There are various possible courses of action that you envisage – represent these with lines to the right of the square. Each course of action leads to an outcome. If the outcome is risky draw a circle, and if the outcome is another decision draw a square. Draw new lines out from the squares and circles to represent new decision options or further risky outcomes. Label lines and symbols (squares or circles) with descriptions. Continue in this way until you have exhausted all of the relevant options and outcomes for solving the problem.

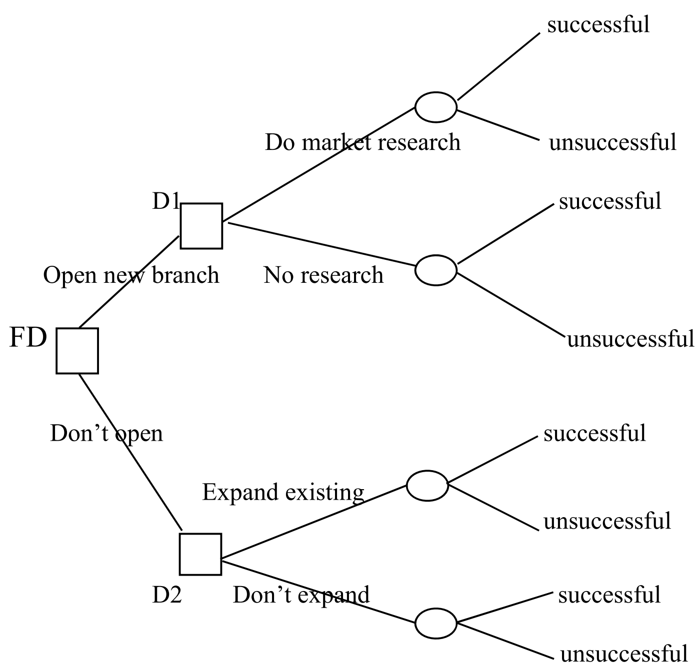
Example 4

The decision you face is whether or not to open a new branch of your business in another town. Whatever your decision, the final result will be success or failure.

If you decide on opening a new branch, the next decision is whether you should first do market research or just open a branch and see what happens. The outcomes in either case are risky – they will either be successful or unsuccessful.

If you decide to not open a branch, the next decision is whether to expand your existing business or just continue as before. Either case will lead to risky outcomes: Successful or unsuccessful.

At this stage, the decision tree will look like this:



Note the difference between decision squares and risky outcome circles. The decision nodes are labelled as FD (final decision) and D1 and D2.

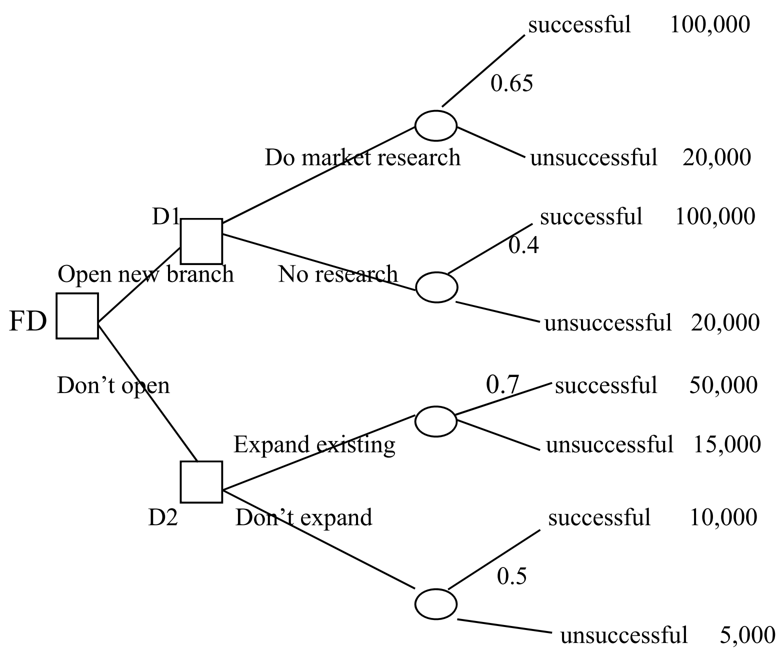
Evaluating the tree

To come to a decision, you need to attach values to the nodes. These values may be monetary values or perhaps just scores. Start at the far right by assigning values to the final outcomes. The values are the income or benefit you will get from that outcome. These values can either be informed guesses or based on researching other businesses.

The lines from risky nodes must be assigned probabilities. Again, these can either be guesses or based on research. With these numbers, work backwards and calculate the worth or value of the various risky nodes and decision nodes.

Assume that you assign values as shown below. The numbers at the far right are in the monetary units of your particular country and represent the cash value (income) to you of that outcome. For example, if you open a new branch after doing market research and it is successful, you estimate the value to your company will be 100,000.

The decimal fractions are the probabilities of the outcomes being successful or unsuccessful.



Note: If the probability of successful outcomes is 0.65, then the probability of unsuccessful outcomes is 0.35 (Why?)

Values of risky outcome nodes

The value assigned to a risky node (circle) is the **expected value** of outcomes at that node. Therefore in your tree the value of the top risky node is $\sum ap(a)$, summed over the two outcomes *a*:

$(100{,}000 \times 0.65) + (20{,}000 \times 0.35) = 72{,}000$

The value of the three risky nodes below that are, respectively:

$(100{,}000 \times 0.4) + (20{,}000 \times 0.6) = 52{,}000$

$(50{,}000 \times 0.7) + (15{,}000 \times 0.3) = 39{,}500$

$(10{,}000 \times 0.5) + (5{,}000 \times 0.5) = 7{,}500$

Values of decision nodes

You now need to attach a cost or expense to each course of action following from a decision node. Each possible course of action will cost you some money.

Assume these action costs

Market research:	5,000	No research:	0
Expand existing business:	15,000	Don't expand:	0
Open new branch:	35,000	Don't open:	0

Write the costs in brackets along the lines emanating from each decision node.

A value is then attached to the decision node itself, by deducting each action cost from the value of the risky node attached to that action line and taking the *maximum of the two values.* This gives the benefit of taking that particular decision (course of action).

Working backwards, you see that the value of the decision node D1 is: Maximum of (72,000 – 5,000) and (52,000 – 0)

= Max of (67,000 and 52,000)

= 67,000

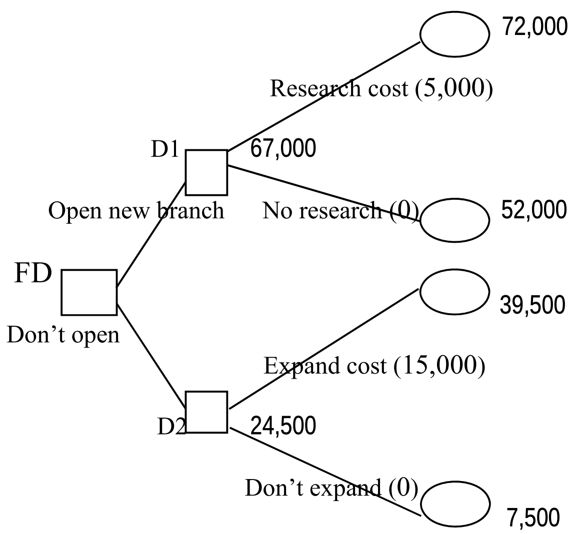
Value of decision node D2 is:

Maximum of (39,500 – 15,000) and (7,500 – 0)

= Max of (24,500 and 7,500)

= 24,500

The evaluated tree can now be pared down and redrawn to look like this:



Final analysis

Finally, working back to decision node FD, you choose between decisions D1 and D2 by taking the node with the biggest value. The course of action that gave the greatest benefit (67,000) is clearly D1: Open a new branch**.** Then it would also be better to do market research, since this course of action gives an expected value of (72,000 – 5,000). Not doing research gives a smaller value of (52,000 – 0).

If you can't open a new branch, you should expand the existing branch, with a value of (39,500 – 15,000).

The worst decision would be to not open a new branch and not expand. It has a value of only (7,500 – 0).

Note: Decision trees make all options and outcomes clearly visible. They allow you to assign values and probabilities and come to a final decision quantitatively. However, they are *models* of the real situation and should be used wisely.

Activity 5.4

Activity

Farm or mine?

