Coordinate Geometry and the Straight Line

6

This unit aims to explain the concept of the coordinate geometry and the straight line. The unit are prepared with topics such as quadrants and coordinates of midpoints, distance between two points, the straight line, different forms of equation of a straight line and their application is solving business problems.

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Lesson-1: Coordinate Geometry

After studying of this lesson, you should be able to:

- Explain the nature of coordinate geometry;
- Identify the quadrants;
- Identify the coordinates of any point;
- Calculate the coordinates of mid points;
- Calculate the distance between two points.

Nature of Coordinate Geometry

Coordinate geometry is that branch of geometry in which two real numbers, called coordinates, are used to indicate the position of a point in a plane. The main contribution of coordinate geometry is that it has enabled the integration of algebra and geometry. This is evident from the fact that algebraic methods are employed to represent and prove the fundamental properties of geometrical theorems. Equations are also employed to represent the various geometric figures.

Quadrants

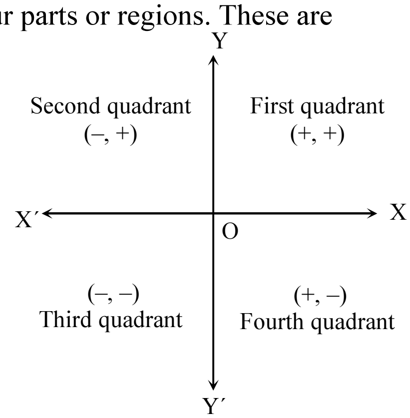
The two directed lines, when they intersect at right angles at the point of origin, divide their plane into four parts or regions. These are

XOY : First quadrant

X´OY : Second quadrant

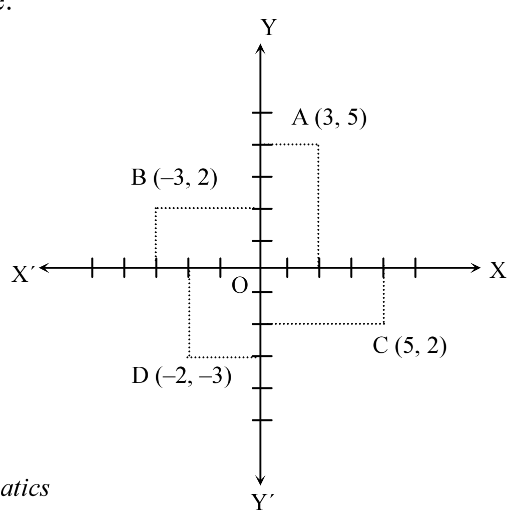
X´OY´ : Third quadrant.

XOY´ : Fourth quadrant



The position of the coordinates in a particular quadrant would depend on the positive and negative values of the

The position of the coordinates in a particular quadrant would depend on the positive and negative values of the coordinates shown in the following figure:



Coordinates

In a two-dimensional figure a point in plane has two coordinates.

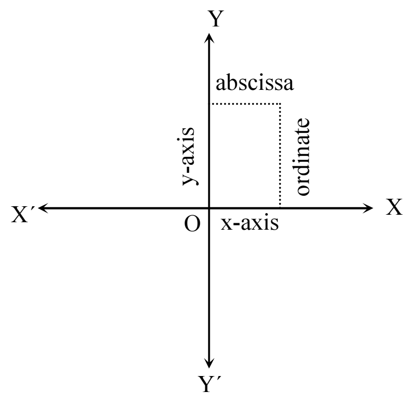
In a two-dimensional figure a point in plane has two coordinates. Generally, the first coordinate is read on the X′OX axis and the second coordinate on the Y′OY axis. Various methods of expressing these pairs of coordinates are:

(a) Varying alphabets *(x, y) (a, b)* etc.

(b) Varying subscripts (x_1, y_1) (x_2, y_2) etc.

(c) Varying dashes (x′′, y′′)

The diagrammatic presentation of the two coordinates is as follows:



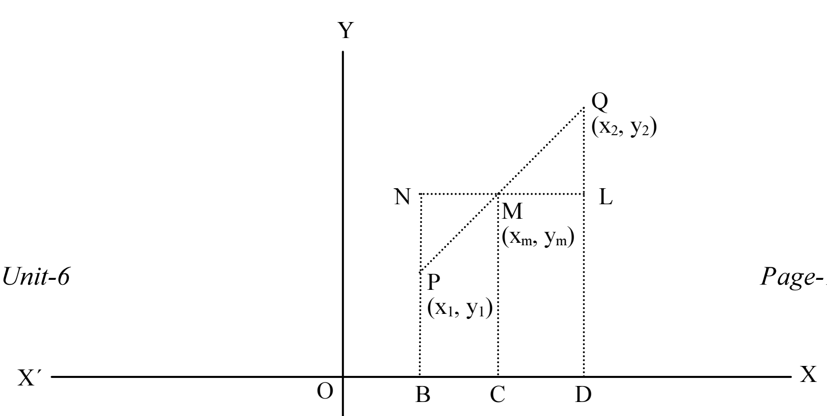
It is observed that the horizontal distance of the point from the Y′OY axis is called the x-coordinate or the abscissa and the vertical distance of the point from the X′OX axis is called the y-coordinate or the ordinate.

