Module 5

Decision-making, risk and challenges

Introduction

Outcomes

Upon completion of this module students will be able to:

- **evaluate** risk and risk aversion;
- **apply** utility theory to risk management;
- **solve** decision-making problems under uncertain conditions;
- **construct** and **use** decision trees for risky decision making;
- **analyse** and **evaluate** strategies using game theory;
- **appreciate** new challenges in management and economics (black swans and fat tails); and
- **critique** philosophies of economists and the new theory of behavioural economics.

Unit 17

Risk and decision-making

Outcomes

Upon completion of this unit students will be able to:

- **be aware of** the changes and challenges of the business in the 21st century;
- **understand** and **apply** the criteria of decision-making under uncertainty (Laplace, maximin, minimax and maximax); and
- **model** risky decision-making processes using decision trees.

Decision-making under certainty, risk and uncertainty

Activity 5.1

Activity
Apply maximax and minimax

What will you do?

Consider the situation of the events manager ordering food hampers. Apply the maximax and maximin decision criteria and give the chosen decision in each case. Discuss your results.

What do you think of the three criteria? Which suits your personality or business style?

Solutions

Pay-off table of profits

Profit pay-offs	Weather outcomes			
Strategies		Warm	Cool	Cold
	1. Buy large quantity	1200	600	−560
	2. Buy medium quantity	750	650	−120
	3. Buy small quantity	590	300	120

The maximax rule: Choose Strategy 1: Buy a large quantity for maximum profit of 1,200.

The maximin criterion: Minimum profit for strategies: −560, −120, 120. Maximum of these minima = 120. Choose Strategy 3: Buy a small quantity for profit of 120.

Activity 5.3

Activity

Final share decision

What will you do?

1. Calculate the volatilities of shares X, Y and Z. (Refer to Modules 2 and 4 if you have forgotten the definition of volatility.)
2. Repeat the calculations using Excel or Open Office.
3. What is your final decision about investing in one of the shares? Justify your answer.
4. Research the concept of diversification. Discuss the benefits of investing in more than one share.

Solutions

1. Volatility = standard deviation = $\sqrt{\text{var}}$

Mean = $\overline{X} = \sum x_i p(x_i)$ summed over all possible values x_i

Variance = $\text{var}(X) = \sigma^2(X) = \overline{X^2} - (\overline{X})^2$ where $\overline{X^2} = \sum x_i^2\, p(x_i)$

In this case, the variable x_i of i=1, 2, 3 refers to the possible rates of return for a share.

vol(X) = 0.016, vol(Y) = 0.0181, vol(Z) = 0.0198

2. The answers should be the same.
3. Z has the highest volatility and is the riskiest. In general, higher returns imply higher risk. If you are very risk-averse, you may choose X instead.
4. It may be better to invest in a portfolio with a combination of the shares, provided they are negatively correlated. In such a case, you can decrease the overall volatility and still earn a good return.

Decision trees

Activity 5.4

Activity

Farm or mine?

What will you do?

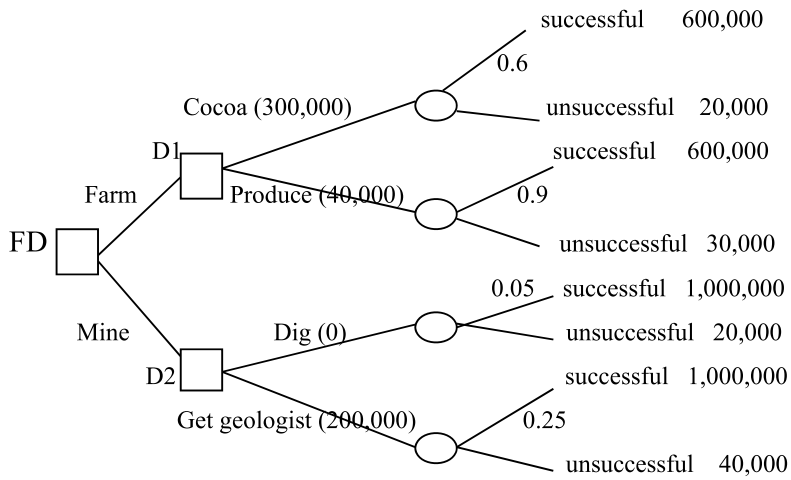
You have a piece of farmland where you think there may be diamonds. You have to decide whether to farm there or start mining. If you decide to farm, you can either plant cocoa for export or you can grow various produce for your own use and to sell locally. If you want to dig for diamonds, you can either get a geologist in to test for diamonds or just start digging.

The probability of a good outcome from deciding on diamonds and getting a geologist is 0.25. The value of this outcome is 1,000,000 and the value of a poor outcome is 40,000. The cost involved with a geologist is 200,000. The value of a positive outcome without a geologist is also 1,000,000 and the probability of a good outcome is 0.05. The value of a poor outcome is 20,000.

With cocoa, the costs are 300,000 and the probability of success is estimated at 0.6. The value of success here is 600,000. The value of no success here is 20,000. With produce, the costs are 40,000 and the probability of success is 0.9, with a final value of 600,000. The value of an unsuccessful outcome is 30,000.

Use a decision tree to analyse the situation. Explain your final decision and justify the course of action you will take.

Solution



Values of risky nodes (top to bottom): 368,000, 543,000, 59,000, 280,000

Values of decision nodes: D1 = 503,000, D2 = 80,000

Farming has far greater value than mining. Between cocoa and various produce, it is more expensive to get involved in cocoa farming and the chances of success are less.