What will you do?

You have a piece of farmland where you think there may be diamonds. You have to decide whether to farm there or start mining. If you decide to farm, you can either plant cocoa for export or you can grow various produce for your own use and to sell locally. If you want to dig for diamonds, you can either get a geologist in to test for diamonds or just start digging.

The probability of a good outcome from deciding on diamonds and getting a geologist is 0.25. The value of this outcome is 1,000,000 and the value of a poor outcome is 40,000. The cost involved with a geologist is 200,000. The value of a positive outcome without a geologist is also 1,000,000 and the probability of a good outcome is 0.05. The value of a poor outcome is 20,000.

With cocoa, the costs are 300,000 and the probability of success is estimated at 0.6. The value of success here is 600,000. The value of no success here is 20,000. With produce, the costs are 40,000 and the probability of success is 0.9, with a final value of 600,000. The value of an unsuccessful outcome is 30,000.

Use a decision tree to analyse the situation. Explain your final decision and justify the course of action you will take.

Activity 5.5

Activity

Understand the terminology

What will you do?

Use this terminology table to record any terms or words you're uncertain about.

This activity is an opportunity to consolidate your understanding of new terminology and concepts you encountered in Unit 17. Fill in the terms you have learned, and then write your own descriptions of them.

Terminology

Term		Description
	:	
	:	
	:	
	:	
	:	

Remember these key points

- Decision making under conditions of uncertainty depends on the attitude of the decision maker. There are four criteria that can be used: Laplace (optimistic), Wald (pessimistic or risk averse), maximin (risk averse) and maximax (risk seeking).
- Decision making under conditions of risk can be modelled and analysed using decision trees. Decision trees consist of nodes (squares and circles) joined by lines in a tree-like structure. Square nodes represent decisions and circles represent risky outcomes. Lines emanating from squares are different options (courses of action) and lines emanating from circles are possible outcomes. Benefit values and costs are attached to nodes and lines to help find the optimal decision.

Unit summary

Summary

You have successfully completed this unit if you can:

- **differentiate** between certainty, risk and uncertainty;
- **apply** the Laplace, Wald, maximax and maximin criteria to uncertain decision-making problems;
- **explain** the modelling of sequences of decisions using decision trees;
- **analyse** the tree and make informed decisions; and
- **be aware of** the changes and challenges of the business world in the 21st century.

Unit 18

Utility theory

Introduction

Decisions are often expressed or measured in terms of monetary values. One basic criterion for decision making is to choose the option that gives the greatest expected monetary value. In a sense, that is what was done in using decision trees. The concept of expected values was also used in choosing between shares or constructing portfolios of shares. In that case, you did not merely choose the portfolio with the highest expected return – you were influenced by the idea of risk as measured by standard deviation of returns (volatility).

Not all risk can be measured in terms of volatility. In this section, you will learn how to express the value or worth of an outcome in terms of the satisfaction of the decision maker. This satisfaction is called **the utility of the outcome** and is based on the decision maker's feelings about risk. The philosophy in utility theory is that people want to maximise their satisfaction or utility, but at the same time most people are risk averse.

Upon completion of this unit you will be able to:

Outcomes

- **express** values of outcomes in terms of utility functions;
- **rank** possible decisions with the help of utility theory;
- **calculate** certainty equivalents; and
- **determine** the Arrow-Pratt co-efficients for decision makers.

Example 5

Using expected values to make decisions under conditions of risk can sometimes lead to bad decisions. Consider a situation where you have to decide whether to invest 1,000 money units in a project or not. There are two outcomes if you invest: with very good economic conditions, you can treble your money with probability 0.21; and for very bad economic conditions, you lose half your money with probability 0.79.

The decision table looks like this:

Decision			
Invest	Profits	2,000	−500
	Probability	0.21	0.79
Do not invest	Profits	0	0
	Probability	0.21	0.79

Figure 4

A simple analysis along the lines of expected values yields:

Expected monetary value EMV (profit) of investing in project:

$$= 0.21 \times 2000 + 0.79 \times (-500)$$

$$= 25$$

Expected monetary value EMV of not investing in project:

$$= 0.21 \times 0 + 0.79 \times 0$$

$$= 0$$

Because the expected return for investing is bigger than the expected return for not investing, you should then decide to invest in the project. Or should you? There is a high risk (79 per cent) of losing half of your money if you invest. On the other hand, there is a possibility of trebling your money. What would you do?

If you have a lot of money, losing 500 units may not cause you unhappiness. But if you have little money, losing 500 units could cause great regret and dissatisfaction.

Example 6

You have won a competition where you can either take 50,000 pounds or a coin will be flipped and you will then win either 100,000 pounds or nothing. What will you do? What if the options are 5,000 pounds with certainty or flipping the coin for either 100,000 pounds or nothing?

Here, you have nothing to lose, but for many people receiving something like 50,000 with certainty is better than taking a chance. However, it depends how much you receive with certainty. Perhaps 5,000 is too little and you would rather flip a coin.

In these cases, in the absence of other information the decision maker's attitude towards risk and wealth becomes relevant. If you are risk-averse, you do not like to take risks. If you are risk-seeking, you love the excitement of taking risks. The levels of your wealth play a role. If you have a lot of money, losing some may not be as important as when you have little money.

How do you quantify this?

Utility functions

The utility function for an individual is denoted by $U(W)$ where variable W is wealth. The value $U(W)$ is the utility (happiness or satisfaction) of the individual with that wealth W. The typical utility function has the following shape:

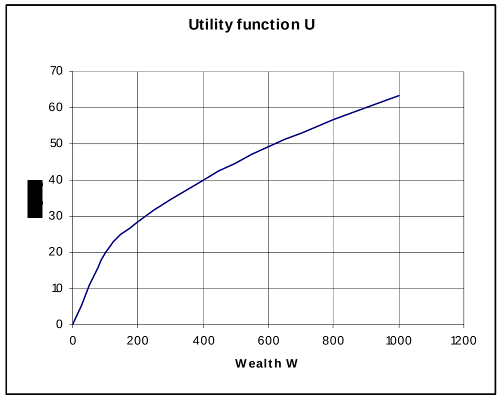


Figure 5

The shape of the graph is called concave. It has a positive slope, showing that utility or satisfaction increases with increasing wealth.

Note: The slope decreases as wealth increases. At high levels of wealth, a change in wealth of 200 units has a smaller change in utility (satisfaction) than at low levels of wealth. This seems to be true of most people.

Activity 5.6

Activity

Analyse the utility function

What will you do?

Consider the graph in Figure 5 as representing the utility function of one of your clients.

1. What is the satisfaction of the client at $W = 400$?
2. What is the satisfaction of the client at $W = 600$?
3. How does the utility change if the wealth changes from 0 to 200?
4. How does the utility change if the wealth changes from 400 to 600?
5. Discuss your answers.

Here's our feedback

1. Read from the graph: $U(400) = 40$
2. $U(600) = 50$
3. Change in utility: $U(200) - U(0) = 28 - 0 = 28$
4. Change in utility: $U(600) - U(400) = 50 - 40 = 10$
5. An increase in wealth from 0 to 200 or a decrease in wealth from 200 to 0 causes a far bigger change in satisfaction (28 units) than a change in wealth between 400 and 600. In the latter case, the change in satisfaction is only 10 units. This probably describes human behaviour. At high levels of wealth, winning or losing a certain amount causes less change in satisfaction than at low levels of wealth.

Activity 5.7

Activity

Graph the utility

What will you do?

Assume your utility function is given by $U(W) = \sqrt{W}$

1. Draw a table of values for different wealth levels between 0 and 10,000 USD.
2. Use Excel to draw a graph of the utility.
3. Suppose you have 6,000 USD and are willing to accept risk as long as your utility (satisfaction) does not decrease by more than five units. Will you invest in a product where you may make 2,000 USD but at the same time stand to lose 1,000 USD?

Here's our feedback

1.

0	0
1,000	31.62
2,000	44.72
3,000	54.77
4,000	63.24
5,000	70.71
6,000	77.45
7,000	83.66
8,000	89.44
9,000	94.86
10,000	100
11,000	104.88
12,000	109.54

Figure 6

2.

