6

web¨v‡mi i¦c I AvKwZ.
Forms and Shapes of a Distribution

cwimsL¨vb c×wZi gva¨‡g DcvË iwkgvjvi †K›`xfz- nevi ceYZv I we¯wZi gvÎv cwigv‡ci gva¨‡g GKwU web¨v‡mi Ašfy³ gvb¸‡jvi g‡a¨ mgi¦cZv I wfbZvi wPÎwU †evSv hvq| wKš web¨vmwUi i¦c I AvKwZ. m¤ú‡K ¯úó †Kvb avivYv cvIqv hvq bv| GKwU MYmsL¨v web¨vm m¤ú‡K eYbv †`evi Rb¨ †K›`xq ceYZv I we¯wZi cwigvc¸‡jv h‡_ó bq| `ÿwU web¨v‡mi GKB Mo I cwiwgZ e¨eavb _vKv m‡Ë I web¨vm `ÿwUi i¦c I AvKwZ. wfb n‡Z cv‡i| A_vr, web¨vmwU mÿgfv‡e web¨¯—, bwK Wvb w`‡K ew¼g, bwK evg w`‡K ew¼g, Zv Rvbv hvq bv| GKwU MYmsL¨v web¨vm ew¼g bv n‡jB †h mÿg n‡e GgbwU wbwðZ K‡i ejv hvq bv| A‡bK mgq GKwU web¨vm AwZmPvjv ev AbwZmPvjv n‡Z cv‡i| AwfÁZvjä web¨vm¸‡jv ¯'vfweK web¨v‡mi KZUkÿymwbKUeZx n‡q‡Q Zv Rvb‡Z cvi‡j bgybv gvb¸‡jvi wfwË‡Z civgvb‡K cv°j‡bi †¶‡Î Avgv‡`i Av¯'v A‡bK †ekx ewo‡q †`q| Dciš, A‡bK mgq †`Lv hvq †h, †K›`xq ceYZv, we¯wZ, ew¼gZv I mPvjZvi cwigvc¸‡jv‡K HK¨e× K‡i GKwU mÿeavRbK cwigvc wbY‡qi c‡qvRb nq| cwiNvZ Avgv‡`i †m iKg GKwU cwigvc c`vb K‡i|

GB BDwb‡U Avgiv †h cvV¸‡jv Aa¨qb Ki‡ev †m¸‡jv n‡jv:

- cvV - 1 : ew¼gZv
- cvV - 2 : cwiNvZ
- cvV - 3 : mPvjZv
- cvV - 4 : ¯'vfweK web¨vm

cvV- 1

e¨¹gZv
Skewness

GB cvV †k‡l hv Rvb‡Z cvi‡eb —

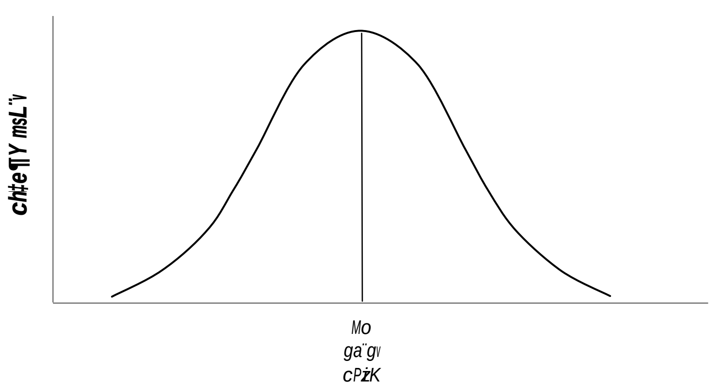
- e¨¹gZv wK
- e¨¹gZvi cwigvc – wbiƒcY I Av‡cw¶K
- Kvj© wcqvim‡bi e¨¹gZvi mnM
- evD‡jii e¨¹gZvi mnM
- ‡Kwji e¨¹gZvi mnM

e¨¹gZv wK (What is Skewness)?

GKwU DcvËiwki wbeb¨vm‡K eYbv Ki‡Z wM‡q Avgiv DcvËiwki †K›`ªxq cewYZvi K_v †R‡bwQ| †K›`ªxq cewYZvi cwigvc¸‡jvi gva¨‡g Avgiv eyS‡Z cwii †h, GKwU wbeb¨v‡mi DcvË iwkgvjv GKwU wbw`ó gv‡bi Pvicv‡k KZUzKz †K›`ªxfzZ i‡q‡Q| we¯—wZi cwigvc¸‡jvi gva¨‡g Avgiv Rvb‡Z cwii DcvËiwk KZUzKz we¯—Z. i‡q‡Q Ges DcvËiwki AvKvi GKwU †_‡K Av‡iKwU KZUzKz wfb| wKš' GKwU wbeb¨v‡mi i|c I AvKwZ. m¤ú‡K Avgiv GLbI wKQyB Rvwb bv| KviY, GKwU MYmsL¨v wbeb¨vm m¤ú‡K eYbv †`evi Rb¨ †K›`ªxq cewYZv I we¯—wZi cwigvc¸‡jv h‡_ó bq| DcvË iwkgvjv wKfv‡e M‡oi Pvicv‡k wb¨¯— i‡q‡Q A_ev M‡oi Dc‡i †ekx DcvËiwk i‡q‡Q, bwK bx‡P †ekx i‡q‡Q Zv Rvbvi c‡qvRb i‡q‡Q| `ywU wbeb¨v‡mi GKB Mo I cwigvZ e¨eavb _vKv m‡Ë¡I wbeb¨vm `ywUi i|c I AvKwZ. wfb n‡Z cv‡i| DcvËiwki eYbvi Rb¨ wbeb¨v‡mi Av‡iv ˆewkó¨ m¤ú‡K Avgv‡`i aviYv †bevi c‡qvRb i‡q‡Q| Avi GB aviYv ‡bevi Rb¨ Avgiv †h cZ¨qwU wb‡q Av‡jvPbv Ki‡ev Zv n‡jv e¨¹gZv|

DcvË iwki gvb¸‡jv M‡oi Pvicv‡k mylgfv‡e wb¨¯— i‡q‡Q wKbv e¨¹gZv †mwU‡KB eYbv K‡i|

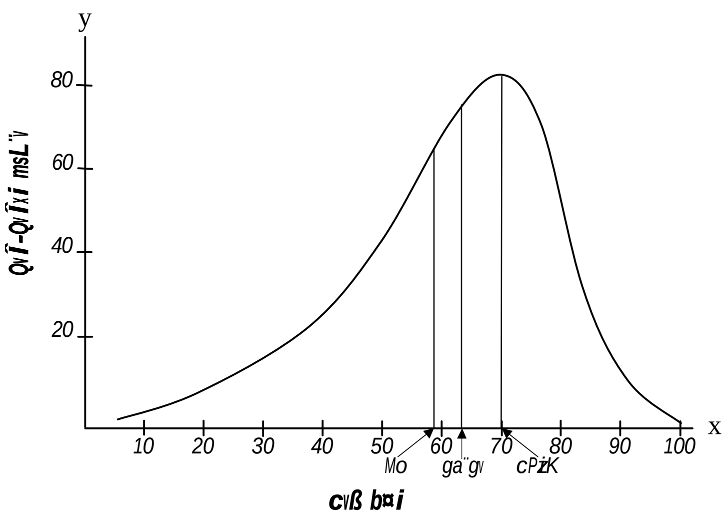
e¨¹gZv ej‡Z GKwU wbeb¨v‡mi Amgi|cZv‡K (asymmetry) †evSvq| A_vr, GKwU wbeb¨v‡mi g‡a¨ mylgZvi Afve _vK‡j †mB wbeb¨vm‡K e¨¹g wbeb¨vm e‡j| Ab¨ K_vq, DcvËiwki gvb¸‡jv M‡oi Pvicv‡k mylgfv‡e wb¨¯— i‡q‡Q wKbv e¨¹gZv †mwU‡KB eYbv K‡i|



wPÎ 6.1.1: mylg MYmsL¨v †iLv

GKwU D`vniY †`qv hvK| aiv hvK, GKwU ebjr msL¨vi DcvËiwki MwYwZK Mo, ga¨gv Ges cPzK-Gi gvb GKB n‡q‡Q| cvß web¨vmwU‡K GKwU MYmsL¨v ‡iLvi (frequency curve) g‡a¨ Dc¯vcb Ki‡j †`Lv hv‡e †h, †iLwPÎwU web¨vmwU‡K m‡ev”P we›`yZ wVK mgvb `yfv‡M fvM K‡i‡Q (wPÎ 6.1.1 `óe¨)| A_vr, cPz‡Ki Dc‡i Ges bx‡P mgvb msL¨K DcvËiwk we`¨gvb| ïayZvB bq, ga¨gv I MwYwZK M‡oi Dc‡i Ges bx‡PI mgvb msL¨K msL¨v gvb cvIqv hv‡e| Zvi A_ n‡jv, DcvËiwki gvb¸‡jv cPzK, ga¨gv I MwYwZK M‡oi `yÕcv‡k mylgfv‡e web¨¯ i‡q‡Q|