Unit 18

Utility theory

Outcomes

Upon completion of this unit students will be able to:

- **express** values of outcomes in terms of utility functions;
- **rank** possible decisions with the help of utility theory;
- **calculate** certainty equivalents; and
- **determine** the Arrow-Pratt co-efficients for decision-makers.

Activity 5.9

Activity

Work with the utility function

What will you do?

1. A manager is working with utility function $U(W) = W - 0.01W^2$. Two projects are being considered: Project A has outcomes of wealth 40 or 28, with probabilities of 0.45 and 0.55, respectively. Project B has outcomes of wealth 39 or 26, with probabilities of 0.6 and 0.4 respectively.

   a) Use Excel to generate a graph of function U (remembering that the values of W must be restricted to $W < 1/(0.02)$).

   b) Rank the projects and describe which project the manager should accept.

   c) Calculate the certainty equivalent to each project.

2. You are a broker and have analysed your client's risk aversion. Accordingly, you have constructed a utility function for her: $U(W) = -e^{-W}$

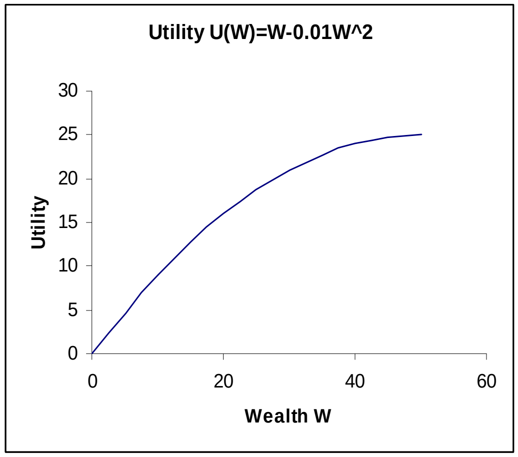
   a) Use Excel to generate a graph of function U.

   b) An investment opportunity offers wealth 2 with probability 0.45 and wealth 3 with probability 0.55. What is the client's certainty equivalent?

   c) Calculate the Arrow-Pratt risk co-efficient for the client. Discuss the implications.

Solutions

1. a)



The reason the graph is restricted to $W < 50$ is because U must remain concave with positive slope.

Slope = $\dfrac{dU}{dW}$ = $1 - 0.02W > 0$ if $W < 1/(0.02)$

b) Calculate expected utilities $E[U(W)]$

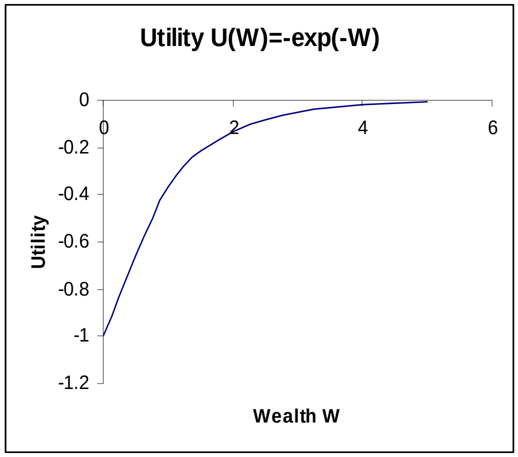
Project A: $E[U(W_A)] = 0.45\ U(40) + 0.55\ U(28)$
$= 10.8 + 11.09$
$= 21.89$

Project B: $E[U(W_B)] = 0.6\ U(39) + 0.4\ U(26)$
$= 14.27 + 7.7$
$= 21.97$

Project B ranks higher than A.

c) Certainty equivalent: $U(C) = E[U(W)]$ implies
$C - 0.01C^2 = 21.89$ for Project A and
$C - 0.01C^2 = 21.97$ for Project B
$C = 32.59$ for Project B and $C = 32.36$ for Project A.

2. a)



b) $U(C) = E[U(W)]$ implies $-e^{-C} = 0.45(-e^{-2}) + 0.55(-e^{-3})$

$$= -0.0883$$
$$e^{-C} = 0.0883$$
$$\ln e^{-C} = \ln 0.0883$$
$$-C = -2.43$$
$$C = 2.43$$

c) A-P $= -\dfrac{U''(W)}{U'(W)} = 1$ (U″ = $(-e^{-W}$ and U′ = e^{-W})

As wealth level W increases or decreases, A-P remains constant. The client will therefore remain at the same level of risk-averseness.

Unit 19

Game theory

Outcomes

Upon completion of this unit students will be able to:

- **set up** pay-off tables for strategies in game theory; and
- **analyse** situations with dominating strategies**.**

Activity 5.11

Activity

Make sure you win

What will you do?

Use the data in the example to answer these questions.

1. Construct the pay-off table for Player Odd.
2. Check that it is a zero-sum game.
3. Is there a specific strategy that a player can follow to definitely win (maximise their utility)? Alternatively, can they co-operate so that both are happy?

Solutions

1.

Pay-off for Odd	Player Odd		
Player Even	Strategy	S1	S2
	S1	−1	+1
	S2	+1	−1

2. The sum of corresponding entries in tables for the two players is zero.
3. There is no strategy and no possible co-operation. One player has to win and the other player will lose, unless both decide not to play at all.

Case study: Retail strategies

Consider the problem of the two managers trying to win customers (the dominating strategies example). Use the same pay-off table or your own data.