Coordinates of Mid-Points

We can find out the coordinates of a mid-point from the coordinates of any two points using the following formula:

$$X_m = \frac{x_1 + x_2}{2} \quad \text{and} \quad Y_m = \frac{y_1 + y_2}{2}$$

This is helpful first in finding out the middle point from a join of any two points and secondly in verifying whether two straight lines bisect each other.



In the above figure, the dotted vertical lines are drawn perpendicular to x-axis and the dotted horizontal lines are parallel to the x-axis. The ΔNMP and ΔQML are the congruent triangles. Therefore NM = ML.

Accordingly BC = CD

or, OC – OB = OD – OC

or, $(x_m - x_1) = (x_2 - x_m)$

$\therefore x_m = \dfrac{x_1 + x_2}{2}$ =........ (1)

Also from the same congruent triangles we set

NP = QL

or, NB – PB = QD – LD

or, MC – PB = QD – MC

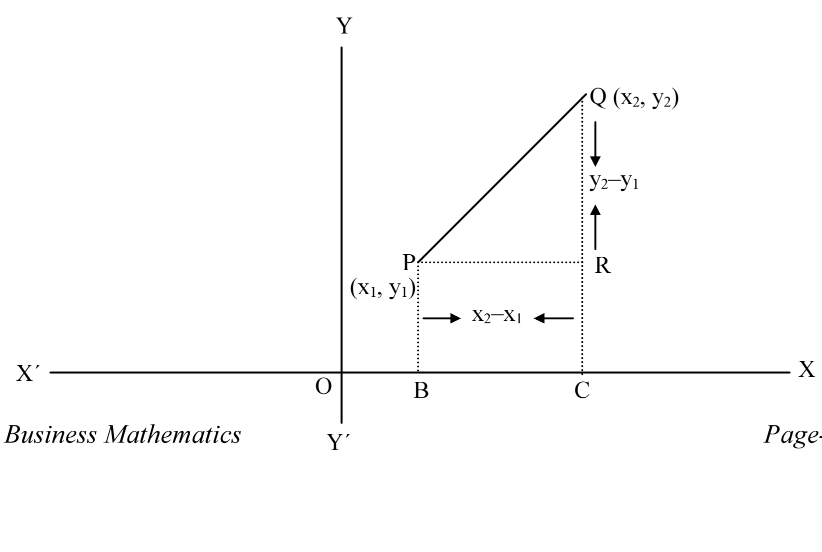
or, $y_m - y_1 = y_2 - y_m$

$\therefore y_m = \dfrac{y_1 + y_2}{2}$ =........ (2)

From (1) and (2), we conclude that the coordinates of the mid-point (x_m, y_m) are $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Distance between Two Points

Consider any two points P and Q with coordinates (x_1, y_1) and (x_2, y_2) respectively. By completing the right-angled triangle PRQ, we have the coordinates of R as (x_2, y_1)



Hence PR = $(x_2 - x_1)$ and QR = $(y_2 - y_1)$

Using Pythagoras theorem,

$PQ^2 = PR^2 + QR^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\Rightarrow$ PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}$

The general formula for the distance between any two given points P(x_1, y_1) and Q (x_2, y_2) is

PQ = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(\textit{difference of abscissa})^2 + (\textit{difference of ordinates})^2}$

Section Formula

The coordinates of a point R (x, y) dividing a line in the ratio of m : n connecting the points P (x_1, y_1) and Q (x_2, y_2). The coordinates of the point R using the following formula:

$$x = \frac{mx_2 + n\ x_1}{m+n} \text{ and } y = \frac{my_2 + n\ y_1}{m+n}$$

The following examples illustrate the model applications of coordinate geometry.