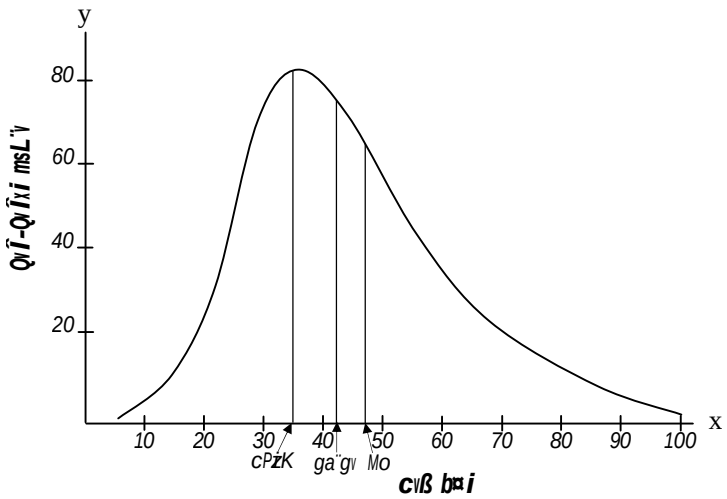
wPÎ 6.1.2-G GKwU cix¶vq QvÎ-QvÎx‡`i cvß b¤‡ii web¨vm‡K †jLwP‡Îi gva¨‡g Dc¯vcb Kiv n‡q‡Q| GLv‡b j¶¨ Kiv hv‡”Q †h, QvÎ-QvÎx‡`i msL¨v Ges cix¶vq cvß b¤i µgvMZ e„× †c‡q‡Q hZ¶Y bv b¤‡ii gvbwU 70-G wM‡q †cŠu‡Q‡Q| Gici, 70-Gi †ekx b¤i cvIqv QvÎ-QvÎxi msL¨v A‡bK Kg e‡j †iLwU `ªZZvi mv‡_ wbgMvgx n‡q‡Q| j¶¨Yxq welqwU n‡jv †h, hw` I me‡P‡q ‡ekx msL¨K QvÎ-QvÎx 70 b¤i †c‡q‡Q, wKš' 70-Gi †P‡q Kg b¤i cvß QvÎ-QvÎxi msL¨v 70-Gi †P‡q †ekx b¤i cvIqv QvÎ-QvÎx‡`i Zzjbvq A‡bK †ekx| Ab¨ K_vq, 70 msL¨v gvbwU n‡jv cPzK Ges ga¨gv I Mo gvbwU Gi bx‡P Ae¯vb Ki‡Q| A_vr, cix¶vq cvß b¤‡ii GB web¨vmwU GKwU e¤¼g web¨v‡mi iƒc wb‡q‡Q|



wPÎ 6.1.2: e¤¼g MYmsL¨v †iLv

e¤¼gZvi A¯‡jPbvq `yU cavb w`‡eP¨ welq i‡q‡Q| c_gZt, e¤¼gZvi w`K (direction)| A_vr, web¨v‡mi †iLwU wK Wvbw`‡K bvwK evg w`‡K †ehK i‡q‡Q †m welqwU Rvbv ¸iæZc~Y©| wØZxqZt, e¤¼gZvi gvÎv (degree)| GB `yU welq m¤úª‡K ¯úó aviYv †c‡Z n‡j Gi cwigvc¸‡jv m¤ú‡K Avgv‡`i Rvb‡Z n‡e| cwigv‡ci Av‡jvPbvq hvevi c‡e e¤¼gZvi iƒc m¤ú‡K Av‡jvPbwU †kl Kiv hvK| wPÎ 6.1.2-G Avgiv †`‡LwQ †h, web¨vmwU evgw`‡K †ehK i‡q‡Q| mybw`ófv‡e ejv hvq †h, QvÎ-QvÎx‡`i cix¶vq cvß b¤‡ii web¨vmwU FYvZ¥Kfv‡e e¤¼g| wKš' hw` †`Lv hvq †h, Ab¨ GKwU cix¶vq AwaK msL¨K QvÎ-QvÎx 40 b¤i †c‡q‡Q Ges 40 b¤‡ii †P‡q †ekx b¤i cvIqv QvÎ-QvÎxi msL¨v 40-Gi †P‡q Kg b¤i cvIqv QvÎ-QvÎx‡`i Zzjbvq A‡bK †ekx i‡q‡Q, Zvn‡j †evSv hv‡e †h, web¨vmwU abvZ¥Kfv‡e e¤¼g n‡q‡Q| cvß web¨vmwU †jLwP‡Îi gva¨‡g Dc¯vcb Ki‡j ‡h iƒc †b‡e, Zv wPÎ 6.1.3-G Dc¯vcb Kiv n‡jv|

e¤¼gZvi Av‡jvPbvq `yU cavb we‡eP¨ welq i‡q‡Q| c_gZt e¤¼gZvi w`K| wØZxqZt, e¤¼gZvi gvÎv|



wPÎ 6.1.3: abvZ¥Kfv‡e e¨¼g web¨vm

G chš Avgv‡`i Av‡jvPbvq wZbwU welq ¯úó n‡q D‡V‡Q| c_gZt, GKwU mylg web¨v‡m cPzK, ga¨gv I MwYwZK M‡oi gvb GKB n‡e| wØZxqZt, GKwU abvZ¥Kfv‡e e¨¼g web¨v‡m MwYwZK Mo cPy‡Ki †P‡q eo nq Ges ga¨gv Mo I cPz‡Ki gvSLv‡b †Kvb GK ¯v‡b Ae¯vb K‡i| abvZ¥Kfv‡e e¨¼g web¨v‡m wKQygvb _v‡K hv AwaKvsk gv‡bi Zzjbvq A‡bK eo nq| ZZxqZt, GKwU FYvZ¥Kfv‡e e¨¼g web¨v‡m cPzK MwYwZK M‡oi †P‡q eo nq Ges ga¨gv MwYwZK Mo I cPz‡Ki g‡a¨ †Kvb GK ¯v‡b Ae¯vb K‡i| MwYwZK Mo wbg gvb m¤úb msL¨vi w`‡K Ae¯vb K‡i| GKwU FYvZ¥Kfv‡e e¨¼g web¨v‡m wKQygvb _v‡K †h¸‡jv AwaKvsk gv‡bi †P‡q †QvU _v‡K| GKwU gvSwi ai‡Yi e¨¼g web¨v‡m MwYwZK Mo Ges ga¨gvi g‡a¨ cv_K¨wU n‡jv cPzK I MwYwZK M‡oi g‡a¨Kvi cv_‡K¨i GK-ZZxqvsk| cPzK, ga¨gv I MwYwZK M‡oi GB m¤úKwU e¨¼gZvi gvÎv cwigv‡ci wfwË c`vb K‡i _v‡K|

e¨¼gZvi cwigvc (Measures of Skewness)

MwYwZK Mo †_‡K cPz‡Ki cv_K¨B n‡jv e¨¼gZvi gvÎv|

we¯wZi cwigv‡ci gZ e¨¼gZvi cwigv‡ci I wbi¼k I Av‡cw¶K cwigvc i‡q‡Q| †h‡nZz GKwU mylg web¨v‡m MwYwZK Mo, ga¨gv I cPz‡Ki gvb GKB nq, †m‡nZz MwYwZK Mo cPzK †_‡K hZ`‡i Ae¯vb Ki‡e web¨vmwU ZZ ‡ekx e¨¼g n‡e| A_vr, MwYwZK Mo †_‡K cPz‡Ki cv_K¨B n‡jv e¨¼gZvi gvÎv| AZGe,

e¨¼gZv = MwYwZK Mo – cPzK|

A_vr, $s_k = \bar{x} - m_o$

hw` web¨vmwU evg w`‡K e¨¼g nq Z‡e MwYwZK M‡oi gvb cPz‡Ki gv‡bi †_‡K †QvU n‡e| MwYwZK Mo †_‡K cPz‡Ki cv_K¨mPK gvbwU FYvZ¥K n‡e| Avi hw` web¨vmwU Wvbw`‡K e¨¼g nq Z‡e MwYwZK M‡oi gvb cPz‡Ki gv‡bi †_‡K eo n‡e Ges cvß cv_‡K¨i gvbwU abvZ¥K n‡e|

e¨¼gZvi gvÎv wbi¡c‡Yi Av‡iKwU wbi¼k cwigvc GB avivYvi Dci cwZwôZ ‡h, GKwU e¨¼g web¨v‡m ga¨gv `ywU PZz_‡Ki wVK gvSLv‡b Ae¯vb K‡i bv| GB cwigvcwU‡K e¨¼gZvi PZz_K cwigvc e‡j| mÎwU n‡jv,

$$\text{ew¼gZv} = \text{PZz©Kmg‡ni mgwó} - \text{ga¨gvi wØ¸Y}$$

$$\text{A\_vr,} \quad s_k = (Q_3 + Q_1) - 2(Md)$$

kZmwiK I `kmwiK-Gi Dci wfwË K‡i ew¼gZvi gvÎv wbiƒc‡Yi Av‡iKwU cwigvc i‡q‡Q| kZmwi‡Ki Dci wfwË K‡i m~ÎwU n‡jv,

$$\text{ew¼gZv} = P_{90} - 2P_{50} + P_{10} \text{ (kZmwi‡Ki Dci wfwË K‡i)}$$

`kmwi‡Ki Dci wfwË K‡i m~ÎwU n‡jv,

$$\text{ew¼gZv} = P_9 - 2\,Md + D_1 \text{ (`kmwi‡Ki Dci wfwË K‡i)}$$

†h wZbwU wbi¼k cwigv‡ci D‡jL Kiv n‡jv †m¸‡jv †_‡K cvß djvdj¸‡jv GKwUi †_‡K Av‡iKwU wfbœ n‡e| KviY, †m¸‡jv wewfbœ bxwZgvjvi wfwË‡Z wbwgZ| Gi d‡j, Zzjbvi †¶‡Î mgm¨vi m„wó nq| Zv QvovI, ew¼gZvi gvÎvi GKB cwigvc ¯^í wfbœZv m¤úbœ web¨v‡mi †¶‡Î †h A_ enb Ki‡e †ekx wfbœZv m¤úbœ web¨v‡mi †¶‡Î †mB GKB A_ enb Ki‡e bv| Kv‡RB `yB ev Z‡ZvwaK web¨v‡mi g‡a¨ h\_vh\_ Zzjbvi j‡¶¨ wfbœZvi mgm¨v m„wóKvix cfveK‡K (disturbing influence of variation) Avgv‡`i AcmviZ Ki‡Z n‡e| Avi Zv Ki‡Z n‡j Avgv‡`i ew¼gZvi Av‡cw¶K cwigvc wbY©q Ki‡Z n‡e| ew¼gZvi †h wZbwU bxwZgvjvi wfwË‡Z wbi¼k cwigvc¸‡jv D‡jL Kiv n‡q‡Q †mB wZbwU bxwZgvjvi wfwË‡Z Avgiv Av‡cw¶K cwigvc¸‡jv Av‡jvPbv Ki‡ev| †m¸‡jv n‡jv,