1. Manager 1 chooses to ignore the fact that there is a second competing manager who can strategise. She sees her pay-off table purely in terms of uncertain decision making, as in Unit 17. Use the four criteria and find the decision she will make in each case. How do these decisions compare with the case where she sees the situation in terms of game theory?
2. Suppose Manager 2 chooses to ignore the fact that there is a second competing manager who can strategise. He, too, sees his own pay-off table purely in terms of uncertain decision making, as in Unit 17. Use the four criteria and find the decision he will make in each case. How do these decisions compare with the case where he sees the situation in terms of game theory?
3. Compare and discuss the methods of decision making in Unit 17 with the game theory method.
4. *Optional project (not teacher-assessed):* Devise a spreadsheet that calculates the maximin and minimax criteria. Hint: Use Excel functions such as MIN, MAX and IF.

Solution

1.

Pay-off for Manager 1	Manager 2			
Manager 1	Strategy	S1	S2	S3
	S1	100	200	−50
	S2	50	150	−200
	S3	90	90	20

Laplace: Calculate expected pay-offs for each strategy.

Strategy 1: $\frac{1}{3}(100) + \frac{1}{3}(200) + \frac{1}{3}(-50) = 83.33$ (rounded off)

Strategy 2: $\frac{1}{3}(50) + \frac{1}{3}(150) + \frac{1}{3}(-200) = 0$

Strategy 3: $\frac{1}{3}(90) + \frac{1}{3}(90) + \frac{1}{3}(20) = 66.67$ (rounded)
Decision: Follow Strategy 1.

Minimax: Maximum losses are 50, 200 and −20 for strategies 1, 2 and 3 respectively. The minimum is −20.
Decision: Follow Strategy 3.

Maximax: Maximum profits are 200, 150 and 90 for strategies 1, 2 and 3 respectively. The maximum is 200.
Decision: Follow Strategy 1.

Maximin: Minimum profits are −50, −200 and 20 for strategies 1, 2 and 3 respectively. The maximum is 20.
Decision: Follow Strategy 3.

Game theory, minimax and maximin (here) give the same result (Strategy 3).

2. Do the same for Manager 2 with the pay-off table:

Pay-off for Manager 2	Manager 2			
Manager 1	Strategy	S1	S2	S3
	S1	-100	-200	50
	S2	-50	-150	200
	S3	-90	-90	-20

Examples:

Minimax: Maximum losses are 100, 200 and −20 for strategies 1, 2 and 3 respectively. The minimum is −20.
Decision: Follow Strategy 3.

Maximax: Maximum profits are: −50, −90 and 200 for strategies 1, 2 and 3 respectively. The maximum is 200.
Decision: Follow Strategy 3.

Manager 2 always follows Strategy 3.

3. Zero-sum game theory and the minimax criterion should give the same result.

4. Optional question. Lecturers to assess student responses.

Unit 20

New challenges: Risk and human behaviour

Outcomes

Upon completion of this unit students will be able to:

- **understand** the role of rare events of huge impact;
- **critique** some thoughts and philosophies that impact on quantitative techniques;
- **apply** value at risk (VaR) calculations;
- **discuss** the limitations of VaR; and
- **appreciate** the importance of human behaviour in the modelling of economic processes.

Activity 5.13

Activity

What is the maximum loss?

What will you do?

A company plans to invest its profits of 55 million rupees in a financial product for one year. The possible profits are very large, but there may be losses. The chief executive is happy to take the risk as long as the expected loss is not greater than 1 million rupees.

Consider the scenario where a manager identifies three possible events that may cause losses.

Event 1: Loss of 0.5 million rupees with probability 0.551

Event 2: Loss of 1.5 million rupees with probability 0.448

Event 3: Loss of X million rupees with probability 0.001

What is the maximum amount X that the company could lose if the rare Event 3 is realised? (Hint: Set the expected loss equal to 1 million and solve for X.)

Solution

If we leave Event 3 out of the picture, the expected loss $E[L]$ is 0.948 million rupees. This is acceptable to the chief executive. Managers may be tempted to forget about Event 3, since the probability is so small. Let's assume an expected loss of 1 million and include Event 3. The equation is:

$E[L] = 0.5 \times 0.551 + 1.5 \times 0.448 + X \times 0.001 = 1$

Then $X = 52.5$ million rupees. In other words, merely setting a limit of 1 million on expected losses can lead to implicit acceptance of a possible loss of 52.5 million. Event 3 would represent a typical black swan.

Swans and value at risk

Activity 5.14

Activity

Calculate VaR

What will you do?

A company plans to invest its profits of 100 million rupees in shares in an oil company. The annual volatility of the share price is 58 per cent.

1. Calculate the 10-day 99 per cent VaR of the investment.
2. Calculate the 10-day 95 per cent VaR of the investment.

Discuss your answers.

Solutions

1. We first calculate the 1-day 99% VaR. The daily volatility is:

   $\frac{0.58}{\sqrt{252}} = 0.0365$. This means that over 1 day the standard deviation of our investment is 3.65% of 100 million or:

   $0.0365 \times 100{,}000{,}000 = 3{,}650{,}000$ rupees

   The 1-day 99% VaR for our position is therefore:

   $2.33 \times 3{,}650{,}000 = 8{,}504{,}500$

   The 10-day 99% VaR for our position is therefore:

   $8{,}504{,}500 \times \sqrt{10} = 26{,}893{,}590$ rupees (rounded off)

   We can be 99% sure that we will not lose more than 26,893,590 rupees over a period of 10 days, or there is a 1% chance that we will lose that amount or more over a period of 10 days.