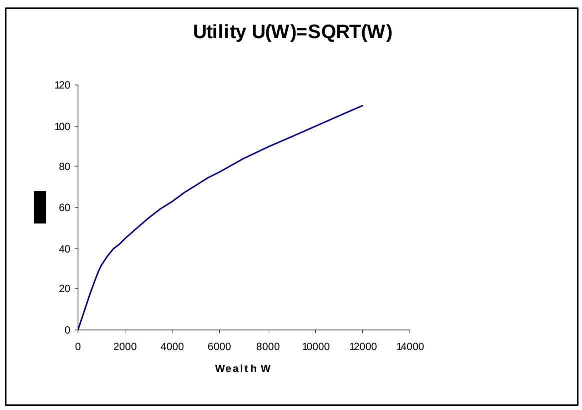


Figure 7

3. The utility at $W = 6{,}000$ is $U(6000) = 77.5$ (rounded off).

   $U(5000) = 70.71$ and $U(8000) = 89.44$

   If the product does well your utility increases by 11.94 units, but if you lose money your utility decreases by 6.79 units. However, the decrease is more than five units, and given your risk-averseness you will not invest in the product.

   Utility functions typically are one of the following types:

   $U(W) = 0.5\sqrt{W}$

   $U(W) = W - aW^2$ for $a > 0$ and restricted to $W < 1/(2a)$

   $U(W) = \ln W$

   $U(W) = -e^{-aW}$ for $a > 0$

The choice of function and value of parameter a are determined by asking people questions about risk and using the answers to construct the shape of their particular utility function.

A utility function reflects risk. It describes the risk aversion of a person (or company). The shape of the graph is concave: at any point on the graph, the slope of the graph to the left of the point is bigger than the slope to the right. This reflects risk aversion. A decrease in wealth has a bigger effect on satisfaction than an increase in wealth of similar size. The more the utility function is curved, the more the risk aversion.

Utility functions can also be used to rank different options. The option with the highest utility ranking is the one that will be chosen by the decision maker.

Expected utility and ranking alternative decisions

If the situation concerns future risky wealth W – that is, W is a random variable – you can calculate the *expected* utility of future wealth. This is denoted by $E[U(W)]$. The expected value is calculated by summing over the probabilities and different possible wealth outcomes.

In situations where a decision has to be made between a number of options, the expected utility of each option can be calculated. These values are then used to rank the options. To *rank* means to place in order of importance or satisfaction. The option with the biggest utility value is ranked first.

Example 7

Suppose a manager faces two options. The first option is risky and has a 50 per cent chance that 10 million INR will be paid out and a 50 per cent chance that nothing will be paid out. The second option will result in an amount M being paid out for certain.

The manager can decide between the two options by comparing expected utilities to rank the two possibilities. He has constructed a utility function for the company for decisions of this type of the form:

$\mathrm{U}(W) = W - 0.05W^2$

Option 1 $\quad E[U(W)] = 0.5\ U(10) + 0.5\ U(0)$

$\quad = 0.5\ (10 - 0.05(10)^2) + 0.5(0)$

$\quad = 2.5$

Option 2 $\quad E[U(W)] = 1\ U(M)$ (because M is certain it has probability 1)

$\quad = M - 0.05M^2$

The final decision will depend on the value of M.

If $M = 2$ million INR, then $E[U(W)] = 1.8$ for Option 2 (calculate this) and Option 1 should be ranked higher than Option 2. This is, of course, because $2.5 > 1.8$. There will be greater satisfaction in choosing Option 1 and this should be the manager's decision.

If M = 3 million INR, then $E[U(W)] = 2.55$ for Option 2 (calculate this) and Option 2 should be ranked higher than Option 1. There will be greater satisfaction in choosing Option 2.

Therefore managers can use utility values to rank the satisfaction that different decisions will bring.

Certainty equivalents

You have seen that, in certain cases, the wealth W will be a random variable. If there are two or more options (projects) you have to decide between, you can use the expected utilities to rank the options and then decide. But if there is only one project, what can you compare it with to decide whether you want to follow the option or not?

A manager can decide at the outset that he or she will be satisfied if the project has value equivalent to a certain guaranteed value C. This value C is called the **certainty equivalent** and is defined by:

$U(C) = E[U(W)]$

This equality means the satisfaction from receiving C for sure is the same as the expected satisfaction of receiving random W.

Taking it one step further, if a project has risky wealth outcome W and C is the given certainty equivalent, the project will be accepted if

$E[U(W)] > U(C)$.

Otherwise, the project will be rejected.

Activity 5.8

Activity

Accept or not?

What will you do?

1. A company has utility function $U(W) = 0.5\sqrt{W}$

   A risky project can have two outcomes in wealth: 1,000 with probability 0.6 or 3,000 with probability 0.4. It has been decided that the company will be satisfied if the project value is equivalent to 2,100 for certain. Will it accept the project or not?

2. You are offered a choice between a risky offer and a guaranteed amount of 6.25 (amounts are in thousands).

   The risky offer has value 5 with probability 0.65 and value 9 with probability 0.35, and your utility function has been determined as given by $U(W) = \ln W$.

   Determine the certainty equivalent C of the offer.
   Discuss the meaning of the answer and give your decision.

Here's our feedback

1. Expected satisfaction for project:

$$E[U(W)] = 0.6(0.5\sqrt{1{,}000}) + 0.4(0.5\sqrt{3{,}000})$$
$$= 20.44$$

Satisfaction from certainty: $U(C) = 0.5\sqrt{2{,}100} = 22.91$

The company will not accept the project. The utility of the risky project is less than the utility of the certain amount.

2. Find C so that $U(C) = E[U(W)]$ — that is, find C so that:

$$\ln C = 0.65 \ln(5) + 0.35 \ln(9)$$
$$= 1.82$$

Apply the exponential function e right through:

$$e^{\ln C} = e^{1.82}$$

$C = 6.17$ (remember that the e and ln functions are inverses and so "cancel each other out").

This means you should be willing to accept 6.17 for certain instead of accepting the risky offer.

Since you were offered 6.25 guaranteed, you should accept this amount and not the risky offer.

Arrow-Pratt co-efficient (optional)

Earlier, you were told that the shape of the utility function reflects risk aversion. The slope decreases with increasing wealth, meaning that at any particular point a decrease in wealth has a bigger effect on satisfaction than an increase in wealth of similar size. The more the utility function is curved, the more unhappiness with loss of wealth and the greater the risk aversion.

Therefore the magnitude of "curvedness" determines the degree of risk aversion. This is quantified by the Arrow-Pratt risk aversion co-efficient:

$$\text{A-P} = -\frac{U''(W)}{U'(W)}$$

Review Module 1, Unit 4. Remember that $U'(W) = \frac{dU}{dW}$ is the first derivative of U and $U'' = \frac{d^2U}{dW^2}$ is the second derivative of U. Second derivatives measure curvedness.

Example 8

Suppose $U(W) = W$. This is a straight line. A person with this utility would be risk-neutral because a loss or increase in wealth would make an equal change in their satisfaction. This is because the slope of the line here is constant.

In fact, the slope is 1: $U'(W) = 1$. So whether their wealth goes up or down with 1 unit, their satisfaction changes by one unit in either case. The risk aversion of this person should be zero. Let's check.

$$U''(W) = \frac{d^2W}{dW^2} = \frac{d}{dW}[1] = 0$$

The Arrow-Pratt risk aversion coefficient is:

$$\text{A-P} = -\frac{U''(W)}{U'(W)} = -0/1 = 0$$

Since utility functions are usually not linear but concave, consider a more realistic example:

Example 9

Let $U(W) = 0.5\sqrt{W}$. Determine the derivatives of $U(W)$:

$U'(W) = (0.5)\ \tfrac{1}{2}W^{-\frac{1}{2}}$ and $U''(W) = (0.5)\ (\tfrac{1}{2})(-\tfrac{1}{2})W^{-1\frac{1}{2}}$

(Review the section on differentiation in Module 1, Unit 4.)

The A-P risk aversion coefficient is: $\text{A-P} = -\dfrac{-\frac{1}{8}(W)^{-1.5}}{\frac{1}{4}(W)^{-0.5}} = 0.5W^{-1}$,

or $\text{A-P} = \dfrac{1}{2W}$

An interesting conclusion can be drawn from this: as wealth level W increases, A-P decreases. Therefore persons with utility $U(W) = 0.5\sqrt{W}$ will become less risk-averse as their wealth increases.