1. Kvj© wcqvim‡bi ew¼gZvi mnM (Karl Pearson's Coefficient of Skewness)
2. evDwji ew¼gZvi mnM (Bowley's Coefficient of Skewness)
3. ‡Kwji ew¼gZvi mnM (Kelly's Coefficient of Skewness)

Kvj© wcqvim‡bi ew¼gZvi mnM (Karl Pearson's Coefficient of Skewness):

GB cwigvcwU ew¼gZvi mnM wbY©‡qi Rb¨ me‡P‡q †ekx e¨eüZ n‡q _v‡K| wcqvimb c`Ë ew¼gZvi mnM wbY©‡qi c×wZwU cPziK wbY©‡qi AwfÁZvj× m¤ú‡Ki wfwË‡Z wbwgZ n‡q‡Q| wbi¼k cwigv‡ci †¶‡Î Avgiv †`‡LwQ †h, ew¼gZvi w`KwU (direction of skewness) wbavwiZ nq MwYwZK Mo †\_‡K cPzi‡Ki cv\_©‡K¨i gvÎvi Dci| hw` MwYwZK M‡oi gvb cPzi‡Ki †P‡q eo nq Z‡e web¨vmwU abvZ¥Kfv‡e ew¼g n‡e, Avi hw` †QvU nq Z‡e web¨vmwU FYvZ¥Kfv‡e ew¼g n‡e| Av‡cw¶K cwigv‡ci †¶‡Î ew¼gZvi gvÎvwU cwiwgZ GK‡K (standard units) cKvwkZ nq| KviY, Av‡cw¶K cwigv‡ci gvb-m~ÎwU n‡jv cwigvc¸‡jv‡K GKK ¯^vaxb (independent of measurement unit) wnmv‡e cKvk Kiv|

wcqvim‡bi ew¼gZvi mnM wbY©‡qi m~ÎwU n‡jv,

$$Sk_p = \frac{\bar{x} - m_0}{\sigma}$$

‡hLv‡b, Sk_p = wcqvim‡bi ew¼gZvi mnM

$\bar{x}$ = MwYwZK Mo

m_o = cPziK

σ = cwiwgZ e¨eavb

wcqvimb c`Ë ew¼gZvi mnM wbY©‡qi c×wZwU cPziK wbY©‡qi AwfÁZvj× m¤ú‡Ki wfwË‡Z wbwgZ n‡q‡Q|

wKš hLb e¨¼gZvi gvÎv Lye †ewk nq, ZLb †h m¤ú‡Ki Dci wfwË K‡i wcqvim‡bi e¨¼gZvi mnMwU wbgZ nq †mB m¤úKwU †f‡½ hvq Ges wcqvim‡bi mnMwU Avi wbfi‡hvM¨ djvdj †`q bv| hw` cPzKwU h_vh_fv‡e msÁvwqZ bv nq, Z‡e m~wU ms‡kvab Kivi c‡qvRb c‡o| †m †¶‡Î wb‡gv³ m~wU e¨envi Kiv n‡q _v‡K:

$$Sk_P = \frac{3\left(\text{Mean} - \text{Median}\right)}{s}$$

mviwY 6.1.1-G c`Ë DcvË e¨envi K‡i wcqvim‡bi e¨¼gZvi mnM wbYq K‡i †`Lv‡bv n‡jv|

mviwY 6.1.1: 100 Rb Qv‡Îi gwmK Li‡Pi wfwË‡Z wcqvim‡bi e¨¼gZvi mnM wbYq

gwmK LiP (UvKvq)	f_i	x_i	$d = \frac{x_i - A}{C.I.}$	d^2	fd	fd^2
1000 - 1500	4	1250	–3	9	–12	36
1500 - 2000	13	1750	–2	4	–26	52
2000 - 2500	19	2250	–1	1	–19	19
2500 - 3000	32	2750 = A	0	0	0	0
3000 - 3500	20	3250	1	1	20	20
3500 - 4000	8	3750	2	4	16	32
4000 - 4500	3	4250	3	9	9	27
4500 - 5000	1	4750	4	16	4	16
‡gvU	**100**				**–8**	**202**

e¨¼gZvi mnM wbY‡qi c‡e Avgv‡`i MwYwZK Mo, cPzK Ges cwiwgZ e¨eavb wbYq K‡i wb‡Z n‡e|

$$\bar{x} = A + \frac{\sum fd}{N} \times C.I.$$

$$= 2750 + \frac{-8}{100} \times 500$$

$$= 2750 - 0.08 \times 500$$

$$= 2750 - 40$$

$$= 2710$$

∴ wbYxZ MwYwZK Mo n‡jv 2710

$$m_o = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times C.I.$$

$$= 2500 + \frac{13}{13 + 12} \times 500$$

$$= 2500 + \frac{13}{25} \times 500$$

$= 2500 + 0.52 \times 500$

$= 2500 + 260$

$= 2760$

∴ wbYxZ cPźK n‡jv 2760

$$s = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \times C.I.$$

$$= \sqrt{\frac{202}{100} - \left(\frac{-8}{100}\right)} \times 500$$

$$= \sqrt{2.02 - 0.08} \times 500$$

$$= \sqrt{1.94} \times 500$$

$$= 1.39 \times 500$$

$$= 695$$

∴ wbYxZ cwigZ e¨eavb n‡jv 695

†h‡nZzm¤Î c‡qv‡Mi Rb¨ c‡qvRbxq gvb¸‡jv wbYxZ n‡q‡Q, †m‡nZzGevi Avgiv Kvj wcqvim‡bi e¼gZvi mnM wbYq Ki‡Z cwi|

$$\therefore \quad Sk_p = \frac{\bar{x} - m_0}{\sigma}$$

$$= \frac{2710 - 2760}{695}$$

$$= \frac{-50}{695}$$

$$= -0.072$$

∴ wbYxZ Kvj wcqvim‡bi e¼gZvi mnM n‡jv –0.072

A_vr, cwigZ gv‡bi †cw¶‡Z MwYwZK Mo †_‡K cPź‡Ki gvb 7.2 kZvsk †ekx Ges †h‡nZz cvß gvbwU FYvZ¥K, †m‡nZæjv hvq †h, web¨vmwU FYvZ¥Kfv‡e e¼g|

evDwji e¼gZvi mnM *(Bowley's Coefficient of Skewness)*

evDwji e¼gZvi mnM c×wZwU PZz‡Ki avivYvi Dci cwZwôZ|

GB c×wZwU PZz‡Ki avivYvi Dci cwZwôZ| Db¥y³ †kYx m¤wjZ web¨v‡mi †¶‡Î †h‡nZz MwYwZK Mo wbYq Kiv hvq bv, †m‡nZzcqvim‡bi e¼gZvi mnMI wbYq Kiv hvq bv| Db¥y³ †kYx Ges Pig gv‡bi Dcw¯wZ m¤wjZ web¨v‡mi †¶‡Î evDwji e¼gZvi mnM wbY‡qi c×wZwU we‡klfv‡e Dc‡hvMx| Ae¯vbMZ cwigv‡ci †¶‡Î evDwji c×wZ‡Z e¼gZvi mnM wbYq Kiv DwPr| mviwY 6.1.2-G c`Ë DcvË e¨envi K‡i evDwji e¼gZvi mnM wbYq K‡i †`Lv‡bv n‡jv|

mvi wY 6.1.2: evDwj c`Ë m‡e¨envi K‡i ew¼gZvi mnM wbYq

‡kYx mxgv	NUbmsL¨v	µg‡hvwRZ MYmsL¨v
0 - 50	25	25
50 - 100	50	75
100 - 150	90	165
150 - 200	30	195
200 - 250	26	221
250 - 301	39	260
‡gvU	260	

c_‡g Avgv‡`i c_g I ZZxq PZz_K Ges ga¨gvi gvb wbYq K‡i wb‡Z n‡e|

c_g PZz_K $Q_1 = L_1 + \frac{\frac{n}{4} - f_c}{f_{Q_1}} \times C.I.$

‡hLv‡b, Q_1 = c_g PZz_K

L_1 = PZz_K †kYx‡i wbg mxgv

n = †gvU NUbmsL¨v

f_c = µg‡hvwRZ NUbmsL¨v

f_{Q_1} = c_g PZz‡Ki NUbmsL¨v

Gevi c_g PZz_K †kYx wbYq Ki‡Z n‡e|

$\therefore \quad \frac{n}{4} = \frac{260}{4} = 65$

†h‡nZzc wZwU PZz‡Ki gvb 65, †m‡nZzc_g PZz_K †kYxwU n‡jv 50 - 100 †kYx|

$\therefore \quad Q_1 = L_1 + \frac{\frac{n}{4} - f_c}{f_{Q_1}} \times C.I.$

$= 50 + \frac{65 - 25}{50} \times 50$

$= 50 + \frac{40}{50} \times 50$

$= 50 + 0.8 \times 50$

$= 50 + 40$

$= 90$

$\therefore$ wbYxZ Q_1 n‡jv 90

Gevi ZZxq PZz_K †kYx wbYq Ki‡Z n‡e|

$$\therefore \quad \frac{3n}{4} = \frac{3\times 260}{4} = \frac{780}{4}\times 50 = 195$$