2. The 1-day 95% VaR for our position is therefore:

   $1.96 \times 3{,}650{,}000 = 7{,}154{,}000$

   The 10-day 99% VaR for our position is therefore:

   $7{,}154{,}000 \times \sqrt{10} = 22{,}622{,}934$ rupees (rounded off)

   We can be 95% sure that we will not lose more than 22,622,934 rupees over a period of 10 days or there is a 5% chance that we will lose that amount or more over a period of 10 days.

Activity 5.15

Activity

Make a rational decision

What will you do?

Look at the "Rank the options" example and explain why no majority decision can be made based on the rankings.

Now assume Manager 1's ranking carries weight 1, Manager 2's ranking carries weight 1.5 and Manager 3's ranking carries weight 3.

Can you make a rational decision?

Solution

Example 1: Manager 1's ranking is: $x > y > z$

Manager 2's ranking is: $y > z > x$

Manager 3's ranking is: $z > x > y$

Any pairing of managers leads to a conflicting ranking. Managers 1 and 2 have a conflict on the ranking of *x* and *z*. Managers 2 and 3 have a conflict on ranking *y* and *z.* Managers 1 and 3 have a conflict on ranking *y* and *z.* For this reason, a majority is impossible.

Manager 3 has a vote larger than the combined votes of managers 1 and 2. Her ranking will be accepted.

Case study:
Group research project

This case study involves rational behaviour in people's decision making.

1. Analyse these situations and show that, rationally speaking, people who choose Option 1 in Situation X should also choose Option 1 in Situation Y. Likewise, people who choose Option 2 in Situation X should also choose Option 2 in Situation Y.

   Situation X

   You are given 10,000 units of money and must choose between two courses of action:

   - Option 1: Throw a dice. If it shows an even number, you win 5,000 units. If it shows an odd number, you win nothing.
   - Option 2: Receive 2,000 units with certainty.

   Situation Y

   You are given 15,000 units of money and must choose between two courses of action:

   - Option 1: Flip a coin. If it shows heads, you lose 5,000 units. If it shows tails, you lose nothing.
   - Option 2: Lose 3,000 units with certainty.

2. Send out a questionnaire to a group of people, asking them to make a choice between the options in Situation X. Analyse the results. Then send out another questionnaire to the same group of people, asking them to make a choice between the options in Situation Y. Write up your results as a report.

Discussion

People who choose Option 1 in situation X: Expected pay-off = 12,500

People who choose Option 1 in situation Y: Expected pay-off = 12,500

People who choose Option 2 in Situation X: Expected pay-off = 12,000

People who choose Option 2 in Situation Y: Expected pay-off = 12,000

Assessment

Assessment — Module 5

1. As manager for a retail store, you have to order new stock. There are three ranges to choose from: luxury, middle of the range and inexpensive. The strategic decision on which range to buy depends on the economic outlook for the season, but the economic forecasts are uncertain. Historical data shows that the expected profits are:

Profit pay-offs	Economic outlook			
		Good	Average	Bad
Strategies	1. Buy luxury range	100,000	40,000	−80,000
	2. Buy middle of range	70,750	60,000	−30,200
	3. Buy inexpensive range	−20,200	50,500	50,000

Figure 8 Pay-off table of profits

   a) Use the Laplace criterion to make a decision.

   b) Use the Wald criterion to make a decision.

   c) Use the maximax criterion to make a decision.

   d) Use the maximin criterion to make a decision.

   e) Compare and discuss the results.

2. Assume that further research shows probabilities can be assigned to the possible economic outcomes in Question 1. The probabilities are given in Figure 14.

   Use expected profit values to decide on the strategy you will follow to order stock under the given risks.

Good economy	Average economy	Bad economy
0.2	0.34	0.46

Figure 9 Table of probabilities

3. You have a piece of land where you think there may be oil. You have to decide whether to have a seismic survey done or not. Seismic surveys are reasonable accurate, but not always. The cost of a survey is 100 monetary units (which can be in thousands of pounds or

dollars).

Regardless of the survey, your next decision is to drill for oil or sell the land. The cost of drilling is 200 units and the income from selling is 100. (These two values will have different signs in the tree.)

If you decide to drill after a survey, the outcomes are either a dry hole with probability 0.5 or oil. If you decide to drill without a survey, the outcomes are a dry hole with probability 0.7 or oil. The final pay-off for oil is 1,000 and the final pay-off for a dry hole is −300.

If you decide to sell, there are no further risky outcomes and the final pay-off in all cases is 100.

Use a decision tree to analyse the situation. Discuss the outcomes of the various options. Write up your final decision and justify the course of action you will take.

4. Two managers are in a competitive zero-sum situation. The employees from their divisions can accept new positions in the opposite division. That is, employees from Manager 1's division can accept a job in Manager 2's division and vice versa. Each manager wants to attract as many people from the other division and lose as few as they can. Each manager has three strategies they can follow. The pay-off matrix for a particular manager is constructed as follows: positive numbers indicate the number of employees gained from the other division and negative numbers indicate the number of employees lost to the competing manager's division.

   The strategies and the pay-offs are shown in the table below. Pay-offs are from the perspective of Manager 1. In other words, if Manager 1 follows his strategy 1 and Manager 2 follows her strategy 1, then two employees will leave Manager 1's division and join Manager 2's division. But if Manager 1 follows his strategy 1 and Manager 2 follows her strategy 3, then four people will join Manager 1 and leave Manager 2.