Example-1:

Find the coordinates of the mid-point of the straight line joining P (5, –4) and Q (–1,10)

Solution:

We know that the coordinates of the mid-point of the line joining two of points (x_1, y_1) and (x_2, y_2) are: $X_m = \frac{x_1 + x_2}{2}$ and $Y_m = \frac{y_1 + y_2}{2}$

$\therefore$ The coordinates of the mid-point of the straight line joining P (5, –4) and Q (–1, 10) are

$$X_m = \frac{5-1}{2} = 2 \text{ and } Y = \frac{-4+10}{2} = 3$$

i.e. the midpoint is M (2, 3)

Example-2:

Find the distance between the points (–2, 3) and (1, –3)

Solution:

Let (–2, 3) be denoted by (x_1, y_1) and (1, –3) be denoted by (x_2, y_2)

Therefore the required distance is, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{[1-(-2)]^2 + (-3-3)^2}$$

$$= \sqrt{9+36} = \sqrt{45} = 3\sqrt{5} \text{ units.}$$

Example-3:

Show that the points (6, 6) (2, 3) and (4, 7) are the vertices of a right-angled triangle.

Solution:

Let P, Q, R be the points (6, 6), (2, 3) and (4, 7) respectively, then

$$PQ^2 = [(6-2)^2 + (6-3)^2] = 16 + 9 = 25$$

$$QR^2 = [(2-4)^2 + (3-7)^2] = 4 + 16 = 20$$

$$RP^2 = [(4-6)^2 + (7-6)^2] = 4 + 1 = 5$$

$$\therefore PQ^2 = QR^2 + RP^2$$

$$\Rightarrow \angle PQR = 1 \text{ right angle.}$$

Hence the points P(6, 6), Q (2, 3) and R (4, 7) are the vertices of a right angled triangle.

Example – 4:

Prove that (–2, –1), (1, 0), (4, 3) and (1, 2) be the vertices of a parallelogram.

Solution:

Let P (–2, –1), Q (1, 0), R (4, 3) and S (1, 2) be the vertices of a quadrilateral.

Then the midpoint of PR = $\left(\frac{-2+4}{2}, \frac{-1+3}{2}\right)$ = (1, 1) ... (i)

and the midpoint of QS = $\left(\frac{1+1}{2}, \frac{0+2}{2}\right)$ = (1, 1) ... (ii)

From (i) and (ii), we conclude that PR and QS bisect each other at the same point (1,1) and hence the quadrilateral PQRS is a parallelogram.

Example – 5:

Determine the coordinates of the vertices of the triangle PQR if the middle points of its sides PQ, QR and RP have coordinates (2, 5), (–4, 3) and (4, –1) respectively.

Solution:

Let the coordinates of the point P, Q and R of the triangle PQR are (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively. Therefore, we have:

For coordinates of the

midpoint of PQ : $= \dfrac{x_1 + x_2}{2} = 2; \qquad \Rightarrow x_1 + x_2 = 4 \qquad \text{... (i)}$

$\dfrac{y_1 + y_2}{2} = 5; \qquad \Rightarrow y_1 + y_2 = 10 \qquad \text{... (ii)}$

For coordinates of the

midpoint of QR : $\dfrac{x_2 + x_3}{2} = -4 \qquad \Rightarrow x_2 + x_3 = -8 \qquad \text{... (iii)}$

$\dfrac{y_2 + y_3}{2} = 3; \qquad \Rightarrow y_2 + y_3 = 6 \qquad \text{... (iv)}$

For coordinates of

the midpoint of PR : $\dfrac{x_3 + x_1}{2} = 4; \qquad \Rightarrow x_3 + x_1 = 8 \qquad \text{... (v)}$

$\dfrac{y_3 + y_1}{2} = -1; \qquad \Rightarrow y_3 + y_1 = -2 \qquad \text{... (vi)}$

Adding (i), (iii) and (v), we have $x_1+x_2+x_2+x_3+x_3+x_1 = 4 - 8 + 8$

$\Rightarrow 2(x_1 + x_2 + x_3) = 4$

$\Rightarrow x_1 + x_2 + x_3 = 2 \qquad \text{...(vii)}$

Adding (ii), (iv) and (vi), we have $y_1+y_2+y_2+y_3+y_3+y_1 = 10 + 6 - 2$

$\Rightarrow 2\,(y_1+y_2+y_3) = 14$

$\therefore\ y_1+y_2+y_3 = 7 \qquad \text{... (viii)}$

Now, by (iii) and (vii), we have $x_1 = 10$

by (v) and (vii), we have $x_2 = -6$

by (i) and (vii), we have $x_3 = -2$

Again, by (iv) & (viii), we have $y_1 = 1$,

by (vi) & (viii), we have $y_2 = 9$

by (ii) & (viii), we have $y_3 = -3$

Therefore, the coordinates of the vertices of the triangle PQR are: P (10, 1), Q (–6, 9) and R (–2, –3)