ZZxq PZz_K †kYxwU n‡jv 150 – 200|

$$\therefore \quad Q_3 = L_1 + \frac{\frac{3n}{4} - f_c}{f_{Q_3}} \times C.I.$$

$$= 150 + \frac{195-165}{30}\times 50$$

$$= 150 + \frac{30}{30}\times 50$$

$$= 200$$

∴ wbYxZ Q_3 n‡jv 200

Gici ga¨gvi gvbwU wbYq Ki‡Z n‡e|

$$\therefore \quad Md = L_1 + \frac{\frac{n}{2} - F}{f} \times C.I.$$

$$= 100 + \frac{130-75}{90}\times 50$$

$$= 100 + \frac{55}{90}\times 50$$

$$= 50 + 0.61 \times 50$$

$$= 130.5$$

∴ wbYxZ Md n‡jv 130.5

Gevi cvß gvb¸‡jvi wfwË‡Z evDwji e¼gZvi mnM wbYq Kiv †h‡Z cv‡i|

$$\therefore \quad Sk_B = \frac{Q_3 + Q_1 - 2Md}{Q_3 + Q_1}$$

$$= \frac{200 + 90 - 2\times 130.5}{200 - 92}$$

$$= \frac{290 - 261}{110} = \frac{229}{110}$$

$$= 0.26$$

∴ wbYxZ Sk_B n‡jv 0.26

‡h‡nZzcvß gvbwU abvZ¥K, †m‡nZzæjv hvq †h, web¨vmwU abvZ¥K fv‡e e¼g|

†Kwj i ew¼gZvi mnM (Kelly's Coefficient of Skewness):

†Kwj `knwiK I kZnwi‡Ki avivYvi Dci wfwË K‡i ew¼gZvi mnM wbY‡qi wb‡gv³ m‡e©vb K‡i‡Qb|

`knwi‡Ki Dci wfwË K‡i,

$$Sk_k = \frac{D_1 + D_9 - 2Md}{D_9 + D_1}$$

†h‡nZz Sk_k = †Kjxi ew¼gZvi mnM

D_1 = c_g `knwiK

D_9 = beg `knwiK

Md = ga¨gv

kZnwi‡Ki Dci wfwË K‡i,

$$Sk_k = \frac{P_{10} + P_{90} - 2Md}{P_{90} + P_{10}}$$

†hLv‡b, Sk_k = †Kjxi ew¼gZvi mnM

P_{10} = 10g kZnwiK

P_{90} = 90g kZnwiK

Md = ga¨gv

Dc‡iv³ wZbwU c×wZi g‡a¨ wcqvim‡bi ew¼gZvi mnM wbY‡qi c×wZwUB mewaK e¨eüZ nq| KviY, G c×wZwU mn‡R wbYq Kiv hvq Ges †evSv hvq| †Kwj i c×wZwU †Zgb e¨eüZ nq bv e‡j †mwU wbYq Kivi c‡qvRb †bB|

mvivsk

DcvËiwk wefv‡M M‡oi Pvicv‡k web¨¯— i‡q‡Q Zv RvbviRb¨ cwimsL¨v‡bi †h cZ¨qwU Avgv‡`i mvnvh¨ K‡i ZvB n‡"Q ew¼gZv| ew¼gZv Av‡jvPbvq `ÿZv cavb we‡eP¨ we‡lq i‡q‡Q — ew¼gZvi w`K Ges ew¼gZvi gvÎv| hw` web¨vmwU myZg nq Zvn‡j cPzK, ga¨gv I M‡oi gvb GKB n‡e| abvZ¥Kfv‡e ew¼g web¨v‡m MwYwZK Mo cPz‡Ki †P‡q eo nq Ges ga¨gv Mo I cPz‡Ki gvSLv‡b †Kvb GK ¯v‡b Ae¯vb K‡i| FYvZ¥Kfv‡e ew¼g web¨v‡m cPzK MwYwZK M‡oi †P‡q eo nq Ges ga¨gv MwYwZK Mo I cPz‡Ki g‡a¨ †Kvb GK ¯v‡b Ae¯vb K‡i|

cv‡VwËi gƒj¨vqb

^be¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| ew¼gZv ej‡Z GKwU web¨v‡mi ..... †K †evSvq|
   K. mgi¡cZv
   L. Amgi¡cZv
   M. mgi¡cZv I Amgi¡cZv DfqB
   N. Dc‡ii †Kvbwb UB bq

2| ew¼gZvi gvÎv nj .....|
   K. MwYwZK Mo †_‡K cPz‡Ki cv_K¨
   L. ga¨gv †_‡K cPz‡Ki cv_K¨
   M. MwYwZK Mo †_‡K ga¨gvi cv_K¨
   N. Dc‡ii me¸‡jvB mwVK|

3| evDwji ew¼gZvi mnM c×wZwU †Kvb avivYvi Dci cwZôZ?
   K. cPzK
   L. ga¨gv
   M. MwYwZK Mo
   N. PZzK

msw¶ß ck

1| ew¼gZv wK?
2| ew¼gZvi cwigvc ej‡Z wK †evSvq?

iPbvgƒK ck

1| ew¼gZvi mnM wbY‡q Kvj wcqvimb c`Ë cwigvcwU D`vniYmn Av‡jvPbv Ki‡b|
2| ew¼gZvi mnM wbY‡q evDwj c`Ë cwigvcwU D`vniYmn Av‡jvPbv Ki‡b|

cvV- 2

cwiNvZ
Moments

GB cvV †k‡l hv Rvb‡e —

- cwiNvZ wK
- Awb¨¯Í DcvË †_‡K cwiNvZ wbYq
- wb¨¯Í DcvË †_‡K cwiNvZ wbYq

cwiNvZ wK (What are Moments)?

GKwU wbe¨v‡mi i|c‡K eYbv Kivi Rb¨ cwiNvZ‡K e¨envi Kiv nq _v‡K|

GKwU wbe¨v‡mi i|c‡K eYbv Kivi Rb¨ cwiNvZ‡K e¨envi Kiv n‡q _v‡K| cwiNvZmgn eû eYbvgj‡K cwimsL¨vbK cwigvc‡K (†hgb, †K›`ªxq cеYZv, we¯Íw, eª¼gZv I mPjZv) msw¶ßKi‡Yi Rb¨ cwimsL¨v‡bi GKwU myeavRbK I HK¨e×Ki‡Yi c×wZ‡K cwZwbwaZ K‡i| cwiNvZmgn‡K MÖxK eYgvjvi ÔµÕ (wgD) A¶i w`‡q cKvk Kiv nq|

Awb¨¯Í DcvË‡Ei †¶‡Î Mo $(\bar{x})$ †_‡K †Kvb Pj‡Ki (x) rZg cwiNvZ‡K wbgwjwLZfv‡e cKvk Kiv nq,

$$\mu_r = \frac{1}{N}\sum(x_i - \bar{x})^r$$

‡hLv‡b, μ_r = Mo †_‡K rZg cwiNvZ

N = †gvU NUbmsL¨v

x_i = wbe¨v‡mi cwZwU gvb

$\bar{x}$ = MwYwZK Mo

†h †Kvb AbygxZ Mo ev †¯”QvPwqZ gvb (A) †_‡K †Kvb Pj‡Ki (x) rZg cwiNvZ‡K wbgwjwLZ m‡Î i gva¨‡g cKvk Kiv nq,

$$\mu'_r = \frac{1}{N}\sum(x_i - \bar{x})^r$$

‡hLv‡b, μ'_r = ‡¯”QvPwqZ gvb †_‡K †Kvb Pj‡Ki rZg cwiNvZ

x_i = DcvËiwki cwZwU gvb

A = ‡¯”QvPwqZ gvb

N = ‡gvU NUbmsL¨v

Formatted: Subscript

wb¨¯Í DcvË‡Ei †¶‡Î Mo $(\bar{x})$ †_‡K †Kvb Pj‡Ki (x) rZg cwiNvZ‡K wb‡gv³ m‡Î i gva¨‡g cKvk Kiv nq,

$$\mu'_r = \frac{1}{N}\sum f_i(x_i - \bar{x})^r$$

†h †Kvb Abyl`gZ Mo ev †¯"QvPwqZ gvb (A) †_‡K †Kvb Pj‡Ki (x) rZg cwiNvZ‡K wbgwjwLZ m‡Ii gva¨‡g cKvk Kiv hvq,

$$\mu''_r = \frac{1}{N}\sum f_i (x_i - A)^r$$

‡hLv‡b, μ'_r = Mo †_‡K †Kvb Pj‡Ki rZg cwiNvZ

μ''_r = †¯"QvPwqZ gvb †_‡K †Kvb Pj‡Ki rZg cwiNvZ

N = ‡gvU NUbmsL¨v

f_i = NUbmsL¨v

x_i = †kYx ga¨-we›`y

$\bar{x}$ = MwYwZK Mo

A = †¯"QvPwqZ gvb

r-Gi wewfb gv‡bi Rb¨ Avgiv wewfb cwiNvZ cv‡ev| hw` r = 1 nq, Z‡e Avgiv c_g cwiNvZ cv‡ev; hw` r = 2 nq, Z‡e Avgiv wØZxq cwiNvZ cv‡ev; Ges Gfv‡e r-Gi gv‡bi Dci wfwË K‡i cwiNv‡Zi wewfb Ae¯'vb Rvb‡Z cv‡i‡ev| cwimsL¨v‡b c_g Pviwu cwiNvZ me‡P‡q †ekx c‡qvRbxq Ges ¸i“Zc¥K| wb‡æ Mo †_‡K Pviwu cwiNvZ wbY‡qi m‡Ii ¸‡jv‡K †`Lv‡bv n‡jv|