   The pay-off table for Manager 1 is:

Pay-off for Manager 1	Manager 2			
Manager 1	Strategy	S1	S2	S3
	S1	−2	−2	4
	S2	2	−1	3
	S3	−1	−2	2

Figure 10

   a) Are there any dominating strategies for Manager 1?
   b) Draw the pay-off matrix for Manager 2.
   c) Are there any dominating strategies for Manager 2?

d) Use the process of dominating strategies to identify the strategy each manager should follow.

5. Assume you have utility function $U(W) = 0.5\sqrt{W}$ where W indicates wealth level. You can invest in a risky project for one year.

   The risky project has three outcomes in wealth: 1,600 pounds with probability 0.5, 1,800 pounds with probability 0.3 and 2,200 pounds with probability 0.2.

   a) What is the expected utility of wealth for the project?

   b) Calculate the certainty equivalent for the project.

   c) The project needs an initial investment of 1,500 pounds. As an alternative to investing in the risky project, you can also invest the money in the bank for a year at 16.5 per cent p.a. compound interest.

   Will you accept the project or not?

6. A financial manager is working with utility function $U(W) = W - 0.01W^2$. Find an expression for the Arrow-Pratt risk-aversion co-efficient and discuss the implications. How does the co-efficient change with changing values of W?

7. A company plans to invest profits, of 50 million Malaysian ringgit, in shares in a construction company. The annual volatility of the share price is 36 per cent.

   a) Calculate the daily volatility of the share price.

   b) Calculate the 10-day 99 per cent VaR of the investment.

   c) If the company cannot accept losses on an investment of more than 5 million ringgit, should it invest in the shares? Discuss your answer.

Solutions

1. a) **Laplace:** Calculate expected pay-offs for each strategy. Units in thousands are used:

   Strategy 1: ⅓(100) + ⅓(40) + ⅓(−80) = 20

   Strategy 2: ⅓(70.75) + ⅓(60) + ⅓(−30.2) = 33.52

   Strategy 3: ⅓(−20.2) + ⅓(50.5) + ⅓(50) = 26.77
   Decision: Follow Strategy 2.

   b) **Wald (minimax)**: Maximum losses are 80, 30.2 and 20.2 for strategies 1, 2 and 3, respectively. The minimum is 20.2.
   Decision: Follow Strategy 3.

   c) **Maximax:** Maximum profits are 100, 70.75 and 50.5 for strategies 1, 2 and 3, respectively. The maximum is 100.
   Decision: Follow Strategy 1.

d) **Maximin**: Minimum profits are −80, −30.2 and −20.2 for strategies 1, 2 and 3, respectively. The maximum is −20.2. Decision: Follow Strategy 3.

Discussion: Different criteria give different results. Lecturers should engage students in discussion.

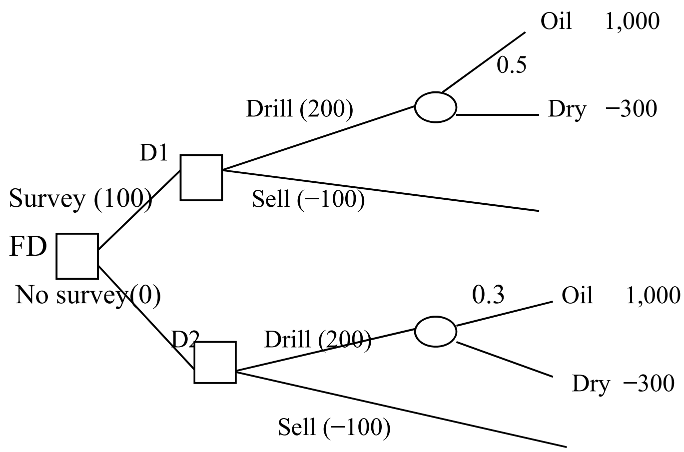
2. Strategy 1: E[profit] = −3.2

   Strategy 2: E[profit] = 20.66

   Strategy 3: E[profit] = 34.11

   Follow Strategy 3.

3.



Values of risky nodes (top to bottom): 350, 90

Value of decision nodes: D1 = max[350−200, −(−100)] = 150
D2 = max[90−200, 100)] = 100

FD = max[150−100, 100 – 0] = 100

We should decide to do a survey and drill rather than no survey and drill. But if we cannot afford a survey, then we should sell.

4. a) Strategy 2 dominates Strategy 3. Remove Strategy 3 for Manager 1.

b)

Pay-off for Manager 2	Manager 2			
Manager 1	Strategy	S1	S2	S3
	S1	+2	+2	−4
	S2	−2	+1	−3

c) Strategy 2 also dominates Strategy 3 for Manager 2.

d) Remove Strategy 3 for Manager 2. Manager 1 will prefer Strategy 2. Manager 2 will also prefer his Strategy 2. Both follow their Strategy 2. Manager 1 loses one employee and Manager 2 gains that employee.

5. a) $E[U(W)] = 0.5(0.5)\sqrt{1600} + 0.3(0.5)\sqrt{1800} + 0.2(0.5)\sqrt{2200}$

$= 21.05$

b) $0.5\sqrt{C} = 21.05$ implies $C = 1{,}772.41$

c) The bank offers 1500(1.165) = 1,747.50 with certainty. This is less than the certainty equivalent, so you should accept the project.