Example – 6:

Find the co-ordinates of a point C dividing a line in the ratio of 7:3 connecting the points P (–2, 9) and Q (8, –1)

Solution:

Let the coordinates of the point C are x and y. Now by the formula

$$x = \frac{m\ x_2 + n\ x_1}{m+n} \text{ and } y = \frac{m\ y_2 + n\ y_1}{m+n}$$

we have, m = 7, n = 3, $x_1 = -2$, $x_2 = 8$, $y_1 = 9$ and $y_2 = -1$

$$\text{Hence } x = \frac{7x\ \ 8 + 3x\ \ (-2)}{7+3} = \frac{50}{10} = 5$$

$$\text{and } y = \frac{7x\ \ (-1) + 3x\ \ 9}{7+3} = \frac{20}{10} = 2$$

Therefore coordinates of the point C are (5, 2).

Questions for Review

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. Plot the points with the following coordinates
   P (–5, -4), Q (–4, 5), R (2, 4), S 1, –5)

2. Prove that the points (6.6), (2.3) and (4, 7) are the vertices of a right-angled triangle.

3. Determine the coordinates of the vertices of the triangle ABC if the middle points of its sides BC, CA, AB have coordinates (3, 2) (–1, –2) nad (5, –4) respectively.

4. If (–3, 2), (1, –2) and (5, 6) are the midpoints of the sides of a triangle, find the coordinates of the vertices of the triangle.

5. The points (3, 4), and (–2, 3) form with another point (x, y) an equilateral triangle. Find x and y.

6. Prove that the triangle with vertices at the points (0, 3) (–2, 1) and (–1, 4) is right angled.

Multiple Choice Questions (√ the most appropriate answer)

1. The distance between the points (2, –3) and (2, 2) is

   (a) 2 unit (b) 3 Unit (c) 4 Unit (d) 5 Unit

2. In which quadrant does (–4, 3) lie?

   (a) First quadrant (b) Second quadrant

   (c) Third quadrant (d) Fourth quadrant

3. The coordinates of a point situated on x-axis at a distance of 3 unit from y-axis is :

   (a) 0, 3) (b) 3, 0) (c) (3, 3) (d) (-3, 3)

4. The coordinates of a point below x-axis at a distance of 4 units from x-axis but lying on y-axis is

   (a) (0, 4), (b) (–4, 0) (c) (0, –4) (d) (4, –4)

5. The points A (0, 6), B (–5, 3), and C (3,1) are the vertices of a triangle which is

   (a) Isosceles (b) Right angled (c) Equilateral (d) None of these.

6. The area of a the triangle whose vertices are (3, 8) B (–4, 2) and C (5, –1) (in square units) is

   (a) 28 .5 (b) 37.5 (c) 75 (d) 57.

7. Which point of x-axis is equidistant from the points A (7, 6) and B (–3, 4)?

   (a) (0,4), (b) (3, 0), (c) (–4, 0), (d) (0, 3)

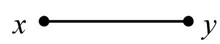
Lesson-2: The Straight Line

After studying this lesson, you should be able to:

- Discuss the nature of straight line;
- State the slope of a straight line;
- Highlights the different forms of equations of the straight line.

The Straight Line

If two points are given and if we join the points by a scale we get a straight line. Thus if x and y are two given points



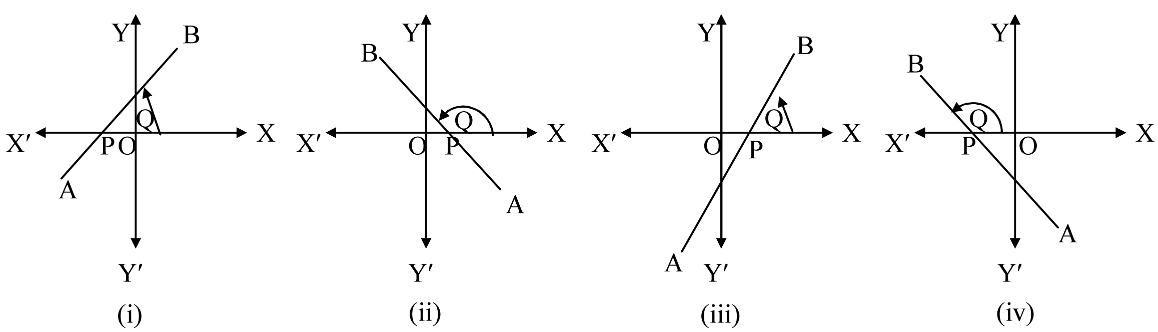
then *xy* is a straight line.