Aweb¨¯— Dcv‡Ëi †¶‡Î,

c_g cwiNvZ $\mu_1 = \frac{\sum (x_i - \bar{x})}{N}$

wØZxq cwiNvZ $\mu_2 = \frac{\sum (x_i - \bar{x})^2}{N}$

ZZxq cwiNvZ $\mu_3 = \frac{\sum (x_i - \bar{x})^3}{N}$

PZz_© cwiNvZ $\mu_4 = \frac{\sum (x_i - \bar{x})^4}{N}$

web¨¯— Dcv‡Ëi †¶‡Î,

c_g cwiNvZ $\mu_1 = \frac{\sum f_i (x_i - \bar{x})}{N}$

wØZxq cwiNvZ $\mu_2 = \frac{\sum f_i (x_i - \bar{x})^2}{N}$

ZZxq cwiNvZ $\mu_3 = \frac{\sum f_i (x_i - \bar{x})^3}{N}$

PZz_© cwiNvZ $\mu_4 = \frac{\sum f_i (x_i - \bar{x})^4}{N}$

‡hLv‡b, f_i = NUbmsL¨v

x_i = †kYx ga¨-we›`y

$\bar{x}$ = MwYwZK Mo

N = †gvU NUbmsL¨v

Mo †_‡K c_g cwiNwZ Avgv‡`i GKwU web¨v‡mi †Kvb bv †Kvb ‰ewkó¨ m¤ú‡K avivYv †`q| Mo †_‡K c_g cwiNwZ Avgv‡`i MwYwZK Mo m¤ú‡K avivYv †`q Ges Gi gvb memgq kZ© nq| wØZxq cwiNwZ †f`v¼ m¤ú‡K, ZZxq cwiNwZ ew¼gZv m¤ú‡K, Ges PZz_© cwiNwZ mPwjZv m¤ú‡K avivYv †`q|

GLv‡b D‡jL¨ †h, cwiZwU cwiNwZ Avgv‡`i GKwU web¨v‡mi †Kvb bv †Kvb ‰ewkó¨ m¤ú‡K avivYv †`q| Mo †_‡K c_g cwiNwZ Avgv‡`i MwYwZK Mo m¤ú‡K avivYv †`q Ges Gi gvb memgq kZ© nq| wØZxq cwiNwZ †f`v¼ m¤ú‡K, ZZxq cwiNwZ ew¼gZv m¤ú‡K, Ges PZz_© cwiNwZ mPwjZv m¤ú‡K avivYv †`q|

hLb MwYwZK Mo cÖmsL¨v bv n‡q fMvsk nq, ZLb cwiNwZ¸‡jv‡K †Kvb †¯”QvPwqZ gvb ev Abyôv‡gZ Mo †_‡K wbYq K‡i c‡i cKZ.cwiNv‡Z i¡cvšwiZ Ki‡Z nq| †¯”QvPwqZ gvb †_‡K wbYxZ cwiNwZ‡K A‡kvwaZ cwiNwZ e‡j Ges †m¸‡jv‡K MxK eYgvjvi ÔvôÕ A¶i Øviv wPwýZ Kiv nq| c_g PviwU A‡kvwaZ cwiNwZ wbY‡qi mÂ‡¸‡jv n‡jv,

c_g cwiNwZ $\nu_1 = \frac{\sum(x_i - A)}{N}$

wØZxq cwiNwZ $\nu_2 = \frac{\sum(x_i - A)^2}{N}$

ZZxq cwiNwZ $\nu_3 = \frac{\sum(x_i - A)^3}{N}$

PZz_© cwiNwZ $\nu_4 = \frac{\sum(x_i - A)^4}{N}$

‡¯”QvPwqZ gvb †_‡K wbYxZ A‡kvwaZ cwiNwZmgn¥K wbgwjwLZ m¤úK m¤Î i gva¨‡g cKZ. cwiNv‡Z i¡cvšwiZ Ki‡Z nq| †m¸‡jv n‡jv,

$\mu_1 = 0$

$\mu_2 = \nu_2 - \nu_1^2$

$\mu_3 = \nu_3 - 3\nu_2\nu_1 + 2\nu_1^3$

$\mu_4 = \nu_4 - 4\nu_3\nu_1 + 6\nu_2\nu_1^2 - 3\nu_1^4$

Avgiv Rwb †h, cwiNwZ GKwU web¨v‡mi AvKvi I cKwZ m¤ú‡K eYbv †`q| †Kvb web¨v‡mi ew¼gZv Ggb GKwU ˆewkó¨ hvi cwieZ‡bi d‡j web¨v‡mi cKwZi cwieZb N‡U| Z‡e G welq‡qi Mfx‡i hvevi c‡e Avgv‡`i `ÿZv a¡eK (constants) m¤ú‡K Rvb‡Z n‡e hv wØZxq, ZZxq I PZz_© cwiNwZ †_‡K wbYq Kiv nq| †mB a¡eK `ÿZv n‡jv, β_1 (weU$_1$) Ges β_2 (weU$_2$)|

β_1 n‡jv,

$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$

β_1 ev¼gZvi cwigvc wnmv‡e e¨eüZ nq| GKwU m~lg web¨v‡m β_1 memgq kb¨ n‡e| Z‡e ev¼gZvi cwigvc wnmv‡e β_1-Gi mxgve×ZwU n‡jv GwU ev¼gZvi w`K (direction) m¤ú‡K Avgv‡`i †Kvb aviYv †`q bv| KviYwU n‡jv, $\mu_3$-Gi gvbwU abvZ¥K ev FYvZ¥K DfqB n‡Z cv‡i| wKš‘ $\mu_3^{\,2}$-Gi gvb memgq abvZ¥K n‡e| AZGe, $\beta_1 = \dfrac{\mu_3^{\,2}}{\mu_2^{\,3}}$ memgq FYvZ¥K n‡q _v‡K| GB mxgve×ZvU `~ixf~Z nq hw` Avgiv Kvj© wcqvimb-Gi γ_1 (M¨vgv$_1$) wbYq Kwi| γ_1-†K β_1-Gi eMg~j wnmv‡e msÁwqZ Kiv hvq| A_©vr,

β_1 ev¼gZvi cwigvc wnmv‡e e¨eüZ nq| GKwU m~lg web¨v‡m β_1 memgq kb¨ n‡e|

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = \frac{\mu_3}{\sigma^3}$$

ev¼gZvi wPýwU wbfi K‡i μ_3-Gi gv‡bi Dci| hw` $\mu_3$-Gi gvb abvZ¥K nq, Z‡e web¨vmwU abvZ¥Kfv‡e ev¼g n‡e| Avi hw` FYvZ¥K nq, Z‡e web¨vmwU FYvZ¥Kfv‡e ev¼g n‡e|

ev¼gZvi wPýwU wbfi K‡i μ_3-Gi gv‡bi Dci| hw` $\mu_3$-Gi gvb abvZ¥K nq, Z‡e web¨vmwU abvZ¥Kfv‡e ev¼g n‡e, Avi hw` FYvZ¥K nq, Z‡e web¨vmwU FYvZ¥Kfv‡e ev¼g n‡e|

β_2-‡K mP~jZvi cwigvc wnmv‡e e¨envi Kiv nq| AZGe,

$$\beta_2 = \frac{\mu^4}{\mu_2^{\,2}}$$

Awebb¨¯— Dcv‡Ëi Mo †_‡K cwiNvZ wbYq (Computing Moments about Mean from Ungrouped Data)

mviwY 6.2.1-G cwPwU Dcvˇ gvb – 2, 3, 7, 8, I 10 – e¨envi K‡i Mo †_‡K cwiNvZ wbYq K‡i †`Lv‡bv n‡jv|

mviwY 6.2.1: Awebb¨¯— Dcv‡Ëi Mo †_‡K cwiNvZ wbYq

x_i	$d=(x_i-\bar{x})$	$d^2=(x_i-\bar{x})^2$	$d^3=(x_i-\bar{x})^3$	$d^4=(x_i-\bar{x})^4$
2	–4	16	–64	256
3	–3	9	–27	81
7	1	1	1	1
8	2	4	8	16
10	4	16	64	256
‡gvU = 30	**0**	**46**	**–18**	**610**

c`Ë Dcvˇmg~‡ni MvwYwZK Mo n‡jv,

$$\bar{x} = \frac{\sum x_i}{N} = \frac{30}{5} = 6$$

wbY©xZ Mo †_‡K Gevi Avgiv c_g PviwU cwiNvZ wbYq Ki‡ev|

c_g cwiNvZ $\mu_1 = \frac{\sum(x_i - \bar{x})}{N} = \frac{0}{5} = 0$

wØZxq cwiNvZ $\mu_2 = \frac{\sum(x_i - \bar{x})^2}{N} = \frac{46}{5} = 9.2$

ZZxq cwiNvZ $\mu_3 = \frac{\sum(x_i - \bar{x})^3}{N} = \frac{-18}{5} = -3.6$

PZz_ cwiNvZ $\mu_4 = \frac{\sum(x_i - \bar{x})^4}{N} = \frac{610}{5} = 122$

mviwY 6.2.1-G c`Ë DcvË e¨envi K‡i mviwY 6.2.2-G †¯"QvPwqZ gvb 4 †_‡K cwiNvZ wbYq K‡i †`Lv‡bv n‡jv|

mviwY 6.2.2: Awe‡'¯ Dcv‡Ëi †¯"QvPwqZ gvb (4) †_‡K cwiNvZ wbYq

x_i	$d = (x_i - A)$	$d^2 = (x_i - A)^2$	$d^3 = (x_i - A)^3$	$d^4 = (x_i - A)^4$
2	–2	4	–8	16
3	–1	1	–1	1
7	3	9	27	81
8	4	16	64	256
10	6	36	216	1296
‡gvU	**10**	**66**	**–298**	**1650**

†¯"QvPwqZ gvb wnmv‡e 4-‡K a‡i Avgiv c_g PviwU cwiNvZ wbYq Ki‡ev|

c_g cwiNvZ $\nu_1 = \frac{\sum(x_i - A)}{N} = \frac{10}{5} = 2$

wØZxq cwiNvZ $\nu_2 = \frac{\sum(x_i - A)^2}{N} = \frac{66}{5} = 13.2$

ZZxq cwiNvZ $\nu_3 = \frac{\sum(x_i - A)^3}{N} = \frac{298}{5} = 59.6$

PZz_ cwiNvZ $\nu_4 = \frac{\sum(x_i - A)^4}{N} = \frac{1650}{5} = 330$

GLb Avgv‡`i †¯"QvPwqZ gvb †_‡K wbYvZ A‡kvaZ cwiNvZ¸‡jv‡K wbgwjwLZ m¤úK© m‡Î i gva¨‡g cKZ.cwiNv‡Z i¡cvšwiZ Ki‡Z n‡e|