Activity 5.9

Activity

Work with the utility function

What will you do?

1. A manager is working with utility function $U(W) = W - 0.01W^2$. Two projects are being considered: Project A has outcomes of wealth 40 or 28, with probabilities of 0.45 and 0.55, respectively. Project B has outcomes of wealth 39 or 26, with probabilities of 0.6 and 0.4 respectively.
   a) Use Excel to generate a graph of function U (remembering that the values of W must be restricted to $W < 1/(0.02)$).
   b) Rank the projects and describe which project the manager should accept.
   c) Calculate the certainty equivalent to each project.
2. You are a broker and have analysed your client's risk aversion. Accordingly, you have constructed a utility function for her: $U(W) = -e^{-W}$
   a) Use Excel to generate a graph of function U.
   b) An investment opportunity offers wealth 2 with probability 0.45 and wealth 3 with probability 0.55. What is the client's certainty equivalent?
   c) Calculate the Arrow-Pratt risk co-efficient for the client. Discuss the implications.

Activity 5.10

Activity

Understand the terminology

What will you do?

Use this terminology table to record any terms or words you're uncertain about.

This activity is an opportunity to consolidate your understanding of new terminology and concepts you encountered in Unit 18. Fill in the terms you have learned and then write your own descriptions of them.

Terminology

Term		Description
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Remember these key points

- The value or worth of an outcome can be expressed in terms of the satisfaction of the decision maker. This satisfaction is called the utility of the wealth associated with an outcome and is denoted by $U(W)$. It is based on the decision maker's feelings about risk. There are various concave functions that can be used to describe utility.
- Utility functions and expected utility can also be used to rank different options. The option with the highest utility ranking is the one that will be chosen by the decision maker.
- The certainty equivalent is the amount of certain wealth C for which expected satisfaction equals that of risky wealth W. It is defined by
  $U(C) = E[U(W)]$.
- The Arrow-Pratt risk aversion coefficient A-P = $-\dfrac{U''(W)}{U'(W)}$ gives more information about the risk and satisfaction a person feels towards different levels of wealth. It measures the curvature of the utility function.
- Utility functions can be constructed by getting people to fill in questionnaires about their attitudes to risk and getting them to estimate certainty equivalents for risky projects.
- Otherwise, we often assume an exponential type utility function $U(W) = -e^{-aW}$ and, again, question the person on certainty equivalents to try to find the value of a.

Unit summary

Summary

You have successfully completed this unit if you can:

- **use** utility functions to describe risk aversion;
- **apply** utility functions to rank projects or options and make decisions;
- **calculate** certainty equivalents for risky wealth outcomes; and
- **determine** the Arrow-Pratt co-efficient of risk aversion.

Unit 19

Game theory

Introduction

Decision making is not always a case of facing unknown and impersonal situations and risks. Part of managing a business or organisation can involve conflict and competition with other businesses. Managers formulate strategies in order to gain an advantage over other businesses, and the combination of these strategies and decisions will lead to an outcome — there will be losers and winners. The mathematical theory that helps us to analyse competitive situations is called game theory.

Game theory has been used in areas from biology to nuclear proliferation and goes back to the 18th century work of mathematician Georges-Louis Leclerc de Buffon and of the French physicist André-Marie Ampère in the 19th century. In the 20th century, John von Neumann and Oskar Morgenstern wrote the seminal work on the topic, *Theory of Games and Economic Behaviour*.

This unit looks at simple two-person, zero-sum games. There are two people or organisations involved and the one's losses are the other's gains. Because the losses and gains cancel each other, we have a zero-sum game.

Upon completion of this unit you will be able to:

Outcomes

- **set up** pay-off tables for strategies in game theory; and
- **analyse** situations with dominating strategies.

Example 10

One player is called "Even" and the other player is called "Odd". Each player must simultaneously show a hand with either one or two fingers extended. If both players show the same number, the sum is, of course, even (either two or four) and the player called Even wins. If the players show different numbers of fingers, the sum is odd (three) and the player called Odd wins. A player wins by receiving 1 dollar from the other player.

Win is denoted by (+1) and lose by (−1).

Each player has two strategies:

- Strategy 1: show one finger.
- Strategy 2: show two fingers.

Now construct the pay-off table for the game for Player Even.

The strategies are denoted by S1 and S2. Player Even's strategies are in rows and Player Odd's strategies are in the columns. (Note that the table in Figure 8 gives pay-offs for Player Even.)

Pay-off for Even	Player Odd		
Player Even	Strategy	S1	S2
	S1	+1	−1
	S2	−1	+1

Figure 8

The top left cell containing pay-off (+1) corresponds to strategies S1 for both players: Both hold up one finger. Player Even wins.

The bottom right cell containing pay-off (+1) corresponds to strategies S2 for both players: Both hold up two fingers and Player Even wins again. These cells are where Player Even wins 1 dollar.

The cells containing pay-off (−1) correspond to mixed strategies for the players: One holds up one finger and the other holds up two fingers. These cells are where Player Even loses 1 dollar.

A similar table can be constructed for Player Odd with all the signs reversed. If all pay-offs over both tables are added, the sum will be zero. In general, the pay-off tables can give the utility to each player of winning the game.

Zero-sum game theory rests on two assumptions:

1. Both players are rational (reasonable, sane, think logically).
2. Players are selfish and play only to win (no compassion for the opponent).

Note it!

It is important to see that there is a difference between game theory and decision-making tools such as decision trees, mean-variance analysis and utility theory. With these tools, we assume that the decision maker faces various outcomes based on random (natural) events – there is no active opponent. In game theory, we assume that the decision maker faces an active human opponent who may want to prevent the decision maker from maximising his or her satisfaction.

Reflection

Think about and discuss the basics of game theory.

The original assumption of a rational and "selfish" or self-interested participant has been a basis for economic and other theories over many decades, starting perhaps with Adam Smith (born in 1723 and considered the father of modern economics). Is this a reasonable idea?

Game theory has been extended from selfish play to include situations of co-operation. There is research showing that people often co-operate and collaborate, and that conflict and selfishness may not bring the best results. This is particularly true when we are likely to have frequent interactions with people into the future and our current behaviour is affected by what we think people will do in the future as well as right now.

In 2005, Robert Aumann and Thomas Schelling received the Nobel Prize for work in this area, titled "Conflict and Cooperation Through the Lens of Game Theory". Schelling's work began in the time of the nuclear arms race in the late 1950s. He showed that a party can strengthen its position by weakening its strategies and that uncertain retaliation can be more effective than certain retaliation. These insights have proven to be relevant in conflict resolution and have started new developments in game theory.

Activity 5.11

Activity

Make sure you win

What will you do?

Use the data in Example 10 ("Odd or even?") to answer these questions.

1. Construct the pay-off table for Player Odd.
2. Check that it is a zero-sum game.
3. Is there a specific strategy that a player can follow to definitely win (maximise their utility)? Alternatively, can they co-operate so that both are happy?

From Activity 5.11, you should see that there is no strategy for either player that guarantees a win for one or that keeps both happy. Both strategies 1 and 2 contain losses as well as wins.

Dominating strategies

In some games, dominating strategies can be identified. These are strategies that players should choose to optimise the outcome of the game for them. This does not mean, of course, that both players can win, but the losing player can at least minimise his losses and the winner can guarantee some gains or perhaps even maximise his gains.

Example 11

By working through this example, you can identify dominating strategies.

Scenario: There is a big convention planned in a town. Hundreds of people will spend two days in the town and have money to spend. Two managers of competing retail outlets have formulated campaigns to win customers. Each manager has the following three possible strategies. The strategies are limited by their budgets and by town regulations.

Strategy 1: Put up posters at the Convention Hall on the first day only.

Strategy 2: Put up posters at the Convention Hall on the second day only.

Strategy 3: Put up posters at the Convention Hall on both days.

We assume both managers must choose their strategy today so that neither knows what the other will do. Each has access to the same research showing the estimated net number of customers won or lost per strategy per shop. For example, Manager 1 knows that if she follows Strategy 1 she will win 100 or 200, or lose 50 customers from Shop 2 depending on whether Manager 2 has chosen Strategy 1, 2 or 3, respectively. Manager 2 has the same information.