Straight line is the shortest distance between two distinct points.

Mathematically it is defined as the shortest distance between two distinct points. The study of curves starts with the straight line which is the simplest geometrical entity. Each point of a straight line have a slope. Hence let us discuss the slope of a straight line.

Slope of a Straight Line

The slope of the line is the tangent of the angle formed by the line above the x-axis towards its positive direction whatever be the position of the line as shown below:



Slope of a line is generally denoted by M. Thus if a line makes an angle Q with the positive direction of the x-axis, its slope is, M = tanQ.

If Q is acute (i & iii), slope is positive and if Q is obtuse (ii and iv), the slope is negative.

In terms of the co-ordinates, the slope of the line joining two points, say A (x_1, y_1) and B (x_2, y_2) is given by.

$$M = \tan Q = \frac{y_2 - y_1}{x_2 - x_1} = \frac{Difference\ \ of\ ordinates}{Difference\ \ of\ abscissae}$$

Different Forms of Equations of the Straight Line

1. ***Equations of the coordinate axes:*** All points on the x-axis, the value of y ordinates is always zero. Therefore y = 0 is the equation of x-axis. On the other hand, all points on the y-axis, the value of x ordinates is always zero. Hence x = 0 is the equation of y-axis.

2. ***Equations of lines parallel to the coordinate axes:*** Let P (x, y) be any point on a line parallel to x-axis at a distance *b* from it. Hence the equation of this line is y = b. Similarly x = a is the equation of the line parallel to the y-axis and at a distance *a* from it.

3. ***Origin-slope form:*** If the equation of a line passing through the origin and having the slop *m*, then the required equation of this line is, y = mx.

4. ***Slope-intercept form:*** If the equation of the line has the slope m with an intercept *c* on y-axis, then the required equation of this line is, y = mx + c.

5. ***Two-intercept form:*** Let a straight line intersect the coordinate axes making intercepts of *a* and *b* on x-axis and y-axis respectively, then the required equation of the line is;

   $$y = \frac{a}{b}x + b \Rightarrow \frac{x}{a} + \frac{y}{b} = 1$$

6. ***Slope-point form:*** The equation of a straight line having a slope *m* and passing through the point (x_1, y_1) is.

   $y - y_1 = m\ (x - x_1)$

7. ***Two-point form:*** If a straight line is passing through two points (x_1, y_1) and (x_2,y_2), then the equation of the straight line is,

   $$\frac{y - y_1}{x - x_1} = \frac{y_1 - y_2}{x_1 - x_2} \Rightarrow y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}\left(x - x_1\right)$$

Let us take some worked out examples on straight line.

Example-1:

Find the slope of the line joining points (0, 0) and (2, 3)

Solution:

The required slope = $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-0}{2-0} = \dfrac{3}{2}$

Example-2:

Find the slope of the line whose equation is 2x + 3y – 7 = 0

Solution:

The equation 2x + 3y –7 = 0 can be written as 3y = –2x + 7

$\therefore$ y = $-\frac{2}{3}$x + $\frac{7}{3}$.

Hence required slope is $-\frac{2}{3}$

Example-3:

Find the intercepts that the straight line 3x – 2y – 6 = 0 makes on the coordinate axes.

Solution:

Equation of the given line is

3x – 2y – 6 = 0

or, 3x – 2y = 6

or, $\frac{3x}{6} - \frac{2y}{6} = 1$

or, $\frac{x}{2} - \frac{y}{3} = 1$

or, $\frac{x}{2} + \frac{y}{-3} = 1$

Hence the intercepts made on the axes are 2 and –3.

Example-6.4:

What is the intercept that the straight line x – 2y + 4 = 0 makes on y-axis?

Solution:

The equation of the given line is

x – 2y + 4 = 0

or, 2y = x + 4

or, y = $\frac{1}{2}$x + 2

Therefore the required intercept on y-axis is 2.