$\mu_1 = 0$

$\mu_2 = \nu_2 - \nu_1^2$

$\mu_3 = \nu_3 - 3\nu_2\nu_1 + 2\nu_1^3$

$\mu_4 = \nu_4 - 4\nu_3\nu_1 + 6\nu_2\nu_1^2 - 3\nu_1^4$

Avgiv Rwb †h, MwYZK Mo c_g cwiNv‡Zi i¡c †bq e‡j Zvi gvb memgq ÔOÕ nq| AZGe,

$$\mu_1 = 0$$

$$\mu_2 = \nu_2 - \nu_1^2$$
$$= 13.2 - (2)^2$$
$$= 13.2 - 8$$
$$= 9.2$$

$$\mu_3 = \nu_3 - 3\nu_2\nu_1 + 2\nu_1^3$$
$$= 59.6 - 3\,(13.2 \times 2) + 2\,(2)^3$$
$$= 59.6 - 3 \times 26.4 + 2 \times 8$$
$$= 59.6 - 79.2 + 16$$
$$= -3.6$$

$$\mu_4 = \nu_4 - 4\nu_3\nu_1 + 6\nu_2\nu_1^2 - 3\nu_1^4$$
$$= 330 - 4\,(59.6 \times 2) + 6\,(13.2 \times 2^2) - 3\,(2)^4$$
$$= 330 - 4 \times (119.2) + 6\,(13.2 \times 4) - 3 \times 16$$
$$= 330 - 476.8 + 6 \times 52.8 - 48$$
$$= 330 - 476.8 + 316.8 - 48$$
$$= 122$$

†`Lv hv‡"Q †h, †¯"QvPwqZ gvb †_‡K wbYvZ A‡kwaZ cwiNvZ¸‡jv m¤úK m‡Îi gva¨‡g cKZ. cwiNv‡Z i¡cvš‡ii ci Mo †_‡K wbYvZ cwiNv‡Zi gv‡bi mv‡_ wg‡j wM‡q‡Q Ges Gi d‡j m¤úK m‡Îwi h_v_Zv cgwYZ n‡q‡Q|

web¨¯— DcvË †_‡K cwiNvZ wbYq (Computing Moments from Grouped Data)

web¨¯— DcvË †_‡K c_g Pviwi cwiNvZ wbY‡qi m‡¸‡jv n‡jv,

c_g cwiNvZ $\mu_1 = \dfrac{\sum f_i(x_i - \bar{x})}{N}$

wØZxq cwiNvZ $\mu_2 = \dfrac{\sum f_i(x_i - \bar{x})^2}{N}$

ZZxq cwiNvZ $\mu_3 = \dfrac{\sum f_i(x_i - \bar{x})^3}{N}$

PZz_© cwiNvZ $\mu_4 = \dfrac{\sum f_i(x_i - \bar{x})^4}{N}$

mviwY 6.2.3-G c`Ë DcvË †_‡K Dc‡iv³ m‡¸‡jv e¨envi K‡i †K›`ªq cwiNvZ wbYq K‡i †`Lv‡bv n‡jv|

mvivY 6.2.3: wewU DcvË †_‡K †K› `xq cwiNvZ wbYq

C.I.	f_i	x_i	$f_i x_i$	$(x_i - \bar{x})$	$f_i(x_i - \bar{x})$	$f_i(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^3$	$f_i(x_i - \bar{x})^4$
40-45	28	42.5	1190	-8.7	-243.6	2119.32	-18438.084	1604113.3080
45-50	24	47.5	1140	-3.7	-88.8	328.56	1215.672	4497.9864
50-55	17	52.5	892.5	1.3	22.1	28.73	37.349	48.5537
55-60	14	57.5	805	6.3	88.2	555.66	3500.658	22054.1454
60-65	12	62.5	750	11.3	135.6	1532.28	17314.764	195656.8332
65-70	4	67.5	270	16.3	65.2	1062.76	17322.988	282364.7044
70-75	1	72.5	72.5	21.3	21.3	453.69	9663.597	205834.6161
‡gvU	**100**		**5120**		**0**	**6081**	**28185.6**	**870868.17**

AZGe, msÁvMZ m‡Ĥi e¨envi K‡i wbYxZ c_g Pvi wU cwiNvZ n‡jv,

$$\mu_1 = \frac{\sum f_i(x_i - \bar{x})}{N} = \frac{0}{100} = 0$$

$$\mu_2 = \frac{\sum f_i(x_i - \bar{x})^2}{N} = \frac{6081}{100} = 60.81$$

$$\mu_3 = \frac{\sum f_i(x_i - \bar{x})^3}{N} = \frac{28185.6}{100} = 281.856$$

$$\mu_4 = \frac{\sum f_i(x_i - \bar{x})^4}{N} = \frac{870868.17}{100} = 8708.6817$$

msÁvMZ m‡Ĥi gva¨‡g MwYwZK Mo †_‡K cwiNvZ wbYq Kiv n‡q‡Q| wKš Avgiv Rwb †h, msÁvMZ m‡Ĥi gva¨‡g †h †Kvb cwigvc wbYq mgq mv‡c¶ Ges msL¨v¸‡jv ep¤r AvKvi avivY Ki‡j wbYq c×wZwU RwUj n‡q c‡o| †m ‡¶‡Î msw¶ß c×wZi e¨envi Zv mnRmva¨ K‡i †Zv‡j | mvivY 6.2.4-G msw¶ß c×wZ‡Z c_g Pvi wU cwiNvZ wbYq K‡i †`Lv‡bv n‡jv|

mvivY 6.2.4: wewU DcvË †_‡K †K› `xq cwiNvZ wbYq

C.I.	f_i	x_i	$d = \frac{x_i - A}{C.I.}$	fd	fd^2	fd^3	fd^4
40-45	28	42.5	–3	–84	252	–756	2268
45-50	24	47.5	–2	–48	96	–192	384
50-55	17	52.5	–1	–17	17	–17	17
55-60	14	57.5=A	0	0	0	0	0
60-65	12	62.5	1	12	12	12	12
65-70	4	67.5	2	8	16	32	64
70-75	1	72.5	3	3	9	27	81
‡gvU	**100**			**–126**	**402**	**–894**	**2826**

GLv‡b j¶¨Yxq †h, msw¶ß c×wZ‡Z wePÿwZ wbY‡qi †¶‡Î GKwU †¯"QvPwqZ gvb e¨envi Kiv n‡q‡Q| Avgiv Rwb †h, †¯"QvPwqZ gvb e¨eüZ n‡j cwiNwZ¸‡jv A‡kwaZ _v‡K| †m¸‡jv‡K †kvab K‡i cKZ.cwiNv‡Z i¡cvšwiZ Ki‡Z nq| AZGe, Avgiv c_‡g PviwU A‡kwaZ cwiNwZ wbYq Ki‡ev Ges Zvici †m¸‡jv‡K m¤úK m‡Îi gva¨‡g †kwaZ cwiNv‡Z i¡cvši Ki‡ev|

$$\nu_1 = \frac{\sum \mathrm{fd}}{\mathrm{N}} \times \mathrm{C.I.} = \frac{-126}{100} \times 5 = -1.26 \times 5 = -6.3$$

$$\nu_2 = \frac{\sum \mathrm{f(d)}^2}{\mathrm{N}} \times \mathrm{C.I.}^2 = \frac{402}{100} \times 5^2 = 4.02 \times 25 = 100.5$$

$$\nu_3 = \frac{\sum \mathrm{f(d)}^3}{\mathrm{N}} \times \mathrm{C.I.}^3 = \frac{-894}{100} \times 5^3 = 8.94 \times 125 = -1117.5$$

$$\nu_4 = \frac{\sum \mathrm{f(d)}^4}{\mathrm{N}} \times \mathrm{C.I.}^4 = \frac{2826}{100} \times 5^4 = 28.26 \times 625 = 17662.5$$

GLb Avgiv m¤úK m‡Î e¨envi K‡i †kwaZ cwiNwZ wbYq Ki‡ev| AZGe,

$$\mu_1 = 0$$

$$\mu_2 = \nu_2 - \nu_1^2$$
$$= 100.05 - (-6.3)^2$$
$$= 100.5 - 39.69$$
$$= 60.81$$

$$\mu_3 = \nu_3 - 3\nu_2\nu_1 + 2\nu_1^3$$
$$= -1117.5 - 3\{(100.5)(-6.3)\} + 2(-6.3)^3$$
$$= -1117.5 - 3(-633.15) + 2(-250.047)$$
$$= -1117.5 - 1899.45 - 500.094$$
$$= 1899.45 - 1617.594$$
$$= 281.856$$

$$\mu_4 = \nu_4 - 4\nu_3\nu_1 + 6\nu_2\nu_1^2 - 3\nu_1^4$$
$$= 17662.5 - 4\{(-1117.5)(-6.3)\} + 6\{(60.81)(-6.3)\} - 3(-6.3)^4$$
$$= 17662.5 - 4(7040.25) + 6\{(100.5)(39.63)\} - 3(1575.2961)$$
$$= 17662.5 - 28161 + 23933.07 - 4725.883$$
$$= 41595.57 - 32886.883$$
$$= 8708.6817$$

†`Lv hv‡"Q †h, msÁwMZ m‡Îi gva¨‡g wbYxZ cwiNwZ¸‡jvi Ges msw¶ß c×wZ‡Z wbYxZ cwiNwZ¸‡jvi gvb GKB n‡q‡Q| Kv‡RB †h †Kvb GKwU m‡Îi gva¨‡g web¨¯— DcvË †_‡K cwiNwZ wbYq Ki‡jB P‡j|