The pay-off table for Manager 1 is:

Pay-off for Manager 1	Manager 2			
Manager 1	Strategy	S1	S2	S3
	S1	100	200	−50
	S2	50	150	−200
	S3	90	90	20

Figure 9

We assume a zero-sum game, so that all gains here are losses for Manager 2 and vice versa.

For Manager 1, Strategy 1 dominates Strategy 2 because each pay-off in row 1 is larger than the corresponding pay-offs in row 2. She removes Strategy 2 from consideration. Manager 2 also knows Manager 1 will eliminate Strategy 2, because both have access to the same research. Therefore both now look at a reduced pay-off table, as shown below:

Pay-off for Manager 1	Manager 2			
Manager 1	Strategy	S1	S2	S3
	S1	100	200	−50
	S3	90	90	20

Figure 10

At this point, Manager 2 can clearly identify Strategy 2 as his worst option: He will either lose 200 customers or lose 90, whereas his other strategies lead to smaller losses, and even a win of 50 customers. Remember that the pay-off table for Manager 2 is the negative of the above table for Manager 1, and looks like this:

Pay-off for Manager 2	Manager 2			
Manager 1	Strategy	S1	S2	S3
	S1	−100	−200	+50
	S3	−90	−90	−20

Figure 11

He eliminates Strategy 2. Manager 1 realises this. The pay-off table for Manager 1 now reduces to:

Pay-off for Manager 1	Manager 2		
Manager 1	Strategy	S1	S3
	S1	100	−50
	S3	90	20

Figure 12

The table for Manager 2 is the negative of this.

Manager 2 will definitely choose Strategy 3. He will win 50 customers if Manager 1 chooses Strategy 1 and lose only 20 if Manager 1 chooses Strategy 3. Manager 1 will have to choose Strategy 3 as well and win 20 as opposed to losing 50.

Final analysis: Both managers will choose Strategy 3. Manager 2 will lose 20 customers to Manager 1, but this is still his optimum strategy. Strategy 1 may have initially looked better for Manager 1, but since analysis showed that Manager 2 would definitely choose Strategy 3, Manager 1 will choose Strategy 3 as the optimum one. She will still win 20 customers.

Discussion

This dominating strategies example is simple, but it shows how a careful display of data and analysis leads to optimum decision making.

Can you work out immediately which strategies will be chosen? Is this "game" fair? Discuss.

The value of 20 is the maximin of the profit pay-off table for Manager 1 or, equivalently, the minimax value of losses for Manager 2. It is also called the "saddle point" for the problem.

In general, competitive situations will be far more complex. There can be unstable solutions, mixed strategies, non-zero-sum games, *n*-person games, and so on.

Case study:
Retail strategies

Consider the problem of the two managers trying to win customers (the dominating strategies example). Use the same pay-off table or your own data.

1. Manager 1 chooses to ignore the fact that there is a second competing manager who can strategise. She sees her pay-off table purely in terms of uncertain decision making, as in Unit 17. Use the four criteria and find the decision she will make in each case. How do these decisions compare with the case where she sees the situation in terms of game theory?
2. Suppose Manager 2 chooses to ignore the fact that there is a second competing manager who can strategise. He, too, sees his own pay-off table purely in terms of uncertain decision making, as in Unit 17. Use the four criteria and find the decision he will make in each case. How do these decisions compare with the case where he sees the situation in terms of game theory?
3. Compare and discuss the methods of decision making in Unit 17 with the game theory method.
4. *Optional question (not teacher-assessed):* Devise a spreadsheet that calculates the maximin and minimax criteria. Hint: Use Excel functions such as MIN, MAX and IF.

Activity 5.12

Activity

Understand the terminology

What will you do?

Use this terminology table to record any terms or words you're uncertain about.

This activity is an opportunity to consolidate your understanding of new terminology and concepts you encountered in Unit 19. Fill in the terms you have learned and then write your own descriptions of them.

Terminology

Term		Description
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Remember these key points

- Game theory deals with making decisions in a competitive environment. Two (or more) managers have strategies that seek to provide each with the best outcome. The theory of games provides a neat framework for formulating and analysing simple strategy problems.
- A pay-off table is constructed to show profit or loss pay-offs for the players. By successively removing dominating strategies, a choice of strategies remains that maximise minimum profits for one player while minimising maximum losses for the other player.
- Game theory has been extended to include situations of co-operation. Research shows that people often co-operate and collaborate to bring about the optimum long-term results.

Unit summary

Summary

You have successfully completed this unit if you can:

- **analyse** competitive situations;
- **generate** pay-off tables; and
- **evaluate** dominating strategies to arrive at the saddle point solution.

Unit 20

New challenges: Risk and human behaviour

Introduction

Here, you are introduced to a number of people's ideas that are designed to help meet present and future challenges. The topic of "value at risk" is also discussed

Nobel Prize winners Joseph Stiglitz and Amartya Sen have much to share with us, while Nicolas Taleb is a very outspoken contributor to economic debates who put forward the concept of the "black swan". And finally, Daniel Kahneman has done a lot of research on human behaviour and created (with Amos Tversky) the field of behavioural economics. For this, he won the 2002 Nobel Prize in Economics.

Although the units that follow are less quantitative than those in the previous modules, they have great relevance to modelling and decision making.

Upon completion of this unit you will be able to:

Outcomes

- **understand** the role of rare events of huge impact;
- **critique** some thoughts and philosophies that impact on quantitative techniques;
- **apply** value at risk (VaR) calculations;
- **discuss** the limitations of VaR; and
- **appreciate** the importance of human behaviour in the modelling of economic processes.

The behavioural revolution

In October 2008, the columnist David Brooks wrote an article in *The New York Times* called "The Behavioural Revolution". This piece, summarised below, provides a good introduction to this module.

According to Brooks, models in economics and social sciences are usually based on the assumptions that people are rational and aim at maximising their own interests.

Decision making proceeds as follows:

- Observe the situation.
- Identify the different courses of action.
- Calculate which is in your best interest and make the decision.
- Take action.

The 2008 financial crises have challenged these assumptions. Alan Greenspan, chair of the United States Federal Reserve, told a congressional hearing he had been shocked that markets did not work as expected. His assumption that the self-interest of parties who are active in the markets would protect their investments had been proved incorrect.

It appears that the most important step in decision making may not be that of calculating which course of action we should decide on, but rather that of *our perception* of the situation.

Perception is an action that can influence and bias the decision-making process, and economists and psychologists have been working together on this aspect. Their work may offer some explanations as to "why so many people could have been so gigantically wrong about the risks they were taking".

Swans and value at risk

The swans

Nassim Nicholas Taleb has been writing about the problems of perception for quite some time and his "black swan" concept has become quite influential. This idea was first raised in his book *The Black Swan,* published in 2006. (In this book, he already saw that the United States institution Fannie Mae was "sitting on a barrel of dynamite". His first book, published in 2001, was titled *Fooled by Randomness*.)

What is a black swan? It is a rare, highly improbable event with massive impact. It is the type of event that people choose to ignore because, in their perception, they do not think it will ever occur. The problem is that when it does occur, the effects can be disastrous. This is exactly what happened in America with the sub-prime mortgage crisis.

Taleb warns that while statistics and probability theory are at the core of decision making and risk management, they have their limits. We must always remember that probabilities are not observed, they are derived or guessed from limited observations. They are essential, but can be biased and deceive us.

Decisions should be divided into two groups: those that depend on the probabilities or frequencies of events only; and those that also depend on the magnitude or impact of the event. These latter decisions involve not only expected values, but also higher-order moments (such as skewness and kurtosis) of the random variables.

Example 12

Here's a simple example of the effect of a black swan.

A random variable L describes the possible losses on an investment of 1 million pounds. It is suggested that as long as the expected loss is less than 120,000 pounds, the investment is safe. Is this a reasonable risk-management decision?

Let us assume the events causing losses are identified as:

Event 1: Loss L of 200,000 with probability 0.4

Event 2: Loss L of 50,000 with probability 0.6

Call this **Situation A**. Then the expected loss is:

$E[L] = 200{,}000 \times 0.4 + 50{,}000 \times 0.6 = 110{,}000$

This is less than 120,000. The investment will be considered safe.

But what if we change the situation slightly so that there is also a third possible outcome? Call this **Situation B**.