Questions for Review

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. Find the slope of the straight line joining the points (2, 5) and (–1, 3)

2. Find the intercepts that the line $3x - 4y + 12 = 0$ makes on the axes. What is the slope of this line?

3. Find the slope of the line passing through the point (2, –1) and the origin.

4. Find the equation of the line through (1, –3) which is perpendicular to the line $x - 3y + 4 = 0$

5. Find the equation of the line making intercepts 2 and –3 on the x-axis and y-axis respectively.

6. Find the equation of the line through the origin and the point (–2, –6). What is the slope of the line?

Multiple Choice Questions (√ the appropriate answer)

1. The slope of the line joining A (–3, 5) and B (4, 2) is:

   (i) 3/7 (ii) 7/3 (iii) –3/7 (iv) –7/3

2. The equation of a line parallel to y-axis at a distance of 4 units to the right of y-axis is:

   (i) $x = 4$ (ii) $y = 4$ (iii) $x = 4y$ (iv) $y = 4x$

3. he slope of the line $2x + 3y + 5 = 0$ is:

   (i) 2 (ii) 3/2 (iii) -2/3 (iv) 3

4. he equation of a line with slope 4 and passing through the point, (5 –7) is:

   (i) $y = 4x - 35$ (ii) $4y = x - 35$ (iii) $y = 4x - 27$ (iv) $y = 4x + 27$

5. The equation of the line passing through the point (1, 1) and perpendicular to the line $3x + 4y - 5 = 0$, is:

(i) $3x + 4y - 7 = 0$ (ii) $3x + 4y + k = 0$

(iii) $4x - 3y + 1 = 0$ (iv) $4x - 3y - 1 = 0$

6. The equation of a line passing through the origin and parallel to the line $3x - 2y + 1 = 0$, is:

   (i) $3x - 2y = 0$ (ii) $2x - 3y = 0$

   (iii) $2x - 3y - 1 = 0$ (iv) None

7. The equation of a line passing through (5, 1) and parallel to the line $7x - 2y + 5 = 0$, is:

   (i) $2x - 7y + 33 = 0$ (ii) $7x - 2y + 33 = 0$

   (iii) $2x - 7y - 33 = 0$ (iv) None.

8. The equation of a line parallel to y-axis and passing through (3, –7) is:

   (i) $x = 3$ (ii) $y = -7$ (iii) $y = 7x$ (iv) $y = 3x$

9. The length of perpendicular from the point (4, 1) to the line $3x - 4y + 12 = 0$, is:

   (i) 4 units (ii) 4/3 units (iii) 3 units (iv) 1 units.

10. The length of perpendicular from the origin to the line $3x + 4y + 5 = 0$, is:

    (i) 5 units (ii) 4/3 units (iii) 5/3 units (iv) 1 units.

Lesson-3: General Form of the Equation of a Straight Line

After studying of this lesson, you should be able to:

- State the nature of the general equation of a straight line;
- Highlights on some model applications of the problems related to straight line.

General Form of the Equation of a Straight Line

An equation of the form ax + by + c = 0, is called the general equation of the straight line

An equation of the form ax + by + c = 0, where a, b, c are constants and x, y are variables, is called the general equation of the straight line. Thus 2x + 3y + 7 = 0, 5x – y + 1 = 0, 3x + 2y = 0 are the equations of different straight lines in general form.

The equation ax + by + c = 0 can be written in slope intercept form as

by = –ax – c

or, $y = \frac{a}{b}x - \frac{c}{b}$, where $b \neq 0$

or, y = mx + k, where $m = -\frac{a}{b}$ and $k = -\frac{c}{b}$

i.e. slope of the line = $m = -\frac{a}{b} = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$

Here m is the slope of the line whose equation is $y = mx + k$, where k is the intercept of the line on y-axis.

The equation $ax + by + c = 0$ can also be written in two intercept from as follows:

ax + by = – c

or, $\frac{ax}{-c} + \frac{by}{-c} = 1,\ c \neq 0$

or, $\frac{x}{\frac{-c}{a}} + \frac{y}{\frac{-c}{b}} = 1$

or, $\frac{x}{a} + \frac{y}{b} = 1$ where $A = -\frac{c}{a}$ and $B = -\frac{c}{b}$

Here A and B are known as intercepts on the x-axis and y-axis respectively.

If a line passes through a point (x_1, y_1), its equation is $y - y_1 = m(x - x_1)$, where m is the slope of the line.

If a straight line passes through two points (x_1, y_1) and (x_2, y_2), its equation is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

The slope of this line is $\frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{\text{difference of } y \text{ - coordinates of the points}}{\text{difference of } x \text{ - coordinates of the points}}$$

When two points are given, we can find the slope of the line joining the points.

Thus when two points are given, we can find the slope of the line joining the points. For example, the slope of the line joining the points A (2, 3) and B (5, 4) is $\frac{4-3}{5-2} = \frac{1}{3}$

The following examples illustrate the coordinate geometry and the uses of straight line.

Example-1:

Find the equation of the straight line passing through the points Q (1, 2) and R (3, 7)

Solution:

Let a point P (x, y) lie on the same straight line.