6. A-P $= -\dfrac{U''(W)}{U'(W)}$ with $U'(\mathrm{W}) = 1 - 0.02W$ and $U''(\mathrm{W}) = -0.02$

$$\text{A-P} = -\frac{-0.02}{1-0.02W} = \frac{0.02}{1-0.02W}$$

As W increases, the co-efficient increases, slowly at first but very quickly as W tends to 50. Risk aversion thus increases with increasing wealth.

7. a) 1-day volatility $= \dfrac{0.36}{\sqrt{252}} = 0.0227 = 2.27\%$

b) Over 1 day, the standard deviation of our investment is 2.27% of 50 million, or $0.0227 \times 50{,}000{,}000 = 1{,}135{,}000$ ringgit

The 1-day 99% VaR for our position is therefore $2.33 \times 1{,}135{,}000 = 2{,}644{,}550$

The 10-day 99% VaR for our position is therefore $2{,}644{,}550 \times \sqrt{10} = 8{,}362{,}801$ ringgit (rounded off)

c) We can be 99% sure we will not lose more than 8,362,801 ringgit over a period of 10 days, or there is a 1% chance that we will lose that amount or more over a period of 10 days. The VaR is more than 5 million. The company will have to decide whether it wants to take the risk.

Final examinaton: (model questions)

The examination paper below should be seen as an example of an exam. The questions can be adapted to suit particular circumstances. There are many examples of other possible questions in the various activities and summative assessments in each module.

The duration of the exam should also be taken into account, and the paper shortened or lengthened accordingly. It is a good idea for the lecturer to work through the questions to determine how many minutes are needed for each question.

The model exam below is based on the assumption that it is a written closed-book exam, with the use of a pocket calculator permitted.

It is assumed that access to Excel is not available during the exam and for this reason there are no questions based on the use of Excel in this paper. Marks for activities involving Excel can be collected during the run of the course and added to the exam mark if necessary. If Excel is made available for the exam, the lecturer can adapt the paper by including questions on Excel. Once again, there are many examples of questions available in the activities.

Final assessment — Example of an examination

1. Let $C(x)$ be the cost of producing x items. The marginal cost MC (= $\frac{dC}{dx}$) is given by $MC(x) = x - 1$ and the initial start-up cost before production starts is $C(0) = 1$.

   a) Determine the general expression for cost $C(x)$. [3]

   b) Calculate the cost to produce the first 10 items. [2]

   c) The income function $I(x)$ is known to be linear. There are two data points available: (10, 40) and (0, 20). Find the expression for $I(x)$. [3]

   d) Draw graphs of both $C(x)$ and $I(x)$ on the same set of axes. The divisions along the x-axis should be scaled in units of 2. Determine the approximate break-even point graphically. [4]

   e) Determine the break-even point analytically. [4]

   [16]

2. The following frequency table gives the sizes of fishing vessels (boats) and the number of vessels in each class on an island for the years 1992 and 2002.

Length in metres	*1992*	*2002*
3.01 to 10 m	76	27
10.01 to 17 m	237	123
17.01 to 24 m	419	451
24.01 to 31 m	200	245
31.01 to 38 m	115	60
Source: Annual Abstract of Statistics (UK)		

   a) What conclusions can you draw about the state of fishing on the island? [1]

   b) Expand the table by adding a column for midpoints of intervals and a column for cumulative frequencies for the two years. Midpoints should be rounded to 1 decimal. [4]

   c) Find the means for the two years. Discuss your answers. [5]

   d) Draw a histogram for the distributions. [4]

   [14]

3. Random variable X is binomially distributed and describes the number of successful outcomes in five trials. The probability of a success is 0.7.

   a) Calculate the mean and variance of X. [4]

   b) Determine the probability of, at most, two successes. [4]

   [8]

4. Random variable X is normally distributed on a population. Consider a sample of 200 from the population. The sample has mean 0 and variance $s^2 = 16$.

   a) Calculate the 95 per cent confidence interval for X. Discuss the meaning of this interval. [5]

   b) What is the probability that $X > 2$? [3]

[8]

5. Consider the table of observed values y_t for six annual time steps t. The row of forecast values F_t was obtained by the 2-year moving averages method.

t	1	2	3	4	5	6
y_t	352.5	340	341.5	329.8	320	321.3
F_t	350.5	350	346.3	340.8	335.7	324.9

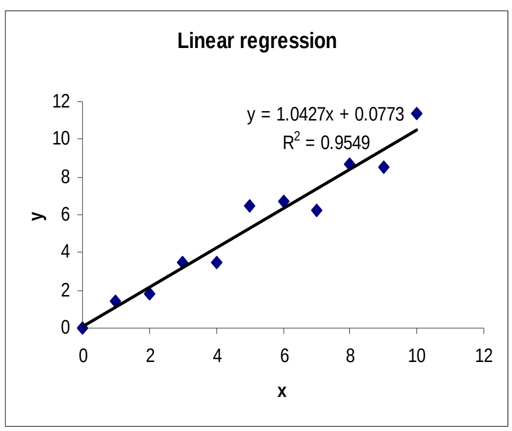
   a) Use the simple averages method to find forecasts for F_1 to F_6. Let $F_1 = 348$. [5]

   b) Use the four-year moving averages method to find a forecast for F_7. [2]

   c) Calculate the mean squared error for each of the simple averages and two-year moving averages method. Compare and interpret your answers. [5]

[12]

6. Consider the following output for a linear regression problem. Data points are indicated by diamond shapes.



   a) Explain the values in the chart. [4]

   b) Estimate a value for y if $x = 20$. [1]

[5]