Event 1: Loss L of 200,000 with probability 0.398

Event 2: Loss L of 50,000 with probability 0.6

Event 3: Loss L of 950,000 with probability 0.002

Nothing much seems to have changed as far as the expected loss is concerned:
$E[L] = 200{,}000 \times 0.398 + 50{,}000 \times 0.6 + 950{,}000 \times 0.002 = 111{,}500$

The investment will still be considered safe – in fact, as far as expected losses are concerned, the two situations seem to be approximately the same.

But note the black swan lurking in Situation B. Although the probability of Event 3 is small, there is a possibility of the loss being massive.

Activity 5.13

Activity

What is the maximum loss?

What will you do?

A company plans to invest its profits of 55 million rupees in a financial product for one year. The possible profits are very large, but there may be losses. The chief executive is happy to take the risk as long as the expected loss is not greater than 1 million rupees.

Consider the scenario where a manager identifies three possible events that may cause losses.

Event 1: Loss of 0.5 million rupees with probability 0.551

Event 2: Loss of 1.5 million rupees with probability 0.448

Event 3: Loss of X million rupees with probability 0.001

What is the maximum amount X that the company could lose if the rare Event 3 is realised? (Hint: Set the expected loss equal to 1 million and solve for X.)

The concept of the black swan is related to that of "fat tails". As mentioned in Module 2, random variables can have probability distributions with fat tails, meaning that the probability of reaching high values is much larger than expected from normally distributed variables.

Studies have shown that the normal distribution gives a reasonably good description of the spread of returns (profits and losses) around the centre of the curve, where most of the gains or losses lie. However, it does not work well at the tails, where the extreme or rare events with very high losses or profits lie. It seems that these events are far more common than we think – the distribution of returns has fat tails.

The mathematician Benoît Mandelbrot has calculated that if the Dow Jones Industrial Index followed a normal distribution, it would have moved by more than 3.4 per cent on only 58 days between 1916 and 2003. However, data shows that the index moved as much on 1,001 days. It should have moved by more than 7 per cent only once in every 300,000 years, but in the 20th century it did so 48 times (http://rs.resalliance.org/2009/). This shows that far from being black swans, extreme events can occur as frequently as white swans.

As you have seen, volatility is measured by standard deviation. If market changes follow the normal distribution, we should be 99 per cent sure that returns will lie within three standard deviations of the mean return.

In 2007, David Viniar, chief financial officer of Goldman Sachs, disclosed that the bank had seen moves as large as 25 standard deviations several days in a row. Therefore the markets were in the extreme tail of their distribution far more often than had been assumed. Not only does this show how violently the markets can swing, it also shows how wrong some models were. New distribution models may well have to have fatter tails.

This has another consequence for risk management. When you insure for different forms of risk (foreign exchange risk, interest rate risk, and so on), you make your position seem safer. You have moved your risk to the insurer. But, in fact, part of the real risk may be hidden. You may have exchanged your everyday risks for the single risk that your insurance company will collapse. Surely such a black swan (extreme event), which would cause you huge losses, is impossible? Unfortunately it is not, and a collapse could happen – recently insurer giant AIG (American International Group, Inc.) did just that.

Value at risk

Value at risk or VaR is a measure that has been very popular in assessing the risk associated with financial investments. It is widely used by financial institutions and central banks, but has recently come under attack precisely because of observations as discussed above. Before looking at the criticism, first consider how VaR works.

VaR made simple

The basic idea is to be able to make these kinds of statements:

> We are 99 per cent sure that that we will lose no more than X amount of money over the next N days.
>
> There is a 1 per cent chance that we can lose X amount or more over the next N days.

The amount we are interested in finding is the value of variable X for the chosen time period N and the chosen confidence interval (99 per cent in this case). Remember that X is actually a random variable.

N is called the time horizon and is often taken as 10 days. The amount X is then the 10-day 99 per cent VaR. We can also take $N = 1$ day or change the confidence level from 99 per cent to 95 per cent.

The relation between the one-day VaR and the 10-day VaR is:

10-day VaR = one-day VaR $\times\sqrt{10}$

We usually calculate the one-day VaR first and then the 10-day VaR from this. Central banks are required to calculate their 10-day 99 per cent VaR and then keep at least three times as much capital to protect them against market risk.

To calculate the value of X (the VaR), we actually look at the *changes in value* over the time period N. These changes are denoted by ΔX (delta X). Positive changes mean profit and negative changes are losses. We then make two important assumptions:

- The changes in the market variable X (share price or portfolio value), namely ΔX, are normally distributed.
- The expected change $E[\Delta X]$ over the short period N is zero.

So we are looking at the losses (or profits) as a normal distribution around a mean value of zero. (Remember that these are assumptions and studies have shown they are usually reasonable assumptions. We have previously mentioned that returns on share prices are often assumed to be normally distributed.)

Case study: Calculate VaR

You are planning to invest 10 million dollars in shares of the Moneywise Corporation (MCorp) and want to know the 10-day 99 per cent VaR. In other words, you want to know the loss over 10 days that you have only a 1 per cent chance of incurring.

You first calculate the one-day 99 per cent VaR. Analysis of MCorp's share price yields the following: the daily volatility is 3 per cent. This means over one day, the standard deviation of your investment is 3 per cent of 10 million or:

$0.03 \times 10{,}000{,}000 = 300{,}000$ dollars

Now, your knowledge of the normal distribution tells you that the 99 per cent confidence interval is an interval of 2.33 standard deviations around the mean.

The one-day 99 per cent VaR for your position is therefore:

$2.33 \times 300{,}000 = 699{,}000$ dollars

The 10-day 99 per cent VaR for your position is therefore:

699,000 dollars $\times \sqrt{10} = 2{,}210{,}432$ dollars (rounded off)

For this reason, you are 99 per cent sure you will not lose more than 2,210,432 dollars over a period of 10 days.

There is a one per cent chance that you will lose 2,210,432 dollars or more over a period of 10 days.

Note: Volatility of a market price is usually given as a percentage per annum. For example, the annual volatility of share XYZ is estimated to be 36 per cent. How do you get the daily volatility from this so that you can calculate the 1-day VaR? The relation is daily

For example, the annual volatility of share XYZ is estimated to be 36 per cent. How do you get the daily volatility from this so that you can calculate the 1-day VaR? The relation is daily volatility = $\frac{annual\ vol}{\sqrt{252}}$ which is based on the assumption that there are 252 trading days in a year.

Problems with VaR

The "Calculate VaR" case study shows there is only a one per cent probability that an event will occur where you will lose more than 2,210,432 dollars over a period of 10 days.

However, it does not tell you how much you could actually lose if that rare event is realised. How much more than 2.2 million could you lose? Could you lose the entire 10 million?

This is one of the reasons why VaR has been criticised as a risk measure. VaR gives an optimistic lower bound for losses. An alternative measure is C-VaR (conditional VaR). This measure allows you to make the statement, "If the worst does happen, we can actually lose amount *W*." So the one-day 99 per cent C-VaR is the expected loss *W* over one day, conditional on the fact that you are in the one per cent left tail of the distribution (that is, conditional on the assumption that the worst has happened).

The calculation of C-VaR lies outside the scope of this course. (To learn about it, consult the website http://www.ise.ufl.edu to look at research report #2001-5 by Rockafellar and Uryasev.)

Activity 5.14

Activity

Calculate VaR

What will you do?

A company plans to invest its profits of 100 million rupees in shares in an oil company. The annual volatility of the share price is 58 per cent.

1. Calculate the 10-day 99 per cent VaR of the investment.
2. Calculate the 10-day 95 per cent VaR of the investment.

Discuss your answers.

Economic development: Choice and behaviour

Amartya Sen

Amartya Sen received the Nobel Prize for Economic Sciences in 1998. According to him, economics and management are about the opportunities people have for good living.

We now live in a world of unprecedented wealth and development, but also of great poverty and oppression. Sen feels strongly that the main object of development should be to give people the freedom to participate, and that individual freedom must be seen as social commitment (*Development as Freedom*, 1999).

Sen believes that social progress can come through rational behaviour and individuals' decision making. Where many people now feel that the rational investor or rational decision maker is a myth, and that good social decisions cannot come from individuals' preferences and self-interest, Sen feels that this view has arisen only because we limit our information bases.