Gradient of PQ = gradient of QR

$$\frac{y-2}{x-1} = \frac{7-2}{3-1}$$

$$\text{or, } \frac{y-2}{x-1} = \frac{5}{2}$$

$$\text{or, } 2y - 4 = 5x - 5$$

$$\text{or, } 2y = 5x - 1$$

$$\text{or, } y = \frac{5x-1}{2}$$

Therefore the equation of the straight line is, $y = \frac{5x}{2} - \frac{1}{2}$

Example-2:

A printer quotes a price of Tk.7,500 for printing 1,000 copies of a book and Tk.15,000 for printing 2,500 copies. Assuming a linear relationship and that 2,000 books are printed. Required:

(a) Find the equation relating to the total cost (y) and the number of books (x) printed.

(b) What is the variable cost of printing 2000 books?

(c) What is the fixed cost?

(d) What is the variable cost per book?

(e) What is the average cost per book to print 2000 books?

(f) What is the marginal cost of the last book printed?

Solution:

(a) Let x coordinate represents number of books and y coordinate represents the cost of printing.

Then the linear relationship between the number of books and cost of printing is the equation of the straight line passing through the points (1,000, 7,500) and (2,500, 15,000).

The equation of a straight line passing thought the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1) \qquad ... (1)$$

Substituting the values of (x_1, y_1) and (x_2, y_2) in equation (1) we get

$$y - 75000 = \frac{75000 - 15000}{1000 - 2500}(x - 1{,}000)$$

$$\text{or, } y - 7{,}500 = \frac{-7500 - 15000}{-1500 - 2500}(x - 1000)$$

or, $y - 7500 = 59x - 1{,}000)$

or, $y - 7500 = 5x - 5000$

or, $y = 5x - 5000 + 7500$

or, $y = 5x + 2500$ ... (2)

The equation no. (2) represents the linear relationship between the number of books printed and cost of printing.

When 2000 books are printed, the cost of printing is given by, y = 5x + 2500.

= 5 (2000) + 2500 = 10,000 + 2500 = Tk.12,500

$\Rightarrow$ Total cost of printing 2000 books is Tk.12,500

(a) Comparing equation no. (2) with the slope-intercept form (y = m x +

(b) we have the variable cost given by, = (5×2000) = Tk.10,000

(c) The fixed cost is, C = 2500

(d) The variable cost per book is = $\frac{10000}{2000}$ = Tk.5

(e) Average cost per book is = $\frac{12500}{2000}$ = Tk.6.25

(f) Marginal cost = (Total cost of 2000 books – Total cost of 1999 books)

= 5(2000) + 2500 – [5(1999) + 2500]

= (12500 – 12495) = Tk.5

Example-3:

If the total manufacturing cost (y) of producing x units of a product is Tk.5,000 at 200 units output and Tk.7,250 at 300 units output and the cost-output relation is linear, then

(a) What is the equation of cost-output relationship in general form?

(b) What is the slope of the cost-output line?

(c) How much does the production of one unit add to total cost?

Solution:

We are given, the cost-output relation is linear and the information given consists of 2 points whose coordinates (x, y) are in the order (units made, total cost). These two points are: (200, 5000) and (300, 7250).

We know that the two-point form of the equation of a straight line is:

$$y - y1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow\ y - 5000 = \frac{7250 - 5000}{300 - 200}(x - 200)$$

$$\Rightarrow\ y - 5000 = \frac{45}{2}(x - 200)$$

$$\Rightarrow 2y - 10{,}000 = 45x - 9{,}000$$

$$\Rightarrow\ -45x + 2y - 1{,}000 = 0$$

(a) Therefore, the equation of the cost-output relationship in general form is:
$-45x + 2y - 1000 = 0$

(b) The slope of the cost-output line is, $m = -\frac{\text{Coefficient of } x}{\text{Coefficient of } y}$

$$= -\frac{(-45)}{2} \qquad = \frac{45}{2} = 22.50$$

(c) Cost (y) of producing x units can be calculated by the cost-output equation as follows: $-45x + 2x - 1000 = 0$

$$\Rightarrow 2y = 45x + 1000$$

$$\Rightarrow y = \frac{45}{2}x + 500$$

And, the cost of producing (x+1) units is, $y = \frac{45}{2}(x+1) + 500$

$$\Rightarrow y = \frac{45}{2}x + \frac{1045}{2}$$

$\Rightarrow$ The cost of production of one unit that adds to the total cost is:

$$\Rightarrow \left[\frac{45}{2}x + \frac{1045}{2}\right] - \left[\frac{45}{2}x + 500\right]$$

$$= \left(\frac{1045}{2} - 500\right) = \frac{45}{2} = \text{Tk.}22.5$$

Example-4:

The total expenses (y) of a mess are partly constant and partly proportional to the number of the inmates (x) of the mess. The total expenses are Tk.1040 when there are 12 members in the mess, and Tk.1600 for 20 members.