†h‡nZz cwiNvZ Avgv‡`i GKwU web¨v‡mi AvKvi I cKyZ m¤ú‡K aviYv †`q, †m‡nZz cwiNv‡Zi cvß gvb¸‡jv e¨envi K‡i Avgiv β_1 Ges β_2 cwigvc `ywU (h_vµ‡g ew¼gZv I mPzjZv cwigvc) wbYq Ki‡Z cwi| AZGe,

$$\beta_1 = \frac{\mu_3^{\,2}}{\mu_2^{\,3}} = \frac{(281.856)^2}{(60.81)^3} = \frac{79442.80474}{224866.6294} = 0.3532$$

$$\beta_2 = \frac{\mu^4}{\mu_2^{\,2}} = \frac{8708.6817}{(60.81)^2} = \frac{8708.6817}{3697.8561} = 2.3550$$

$\therefore \quad \beta_1 = 0.3532$ Ges $\beta_2 = 2.355$

Avgiv Rwb †h, GKwU wb¨vm myg I mg mPzj n‡j $\beta_1 = 0$ Ges $\beta_2 = 3$ n‡e| AZGe, β_1 Ges β_2-Gi wbYxZ gvb `ywU †`‡L Avgiv ej‡Z cwi †h, wb¨vmwU cÖvqcÖvq myg Ges mg mPzj bq| hw`I $\beta_1$-Gi gvb †`‡L wb¨v‡mi ew¼gZvi cKvZwU †evSv hvq bv (KviY, $\mu_3^{\,2}$-Gi gvb memgq abvZ¥K nq), Z‡e μ_3-Gi abvZ¥K gvb Abyhvqx Avgiv ej‡Z cwi †h, wb¨vmwU abvZ¥K fv‡e ew¼gZvi ceYZv c`kb Ki‡Q|

mvivsk

DcvË wb¨v‡mi i|c‡K eYbv Kivi Rb¨ cwiNvZ‡K e¨envi Kiv n‡q _v‡K| cwZwU cwiNvZ wb¨v‡mi †Kvb bv †Kvb ˆewkó¨ m¤ú‡K aviYv †`q| †hgb, c_g cwiNvZ Avgv‡`i MwYwZK Mo m¤ú‡K aviYv †`q, wØZxq cwiNvZ †f`v¼ m¤ú‡K aviYv †`q, ZZxq cwiNvZ ew¼gZv m¤ú‡K Ges PZz_ cwiNvZ mPzjZv m¤ú‡K aviYv †`q| c_g cwiNv‡Zi gvb memgq ÔOÕ nq| wØZxq, ZZxq I PZz_ cwiNvZ †_‡K ew¼gZvi cwigvc (β_1) Ges mPzjZvi cwigvc (β_2) wbYq Kiv hvq|

cv‡VvËi g~j¨vqb

ˆbe¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| GKwU web¨v‡mi i|c‡K eYbv Kivi Rb¨ .... e¨envi Kiv nq|
   K. ew¼gZv
   L. cwiNvZ
   M. mPjZv
   N. ¯^vfvweK web¨vm

2| Awb¨¯— Dcv‡Ëi †¶‡Î c_g cwiNvZ wbY‡qi m‚wU n‡jv:

   K. $\mu_1 = \frac{\sum(x_i - \overline{x})}{N}$

   L. $\mu_1 = \frac{\sum(x_i - \overline{x})^2}{N}$

   M. $\mu_1 = \frac{\sum(x_i - \overline{x})^3}{N}$

   N. $\mu_1 = \frac{\sum f_i (x_i - \overline{x})^4}{N}$

3| GKwU mylg web¨v‡m β_1 memgq .... n‡e|
   K. k~b¨
   L. GK
   M. `yB
   N. wZb

msw¶ß ck

1| cwiNvZ wK?

2| β_1 Ges β_2 wK?

iPbvg~jK ck

1| cwiN‡Zi e¨envi D`vniYmn Av‡jvPbv Kiæb|

2| wbæwjwLZ DcvË¸‡jv e¨envi K‡i c_g PviwU cwiNvZ wbYq Kiæb|

   2 4 6 7 9

cvV- 3

mP¨jZv
Kurtosis

GB cvV †k‡l Avcwb Rvb‡Z cvi‡eb —

- mP¨jZv wK
- mP¨jZvi cKvi‡f`
- mP¨jZvi cwigvc
- Awbw`©ó DcvË †_‡K mP¨jZvi mnM wbYq
- wbw`©ó DcvË †_‡K mP¨jZvi mnM wbYq

mP¨jZv wK (What is Kurtosis)?

Avgiv B‡Zvg‡a¨B †R‡b‡Q †h, GKwU DcvË wbe©vm‡K wewfbœ fv‡e eY©bv Kiv hvq| †hgb, M‡oi wbY©‡qi gva¨‡g GKwU wbe©v‡mi Aš—f©y³ DcvËi wki †K›`ªxq cebYZv Ges cwiwgZ e¨eav‡bi gva¨‡g †K›`ªxq gv‡bi Pvic‡k DcvËi wki we¯—wZ. m¤ú‡K Rvb‡Z cvwi| cwiNvZ wbY©‡qi mgq Avgiv GI †R‡b‡Q †h, c_g cwiNvZ Avgv‡`i Mo, wØZxq cwiNvZ †f`v¼, ZZxq cwiNvZ ew¼gZv Ges PZz_© cwiNvZ mP¨jZv m¤ú‡K aviYv †`q| mP¨jZvi gva¨‡g Avgiv GKwU wbe©v‡mi AvKwZ. m¤ú‡K Rvb‡Z cwi| mP¨jZvi cwigvcwU n‡jv wØZxq cwiN‡Zi e‡Mi Dci PZz_© cwiN‡Zi Abycv‡Z|

> mP¨jZv GKwU MYmsL¨v †iLvi cP‚oK †_‡K mP¨MZvi gvÎvi eYbv †`q|

mP¨jZv GKwU MYmsL¨v †iLvi cP‚oK †_‡K mP¨MZvi (peakedness) gvÎvi eYbv †`q| Ab¨ K_vq, mP¨jZv n‡"Q ¯^vfvweK wbe©v‡mi (normal distribution) Zzjbvq GKwU MYmsL¨v †iLvi D"PZvi gvÎvi cwigvc| mP¨jZvi gva¨‡g GKwU mylg wbe©v‡mi mP¨MZv‡K Av`k a‡i Ab¨vb¨ wbe©v‡mi Zzjbv Kiv nq|

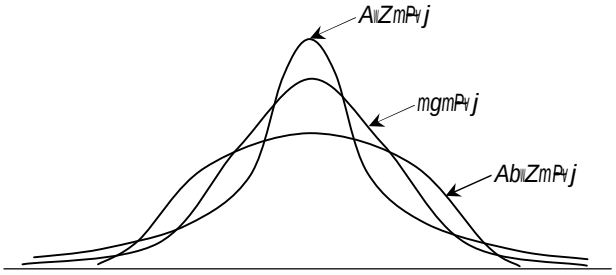
mP¨jZvi cKvi‡f` (Types of Kurtosis)

GKwU MYmsL¨v †iLv hw` ¯^vfvweK ‡iLvi Zzjbvq ‡ekx mP¨M nq, Z‡e †mB †iLwU‡K AwZmP¨j (leptokurtic) †iLv e‡j| hw` †iLwU ¯^vfvweK †iLvi Zzjbvq Kg mP¨M nq, Z‡e Zv‡K AbwZmP¨j (platykurtic) †iLv e‡j| Avi hw` †iLwU myIg nq, A_©vr, ¯^vfvweK †iLvi gZ nq, Z‡e Zv‡K mgmP¨j (mesokurtic) †iLv e‡j (wPÎ 6.3.1 `óe¨)|

mgmP¨j †iLv (Mesokurtic Curve)

> mgmP¨j †iLvi †¶‡I cP‚oK, ga¨gv, I MwYwZK M‡oi gvb GKB nq Ges ‡m¸‡jv †iLvi gvSLv‡b Ae¯'vb K‡i|

GKwU myIg †iLv‡K mgmP¨j †iLv e‡j| GB †iLwU GKwU GK cP‚oK m¤úbœ wbe©v‡mi i~c †bq, †hLv‡b †iLvi gvSLvbwU DP©Zg_v‡K Ges cvš—`ØqwU Xvjyn‡q Abyf~wgK †iLvq †b‡g hvq Z‡e ¯úk K‡i bv| A_©vr, ¯^vfvweK †iLvi mKj ˆewkó¨ mgmP¨j †iLvi g‡a¨ _v‡K| mgmP¨j †iLvi †¶‡Î cP‚oK, ga¨gv I MwYwZK M‡oi gvb GKB nq Ges ‡m¸‡jv †iLvi gvSLv‡b Ae¯'vb K‡i|



wPÎ 6.3.1: mPvjZvi wewfbœ iƒc

AwZmPvj †iLv (Leptokurtic Curve)

GKwU myg †iLvi Zzjbvq AwZ gvÎvq mP¨M †iLv‡K AwZmPvj †iLv e‡j| GwU GKwU GK cPzKwewkó †iLv, Z‡e G ai‡Yi web¨v‡m DcvË iwkgvjv Lye †ekx gvÎvq †K›`xq ¯v‡b ¸"Qe× _v‡K Ges †iLvi cv‡š msL¨v gvb Kg _v‡K|

GKwU myg †iLvi Zzjbvq AwZ gvÎvq mP¨M †iL‡K AwZmPvj †iLv e‡j|

AbwZmPvj †iLv (Platykurtic Curve)