7. A company manufactures garden forks and spades. It costs 10 dollars to make a spade and 8 dollars to make a fork. There is an additional

daily cost of 100 dollars per day for labour. Denote the daily manufactured number of forks by x and the number of spades by y. It takes five minutes to make a spade and five minutes to make a fork. The company has eight hours per day for manufacturing. It wants to minimise costs, but must manufacture at least 30 items per day to supply customers. It must also manufacture fewer than 20 spades per day.

a) Write down an expression for the daily cost function C. [2]

b) Write down the daily constraints. [2]

c) Give a graphical representation of the feasible region. [2]

d) Find the optimal solution using the corner-point method. Discuss. [4]

[10]

8. As manager for a store, you have to embark on new strategies to increase your profits. There are three possible strategies and the profits for each depend on two possible scenarios: high inflation, or low inflation. Historical data shows that the expected profits are:

Pay-off table of profits

Profit pay-offs	Inflation outlook		
Strategies		High	Low
	1.	1,000	2,000
	2.	775	2,800
	3.	−200	3,500

a) Assume complete uncertainty for the inflation outlook. Use the Laplace criterion to make a decision. [3]

b) Use the Wald criterion to make a decision. Compare with the answer in a). [3]

c) Now assume probabilities can be assigned to the inflation outcomes: the probability of high inflation is 0.65 as estimated from data. Which strategy will you follow in this case if you use expected profits as benchmark? [3]

[9]

9. Assume you have utility function $U(W) = \ln W$ where W indicates wealth level. You have 1,000 dollars to invest for two years, either in the bank (risk-free) or in a risky project.

The risky project has three outcomes in wealth: 1,800 dollars with probability 0.1, 1,400 dollars with probability 0.6 and 1,200 dollars with probability 0.3.

a) What is the expected utility of wealth for the risky project? [3]

b) Calculate the certainty equivalent for the risky project. [4]

c) The risk-free investment is in the bank at 10% p.a. interest, compounded semi-annually. Will you accept the risky project or not? [4]

d) Find an expression for the Arrow-Pratt risk-aversion co-efficient and discuss the implications. [4]

[15]

10. A company has invested 12 million rupees in shares. The annual volatility of the share price is 48%.

a) Calculate the daily volatility of the share price. [2]

b) Calculate the 10-day 99% VaR of the investment. What does the figure mean? [4]

[6]

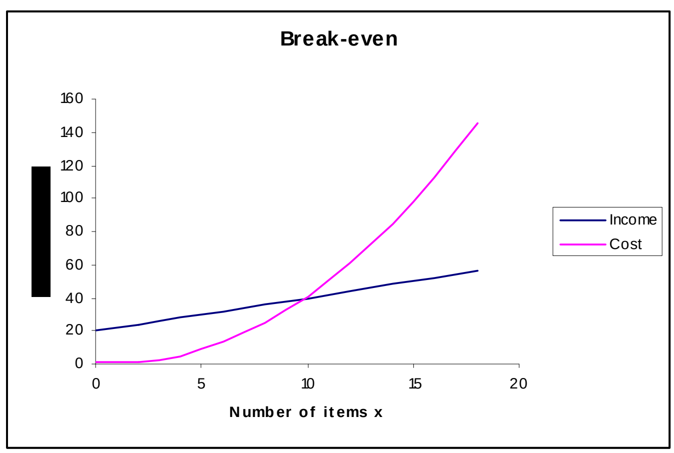
Solutions

1. a) $C(x) = \int(x - 1)dx = \frac{1}{2}(x)^2 - x + k$. Since $C(0) = 1$ we have $k = 1$ and $C(x) = \frac{1}{2}(x)^2 - x + 1$

b) $C(10) = \frac{1}{2}(10)^2 - 10 + 1 = 41$ monetary units

c) Income function I must be of the form: $I(x) = mx + c$. Given data point (0, 20) means $I(0) = 20$ and thus we have $c = 20$. Substitute (10, 40) into equation $I(x) = mx + 20$ gives: $40 = 10m + 20$ and so $m = 2$. Therefore $I(x) = 2x + 20$

d)



e) Break-even means $I(x) = C(x)$. This leads to the quadratic equation: $x^2 - 6x - 38 = 0$

The two roots are $x = -3.86$ and $x = 9.86$.
Break-even occurs when about 10 items are manufactured.

2. a) The fishing industry has decreased slightly over 10 years in terms of numbers of boats. However, there are now more boats of the mid-size range. Small fishermen may be going out of business.

b)

Midpoints	1992 Cum. F	2002 Cum. F
6.5	76	27
13.5	313	150
20.5	732	601
27.5	932	846
34.5	1047	906

c) 1992: Mean = $\frac{1}{N}\sum_{i=1}^{N} x_i f(x_i)$

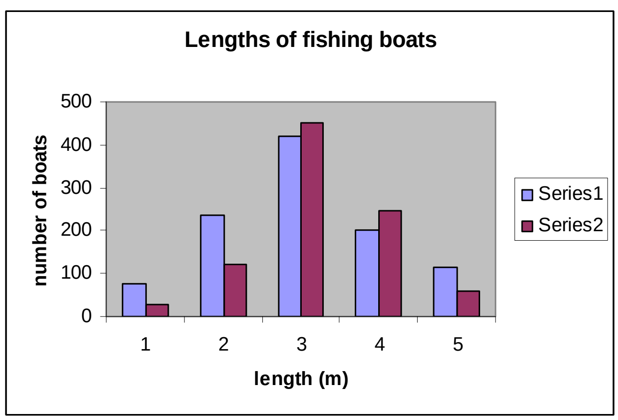
$= \frac{1}{1047}$ [6.5(76) + 13.5(237) + 20.5(419) + 27.5(200) + 34.5(115)] = 20.8 metres (rounded)