(i) Find the linear relationship between y and x

(ii) Find the constant expenses and the variable expenses per member, and

(iii) What would be the total expenditure if the mess has 15 members?

Solution:

Let x coordinate represents the number of inmates of the mess and y coordinate represents the total expenses.

The given two points are (12, 1040) and (20, 1600). The equation of the straight line passing through the points (x_1, y_1) and (x_2, y_2) is given by

$$y - y_1 = \frac{y_1 - y_2}{x_1 - x_2}(x - x_1)$$

$$\Rightarrow y - 1040 = \frac{1040 - 1000}{12 - 20}(x - 12)$$

$$\Rightarrow y - 1040 = \frac{-560}{-8}(x - 12)$$

$\Rightarrow$ y – 1040 = 70 (x - 12)

$\Rightarrow$ y = 70x – 840 + 1040 = 70x + 200 ... (1)

$\Rightarrow$ y = 70x + 200 which is the required relationship between x and y.

(ii) Comparing the equation (i) with slope-intercept form (y = mx + c) we find, the constant expenses (c) = Tk.200 and variable expenses per member (m) = 70

(iii) When the number of members in the mess is 15, the total expenses, y = (70 x 15 + 2000) = Tk.1250.

Example-5:

A firm invested Tk.10 Million in a new factory that has a net return of Tk.5,00,000 per year. An investment of Tk.20 million would yield a net income of Tk.2 million per year. What is the linear relationship between investment and annual income? What would be the annual return on an investment of Tk.15 million.

Solution:

Let x coordinate represents the investment and y coordinate represents the annual income. As the relationship between x and y is linear, we have to find the equation of line through (1,00,00,000; 500000) and (2,00,00,000; 20,00,000)

$\therefore$ The required relationship is $y - y_1 = \dfrac{y_1 - y_2}{x_1 - x_2}(x - x_1)$

$$\text{or,}\quad y - 5{,}00{,}000 = \frac{500000 - 2000000}{10000000 - 20000000}(x - 1{,}00{,}00{,}000)$$

$$\text{or,}\quad y - 5{,}00{,}000 = \frac{-1500000}{-10000000}(x - 1{,}00{,}00{,}000)$$

or, 20y – 1,00,00,000 = 3x – 300,00,000

or, 3x – 20y – 20000000 = 0

Again when investment x = 150,00,000, the annual income (y) can be found by putting the value of x in the equation obtained; i.e. 3 (150,00,000) – 20y – 20000000 = 0

or, –20y = –450,00,000 + 200,00,000
or, –20y = –250,00,000
or, y = 12,50,000.

Questions for Review

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. The total cost of y for x units of a certain product consists of fixed cost and the variable cost. It is know that the total cost is Tk.1,200 for 100 units and Tk.2,700 for 400 units.
   (i) Find the linear relationship between x and y.
   (ii) Find the slope of the line and what does it indicate?
   (iii) If the selling price is Tk.7 per unit, find the number of units that must be produced so that there will be neither profit nor a loss.

2. Find the equation of a straight line passing through the point of intersection of the lines x – 2y + 3 = 0, 2x – 3y + 4 = 0 and parallel to the line joining the points (1,1) and (0, –1).

3. The total cost y, for x units of a certain product, consists of fixed cost and variable cost (proportional to the number of units produced). It is known that the total cost is Tk.6000 for 500 units and Tk.9000 for 1000 units.

   Based on the above information, you are required to find out.
   (i) The linear relationship between x and y;
   (ii) The slope of the line, what does it indicate?

(iii) The number of units that must be produced so that

(a) There is neither profit nor a loss

(b) There is a profit of Tk.1,000.

(c) There is a loss of Tk.300; it is given that the selling price is Tk.8 per unit.

4. A firm invests Tk.10,000 in a business which has a net return of Tk.500 per year. An investment of Tk.20,000 would yield an income of Tk.2,000 per year. What is the linear relationship between investment and annual income? What would be the annual return on an investment of Tk.12,000?

5. If the total manufacturing cost (y) of producing x units of a product is Tk.5,000 at 200 units output and the Tk.7,250 at 300 units output and the cost-output relation is linear, then

(a) What is the equation of cost- output relationship in general form?

(b) What is the slope of the cost-output line?

(c) How much does the production of one unit add to the total cost?