Decision making through preference ranking

Problems arise when we base our choices on simple ranking rules. That is, individuals rank choices according to some simple rules and then apply majority decision making. The "most popular" option is chosen. This can lead to inconsistencies, as well as injustice for the wider society.

In fact, Kenneth Arrow's impossibility theorem implies that social decisions cannot come from individual decisions based on preferences. We will have inconsistencies or we will improve someone's utility (well-being) only by reducing another's well-being.

Example 13

A management situation arises where there are three different courses of action available and a decision has to be made. There are three managers who get together to make the decision, and they decide to do this by each ranking the three options in order of preference. The final decision is then done democratically, by choosing the option ranked at the top by the majority of managers. This sounds fair and efficient.

Suppose the three different actions are denoted by x, y and z. If you rank them in order from most to least preferred, you write:

$x > y > z$ if you mean that x is ranked above y and y above z

Now suppose Manager 1's ranking is: $x > y > z$

Manager 2's ranking is: $y > z > x$

Manager 3's ranking is: $z > x > y$

Then we cannot make a rational decision and choose the most preferred action. This is what Arrow's theorem states.

Sen, on the other hand, feels that the problem is that we have too little information or our information base is too small. If we expand our information to include knowledge such as the fact that the managers are not all equally important or qualified to rank options, we can solve the problem.

Activity 5.15

Activity

Make a rational decision

What will you do?

Look at the "Rank the options" example and explain why no majority decision can be made based on the rankings.

Now assume Manager 1's ranking carries weight 1, Manager 2's ranking carries weight 1.5 and Manager 3's ranking carries weight 3.

Can you make a rational decision?

Example 14

A government has an amount of money (the proverbial pie) that must be shared between three groups. The rule is that they should improve the economic circumstances of the majority of groups.

Groups 2 and 3 get together and so form the majority. The decision is now very simple: take away part of Group 1's share and give it to the other two groups. The majority will indeed have better circumstances, but the decision may lead to great injustice – the government may be taking away from the poorest and weakest group.

Is this type of decision making therefore doomed?

Amartya Sen says no, not necessarily. However, we must expand our information to include knowledge about which group can afford to give up some privileges, which group will benefit most by receiving a bit more, and so on. (However, we must be careful, because this knowledge may also be politicised.)

Sen's social choice theory

It is possible to make social decisions that are just and consistent by using individual decisions. The way to do this is to ensure that individuals look outside the narrow informational base of preference ranking and of self-interest. This will involve education.

Joseph Stiglitz

Joseph Stiglitz is another Nobel laureate who is outspoken on many economic and management issues. We share some of his ideas on risk, and discuss his vision and values (*The Roaring Nineties*, 2003).

According to Stiglitz, the process of creating an economic boom also exposed the economy to more risk and, at the same time, undermined our ability to manage risk. Increases in productivity can lead to job losses as companies have lower loyalty to workers and focus on immediate profits. In many countries, social security and welfare have been downscaled, with workers having to bear the risks of unemployment, retirement and health care.

Risk has become a way of life, and education on how to best manage risk is essential. We cannot turn back the clock on the innovations in economics and labour that continue to take place.

The concept of a lifelong job with a company is no longer realistic. Instead, we must focus on lifelong employability and therefore on lifelong learning. (This is why your decision to study for your MBA is so important.)

Stiglitz feels strongly about equality of opportunity and participatory democracy. His views on rational choice and individual self-interest appear to be narrower and more pessimistic than those of Sen. Stiglitz believes there is no basis for the theories of Adam Smith, whereas Sen believes the basis is sound *as long as we expand the informational bases that inform our decisions and choices***.**

In the opinion of Joseph Stiglitz, governments can do much good by being involved in the economy. The best way to achieve stable, sustainable and fair economic growth would be for governments and markets to work together. There should be some regulations in terms of trust and mutual consideration. Often, however, governments buckle under the pressure from big companies and financial institutions.

Values and ethics are of the utmost importance to Stiglitz. He remains hopeful that what he sees as a present focus on private selfishness and greed may be followed by a period of public-spiritedness, where people focus on the well-being of the entire community.

Behavioural economics

"Models and assumptions" in Unit 13 of Module 4 discussed the modelling process and some of the assumptions that are made in building models. One of the basic assumptions of economics has been that of investor rationality and efficient markets. People are assumed to act in their own best interest and to think carefully about investments so that the whole market then behaves efficiently.

Daniel Kahneman, a psychologist who won the Nobel Prize for Economics in 2002, and Amos Tversky created the theory of behavioural economics or "prospect theory". This relatively new theory has become quite influential and you should be aware of its existence and basic philosophy.

According to this theory, there is only an illusion of control. If situations are framed in terms of losses instead of in terms of gains, even if the outcomes of decisions are the same, people think irrationally. This is explained by the following example, which was published in the Kahneman and Tversky paper "Prospect Theory: An Analysis of Decision under Risk".

Example 15

You will be placed in two situations, Situation A and Situation B, and then asked to make a choice in each case.

Situation A: You are given USD 1,000 and must decide between two actions.

Choose #1 and a coin will be flipped. If it is heads, you win USD 1,000 and if it is tails, you win or lose nothing.

Choose #2 and you receive USD 500 with certainty.

What would you do?

Situation B: You are now given USD 2,000 and must decide between two actions.

Choose #1 and a coin will be flipped. If it is heads, you lose USD 1,000 and if it is tails, you lose nothing.

Choose #2 and you lose USD 500 with certainty.

Which choice will you make?

Now look at the two situations rationally.

Suppose in Situation A you choose #2. This means you would rather end up with USD 1,500 for sure than choose #1 and take the 50 per cent risk of remaining with only USD 1,000.

In Situation B you should then also choose #2 because this will also leave you with USD 1,500 for sure. Choice #1 would have a 50 per cent risk of remaining with only USD 1,000, and we know you are not willing to take that risk.

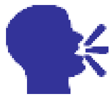
If you are a thinking person, you should choose the same # number action in both situations.

Kahneman and Tversky found the following: in Situation A, most people chose action #2. Receiving something for sure is very attractive. However, when later faced with Situation B, the majority of the very same people suddenly chose #1. Losing some money for sure seemed very unattractive, even if the outcomes in both cases are identical, as shown above. Only 23 per cent of people were rationally consistent and chose #2 in both situations.

Therefore people behave irrationally and can have stronger feelings about losing than winning, even if the outcomes are the same in reality.

Stock markets in the United States in particular are driven by fads and the unpredictable behaviour of people, and this can cause huge excess volatility in individual shares. People are unwilling to "cut their losses" and can hang on to investments or decisions long after it has become clear that the decisions were wrong. This is similar to gamblers who have lost a lot of money but keep on gambling, hoping to make up for the losses with a sudden big win.

Kahneman feels that ordinary utility theory does not adequately explain the behaviour of people under risk. He feels that we should look more at the utility of losses and gains and not at utility of wealth itself. He also feels that a person's utility curve should not just be determined by risk preferences, but also by the present wealth of the person. This is the essence of prospect theory.



Discussion

Humans are complex

There are many instances where people are asked to make a simple decision, but where their decisions are affected by human psychology.

Pair off with another person and tell them you have been given 10 pounds from your university to share with them. You can offer them any amount you wish: 8 pounds, 0 pounds, 10 pounds, whatever you feel like giving. The rest you will keep for yourself. They can accept or reject your offer. If they reject your offer, neither of you gets anything.

Explain this to them and then make them an offer of 1 pound. Repeat this exercise a number of times with different people.

What normally happens is that people are not willing to accept an offer that they perceive as unfair. If you offered the other person 1 pound and kept 9 pounds for yourself, did they accept their offer? Probably not. They may even have become quite angry at you for being selfish. They probably decided to reject the offer and see both of you get nothing rather than letting you keep the biggest share.

But surely this irrational! This is an offer of free money that they didn't have before. Why not accept your offer, walk away and ignore the fact that you have kept the bigger portion? At least they would have received 1 pound.

Research has shown that people would rather go without money and punish both parties when they perceive another person as "unfair". (Or is this behaviour rational, with deeper social implications? Perhaps this will teach the person making the offer to be more just?)

Now suppose you (or they) are faced with a computer screen. The computer has been programmed to offer you an amount from a total of 10 pounds donated to your university by the Commonwealth of Learning (COL). The university computer keeps the rest of the money. You can either accept or reject the computer's offer by pressing key A or key R, respectively. If you reject the offer, the total amount of 10 pounds goes back to COL.