2002: Mean = $\frac{1}{N}\sum_{i=1}^{N} x_i f(x_i)$

$= \frac{1}{906}$ [6.5(27) + 13.5(123) + 20.5(451) + 27.5(245) + 34.5(60)]
= 22 metres (rounded)

The average length of boats has increased.

d)



3. a) Mean = $\overline{X}$ = 5 × 0.7 = 0.35
var(X) = $\sigma^2(X)$ = 5× 0.7 × 0.3 = 1.05

a) The probability of, at most, two successes is:
$P(X = 0 \text{ or } X = 1 \text{ or } X = 2) = p(0) + p(1) + p(2)$

Now $p(0) = \binom{5}{0}(0.7)^{0}(0.3)^{5} = 1\times1\times(0.3)^5 = 0.0024$

$p(1) = \binom{5}{1}(0.7)^{1}(0.3)^{4} = 5\times(0.7)\times(0.3)^4 = 0.0284$

$p(2) = \binom{5}{2}(0.7)^{2}(0.3)^{3} = 10\times(0.7)^2\times(0.3)^3 = 0.1323$

Therefore the probability is 16.31%.

4. a) The 95 per cent confidence interval is:
$(0 - 1.96(4)/\sqrt{200},\ 0 + 1.96\,(4)/\sqrt{200}) = (-0.55, 0.55)$
This means the population mean will lie in this interval around 0 with 95% certainty.

   b) $P(X > 2) = P(X > \sigma) = ½(1 – P(-\sigma < X < \sigma)) = ½(1 – 0.68) = 0.16$ = 16%

5. a)

t	1	2	3	4	5	6
y_t	352.5	340	341.5	329.8	320	321.3
F_t	350.5	352.5	346.3	344.7	340.9	336.8

   b) $F_7 = 328.2$

   c) Two-year moving averages MSE = 84.6
Simple averages MSE = 180.4
The simple averages method is much worse than the moving averages method. The reason is that there is a declining trend and it is therefore better to use only the last few values to capture the trend.

6. a) The R^2 value of 0.9549 shows quite a strong linear relationship between the x- and y-values. The graph also shows the data points lying close to the straight line. The slope of the line is approximately 1, showing an increase of about 1 unit in y for every 1-unit change in x. The y-intercept is close to 0. This, together with the slope of 1.0427, implies that the x- and y-values are almost equal.

   b) $y(20) = 1.0427(20) + 0.0773 = 20.93$

7. a) $C(x, y) = 8x + 6y + 100$

   b) $5x + 5y \le 480$, $x + y \ge 30$, $y \le 20$, $x, y \ge 0$. We can also add integer constraints.

   c) The feasible region is the trapezium-shaped region between the parallel lines $y = 96 - x$ and $y = 30 - x$ and the parallel lines $y = 20$ and $y = 0$.

Lecturers should review graphs for correctness.

d) The corner points are: (10, 20), (30, 0), (76, 20), (0, 96)
C has a minimum at (10, 20). This means that 10 forks and 20 spades should be manufactured to minimise costs. It must be noted that this solution is not efficient in the sense that workers will be busy for only 150 minutes per day

8. a) Strategy 1: E[profit] = ½ [1,000] + ½ [2,000] = 1,500
Strategy 2: E[profit] = ½ [775] + ½ [2,800] = 1,787.50
Strategy 3: E[profit] = ½ [−200] + ½ [3,500] =1,650
Therefore, follow Strategy 2.

b) Rewrite table in terms of losses. Find maximum losses for each strategy and then the minimum. Therefore, follow Strategy 1 (the most conservative strategy).

c) Strategy 1: E[profit] = 0.65[1,000] + 0.35[2,000] = 1,350
Strategy 2: E[profit] = 0.65[775] + 0.35[2,800] = 1,483.75
Strategy 3: E[profit] = 0.65[−200] + 0.35[3,500] =1,095
Follow Strategy 2.

9. a) $E[U(W)] = 0.1[\ln(1800)] + 0.6[\ln(1400)] + 0.3[\ln(1200)] = 7.223$

b) $U(C) = 7.223$
$\ln C = 7.223$
$C = 1{,}370.59$

c) Risk-free: $1000(1 + 0.05)^4 = 1{,}215.51$
This is less than the certainty equivalent. Invest instead in the risky project.

d) A-P co-efficient: $U'(W) = \frac{1}{W}$ and $U''(W) = \frac{-1}{W^2}$

A-P = $\frac{1}{W}$. As wealth increases, risk aversion decreases. The investor will thus be inclined to make more risky investments as her wealth grows.

10. a) 1-day volatility = $\frac{0.48}{\sqrt{252}}$ = 0.0302 = 3.02%

b) Over 1 day the standard deviation of our investment is 3.02% of 12 million, or 0.0302 × 12,000,000 = 362,400 rupees. The 1-day 99% VaR for the position is therefore:

2.33 × 362,400 = 844,392
The 10-day 99% VaR for our position is therefore:
844,392 × √10 = 2,670,202 rupees (rounded off)

We are therefore 99% sure that we will not lose more than 2,670,202 rupees over a period of 10 days, or there is a 1% chance that we will lose that amount or more over a period of 10 days. The company will have to decide whether it wants to take the risk.