Suppose the computer offers you 1 pound. What will your decision be? Research has shown that people would accept any offer from the computer, no matter how small. They don't feel any need to punish the computer. This can be seen as rational behaviour and should have been the behaviour in the first situation as well, but humans are complex creatures.

Case study:
Group research project

This case study involves rational behaviour in people's decision making.

1. Analyse these situations and show that, rationally speaking, people who choose Option 1 in Situation X should also choose Option 1 in Situation Y. Likewise, people who choose Option 2 in Situation X should also choose Option 2 in Situation Y.

   Situation X

   You are given 10,000 units of money and must choose between two courses of action:

   - Option 1: Throw a dice. If it shows an even number, you win 5,000 units. If it shows an odd number, you win nothing.
   - Option 2: Receive 2,000 units with certainty.

   Situation Y

   You are given 15,000 units of money and must choose between two courses of action:

   - Option 1: Flip a coin. If it shows heads, you lose 5,000 units. If it shows tails, you lose nothing.
   - Option 2: Lose 3,000 units with certainty.

2. Send out a questionnaire to a group of people, asking them to make a choice between the options in Situation X. Analyse the results. Then send out another questionnaire to the same group of people, asking them to make a choice between the options in Situation Y. Write up your results as a report.

Activity 5.16

Activity

Understand the terminology

What will you do?

Use this terminology table to record any terms or words you’re uncertain about.

This activity is an opportunity to consolidate your understanding of new terminology and concepts you encountered in Unit 20. Fill in the terms you have learned and then write your own descriptions of them.

Terminology

Term		Description
	:	
	:	
	:	
	:	
	:	
	:	
	:	
	:	
	:	
	:	
	:	

Remember these key points

- In economics, rare black and common white, but vicious, "swans" describe events that may have disastrous consequences but whose probabilities have been ignored. Probability distributions of losses may have fatter tails than many models assume.
- Calculating the VaR (value at risk) for an investment situation from the volatility of the market price is easy to implement, but VaR is not a stable or coherent risk measure. It gives no idea of the magnitude of the loss that may be incurred in the one per cent left-hand tail. For this reason, conditional VaR may be a better measure, but it is far more difficult to apply.
- Nobel Prize winners Amartya Sen and Joseph Stiglitz have a lot to teach us about human involvement in economics. Models are important, but they must be used in a framework that allows room for discussions of a philosophical and psychological nature.
- Behavioural economics is an emerging way of factoring in the often irrational behaviour of humans.
- There are a number of other new theories that have been developed or applied to economics and management, including catastrophe theory, chaos theory and fractals and complexity theory. Time will tell which ideas contribute to the skills of an effective manager.
- As a future decision maker or manager, you should be aware of the value of theories, models and quantitative tools. You should also remember the voices of caution and moderation.

Unit summary

Summary

You have successfully completed this unit if you can:

- **realise** that rare events do occur, with disastrous consequences;
- **explain** that VaR (value at risk) is a measure that gives a bound for the amount of money you may lose with specified probability over a specified period;
- **explain** that although VaR is much used and popular, it is not always a good measure of risk;
- **discuss** that humans have choices in decision making that can affect economic development positively; and
- **explain** that human psychology and behaviour may also cause the markets to behave irrationally.

A last word...

You should regard your studies and their application to management not only in terms of development, but also in terms of freedom. Personal and social development often focuses on increasing personal well-being, wealth or the gross domestic product of a group. However, freedom also concerns creating the opportunities to pursue a course of action and achieve it.

We sincerely hope that you have found this course on quantitative techniques interesting, informative, valuable and stimulating. Make the effort to keep on learning, as lifelong learning brings lifelong employability and the opportunity to contribute to your society.

Assessment

Assessment — Module 5

1. As manager for a retail store, you have to order new stock. There are three ranges to choose from: luxury, middle of the range and inexpensive. The strategic decision on which range to buy depends on the economic outlook for the season, but the economic forecasts are uncertain. Historical data shows that the expected profits are:

Profit pay-offs	Economic outlook			
Strategies		Good	Average	Bad
	1. Buy luxury range	100,000	40,000	−80,000
	2. Buy middle of range	70,750	60,000	−30,200
	3. Buy inexpensive range	−20,200	50,500	50,000

Figure 13 Pay-off table of profits

a) Use the Laplace criterion to make a decision.

b) Use the Wald criterion to make a decision.

c) Use the maximax criterion to make a decision.

d) Use the maximin criterion to make a decision.

e) Compare and discuss the results.

2. Assume that further research shows probabilities can be assigned to the possible economic outcomes in Question 1. The probabilities are given in Figure 14.

   Use expected profit values to decide on the strategy you will follow to order stock under the given risks.

Good economy	Average economy	Bad economy
0.2	0.34	0.46

Figure 14 Table of probabilities

3. You have a piece of land where you think there may be oil. You have to decide whether to have a seismic survey done or not. Seismic surveys are reasonable accurate, but not always. The cost of a survey is 100 monetary units (which can be in thousands of pounds or dollars).

Regardless of the survey, your next decision is to drill for oil or sell the land. The cost of drilling is 200 units and the income from selling is 100. (These two values will have different signs in the tree.)

If you decide to drill after a survey, the outcomes are either a dry hole with probability 0.5 or oil. If you decide to drill without a survey, the outcomes are a dry hole with probability 0.7 or oil. The final pay-off for oil is 1,000 and the final pay-off for a dry hole is −300.

If you decide to sell, there are no further risky outcomes and the final pay-off in all cases is 100.

Use a decision tree to analyse the situation. Discuss the outcomes of the various options. Write up your final decision and justify the course of action you will take.

4. Two managers are in a competitive zero-sum situation. The employees from their divisions can accept new positions in the opposite division. That is, employees from Manager 1's division can accept a job in Manager 2's division and vice versa. Each manager wants to attract as many people from the other division and lose as few as they can. Each manager has three strategies they can follow. The pay-off matrix for a particular manager is constructed as follows: positive numbers indicate the number of employees gained from the other division and negative numbers indicate the number of employees lost to the competing manager's division.

   The strategies and the pay-offs are shown in the table below. Pay-offs are from the perspective of Manager 1. In other words, if Manager 1 follows his strategy 1 and Manager 2 follows her strategy 1, then two employees will leave Manager 1's division and join Manager 2's division. But if Manager 1 follows his strategy 1 and Manager 2 follows her strategy 3, then four people will join Manager 1 and leave Manager 2.

   The pay-off table for Manager 1 is:

Pay-off for Manager 1	Manager 2			
Manager 1	Strategy	S1	S2	S3
	S1	−2	−2	4
	S2	2	−1	3
	S3	−1	−2	2

Figure 15

   a) Are there any dominating strategies for Manager 1?
   b) Draw the pay-off matrix for Manager 2.
   c) Are there any dominating strategies for Manager 2?
   d) Use the process of dominating strategies to identify the strategy

each manager should follow.

5. Assume you have utility function $U(W) = 0.5\sqrt{W}$ where W indicates wealth level. You can invest in a risky project for one year.

   The risky project has three outcomes in wealth: 1,600 pounds with probability 0.5, 1,800 pounds with probability 0.3 and 2,200 pounds with probability 0.2.

   a) What is the expected utility of wealth for the project?

   b) Calculate the certainty equivalent for the project.

   c) The project needs an initial investment of 1,500 pounds. As an alternative to investing in the risky project, you can also invest the money in the bank for a year at 16.5 per cent p.a. compound interest.

   Will you accept the project or not?

6. A financial manager is working with utility function $U(W) = W - 0.01W^2$. Find an expression for the Arrow-Pratt risk-aversion co-efficient and discuss the implications. How does the co-efficient change with changing values of W?

7. A company plans to invest profits, of 50 million Malaysian ringgit, in shares in a construction company. The annual volatility of the share price is 36 per cent.

   a) Calculate the daily volatility of the share price.

   b) Calculate the 10-day 99 per cent VaR of the investment.

   c) If the company cannot accept losses on an investment of more than 5 million ringgit, should it invest in the shares? Discuss your answer.

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http://edge.org/conversation/a-short-course-in-thinking-about-thinking

Further reading

Reading

Where else can I look?

These texts and websites can be consulted for additional information.

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