†h †iLv myg †iLvi Zzjbvq Kg mP¨M _v‡K †mB †iLv‡K AbwZmPvj †iLv e‡j| GwU GK cPzKwewkó †iLv n‡jI web¨v‡mi DcvË iwkgvjv ‡K‡›`i w`‡K †Zgb Nbxfz _v‡K bv Ges web¨v‡mi `ßi cv‡ši †kYx¸‡jv‡Z †ekx cwigv‡Y msL¨v gvb _v‡K| Gi d‡j †iLvi AvKwZwU A‡bKUv fwgi w`‡K Pvcv n‡q _v‡K|

†h †iLv myg †iLvi Zzjbvq Kg mP¨M _v‡K †mB †iLv‡K AbwZmPvj †iLv e‡j|

mPvjZvi cwigvc (Measures of Kurtosis)

PZz_ cwiNv‡Zi gva¨‡g mPvjZvi gvÎv cwigvc Kiv nq|

Aweb¨¯— web¨v‡mi ‡¶‡Î PZz_ cwiNvZ n‡jv,

$$\mu_4 = \frac{\sum x_i^4}{N}$$

Ges web¨¯— Dcv‡Ëi ‡¶‡Î PZz_ cwiNvZ n‡jv,

$$\mu_4 = \frac{\sum f_i x_i^4}{N}$$

A_ev, $$\mu_4 = \frac{\sum fd^4}{N}$$

$$\mu_4 = \nu_4 - 4\nu_3\nu_1 + 6\nu_2\nu_1^2 - 3\nu_1^4$$

wKš web¨v‡mi PZz_ cwiNvZ mPvjZv m¤ú‡K Avgv‡`i GKwU wbi¼k cwigvc c`vb K‡i| wØZxq cwiNv‡Zi gvb w`‡q fvM K‡i cwigvcwU‡K Av‡cwK cwigv‡c i¦cvši Kiv hvq, hv mPvjZvi mnM bv‡g cwiwPZ| mPvjZvi mnM‡K β_2 cZx‡Ki gva¨‡g wPwýZ Kiv nq| AZGe,

$$\beta_2 = \alpha_4 = \frac{\mu_4}{\sigma^4} = \frac{\mu_4}{\mu_2^2}, \quad Ges \quad \gamma_2 = \beta_2 - 3$$

myg web¨vm ev mgmPvj †iLvi †¶‡Î mPvjZvi mn‡Mi gvb n‡e 3| AwZmPvj †iLvi †¶‡Î β_2 < 3 Ges AbwZmPvj †iLvi †¶‡Î β_2 > 3 n‡e|

mPvjZv cwigv‡ci Av‡iKwU cwigvc i‡q‡Q, hv PZyK Ges kZnwi‡Ki Dci wfwË K‡i wbYq Kiv nq| GB cwigvcwU wbY‡qi m¤ÊwU n‡jv,

$$\kappa = \frac{Q}{P_{90} - P_{10}}$$

†hLv‡b, κ = mPvjZvi mnM

Q = PZyK

P_{90} = 90Zg kZnwiK

P_{10} = 10g kZnwiK

cKZc‡¶, PZzKwU n‡jv Avav AvšPZzK cwimi| mPvjZvi mnMwU‡K MxK eYgvjvi †QvU A¶‡ii 'κ' (Kvàv)-Gi gva¨‡g wPwýZ Kiv nq, hv mPvjZvi kZnwiK mnM bv‡g cwiwPZ| GKwU ¯'vfweK web¨v‡mi Rb¨ 'κ'-Gi gvb n‡jv 0.263| Gi †P‡q eo gvb n‡j web¨vmwU n‡e AwZmPvj Ges ‡QvU n‡j web¨vmwU n‡e AbwZmPvj |

Aweb¨¯— DcvË †_‡K mPvjZvi mnM wbYq (Computing Coefficient of Kurtosis from Ungrouped Data)

‡Kvb DcvËi wkí AvKwZ.m¤ú‡K gše¨ Ki‡Z n‡j mPvjZvi mnM wbYq Ki‡Z n‡e| mPvjZvi mnM wbY‡qi m¤ÊwU n‡jv,

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

†hLv‡b, β_2 = mPvjZvi mnM

μ_4 = PZz cwiNvZ

μ_2^2 = wØZxq cwiNv‡Zi eM

AZGe, m¤Ê Abyiƒcvšx Avgv‡`i c_‡g cwiNvZ¸‡jv wbYq K‡i wb‡Z n‡e| mviwY 6.3.1-G Aweb¨¯— DcvË †_‡K cwiNvZ wbYq K‡i †`Lv‡bv n‡jv|

mviwY 6.3.1: Aweb¨¯— DcvË †_‡K cwiNvZ wbYq

x_i	$d = (x_i - \bar{x})$	$d^2 = (x_i - \bar{x})^2$	$d^3 = (x_i - \bar{x})^3$	$d^4 = (x_i - \bar{x})^4$
3	–4	16	–64	256
5	–2	4	–8	16
8	1	1	1	1
9	2	4	8	16
10	3	9	27	81
‡gvU = 35	**0**	**34**	**–36**	**370**

c_‡g Avgv‡`i MwYwZK Mo wbYq K‡i wb‡Z n‡e|

$$\therefore \quad \bar{x} = \frac{\sum x_i}{N} = \frac{35}{5} = 7$$

cvß gvb¸‡jv e¨envi K‡i Gevi cwiNvZ¸‡jv wbYq Ki‡Z n‡e|

$$\mu_1 = \frac{\sum (x_i - \bar{x})}{N} = \frac{0}{5} = 0$$

$$\mu_2 = \frac{\sum (x_i - \bar{x})^2}{N} = \frac{34}{5} = 6.8$$

$$\mu_3 = \frac{\sum (x_i - \bar{x})^3}{N} = \frac{-36}{5} = 7.2$$

$$\mu_4 = \frac{\sum (x_i - \bar{x})^4}{N} = \frac{370}{5} = 74$$

wØZxq I PZz_ cwiNv‡Zi gvb m‡Î c‡qvM K‡i Avgiv mPwjZvi mnM wbYq Ki‡Z cwi| AZGe,

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{74}{(6.8)^2} = \frac{74}{46.24} = 1.60$$

$$\therefore \quad \beta_2 = 1.6$$

Avgiv Rwb †h, hw` $\beta_2 < 3$ nq, Z‡e DcvËiwki web¨vmwUi AvKwZ. n‡e AwZmPwj| AZGe, Avgiv mPwjZvi cvß mn‡Mi gvb (1.6) †`‡L ej‡Z cwi †h, Avgv‡`i DcvËiwki web¨vmwU AwZmPwj AvKwiZi|

web¨¯— DcvË †_‡K mPwjZvi mnM wbYq (Computing Coefficient of Kurtosis from Grouped Data)

mviwY 6.3.2-G c`Ë DcvË †_‡K Avgiv c_‡g cwiNvZ¸‡jv wbYq K‡i †be| GLv‡b cwiNvZ wbY‡qi ms¯¶ß c×wZwU e¨envi Kiv n‡e|

mviwY 6.3.2: web¨¯— DcvË †_‡K cwiNvZ wbYq

C.I.	f_i	x_i	$d = \frac{x_i - A}{C.I.}$	fd	fd^2	fd^3	fd^4
60 - 62	5	61	–2	–10	20	–40	80
63 - 65	18	64	–1	18	18	–18	18
66 - 68	42	67 = A	0	0	0	0	0
69 - 71	27	70	1	27	27	27	27
72 - 74	8	73	2	16	32	64	128
‡gvU	**100**			**15**	**97**	**33**	**253**

c_‡g A‡kvwaZ cwiNvZ¸‡jv wbYq K‡i wb‡Z n‡e|

$$\nu_1 = \frac{\sum fd}{N} \times C.I. = \frac{15}{100} \times 3 = 0.15 \times 3 = 0.45$$

$$\nu_2 = \frac{\sum fd^2}{N} \times C.I.^2 = \frac{97}{100} \times 3^2 = 0.97 \times 9 = 8.73$$

$$\nu_3 = \frac{\sum fd^3}{N} \times C.I.^3 = \frac{33}{100} \times 3^3 = 0.33 \times 27 = 8.91$$

$$\nu_4 = \frac{\sum fd^4}{N} \times C.I.^4 = \frac{253}{100} \times 3^4 = 2.53 \times 81 = 204.93$$

Gev‡i Avgiv †kvwaZ cwiNvZ¸‡jv wbYq Ki‡ev|

$$\mu_1 = 0$$

$$\mu_2 = \nu_2 - \nu_1^2$$
$$= 8.73 - (0.45)^2$$
$$= 8.73 - 0.2025$$
$$= 8.5275$$

$$\mu_3 = \nu_3 - 3\nu_2\nu_1 + 2\nu_1^3$$
$$= 8.91 - 3\{(8.73)(0.45)\} + 2\,(0.45)^3$$
$$= 8.91 - 3(3.9285) + 2\,(0.091125)$$
$$= 8.91 - 11.7855 + 0.18225$$
$$= -2.69325$$

$$\mu_4 = \nu_4 - 4\nu_3\nu_1 + 6\nu_2\nu_1^2 - 3\nu_1^4$$
$$= 204.93 - 4\,\{(8.91)(0.45)\} + 6\{(8.73)(0.45)\} - 3(0.45)^4$$
$$= 204.93 - 4(4.0095) + 6\{(8.73)(0.2025)\} - 3(0.04100625)$$
$$= 204.93 - 16.038 + 6\,(1.767825) - 0.1230$$
$$= 204.93 - 16.038 + 10.6069 - 0.1230$$
$$= 215.5369 - 16.1610$$
$$= 199.3759$$

wØZxq I PZz_ cwiNv‡Zi gvb m‡ŸÏ c‡qvM K‡i Avgiv mPwjZvi mnM wbYq Ki‡Z cwi| AZGe,

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{199.3759}{(8.5275)^2} = \frac{199.3759}{72.7183}$$

$$\therefore \quad \beta_2 = 2.7418$$

‡h‡nZz $\beta_2 = 2.7418$ Ges cvq 3-Gi KvQvKvwQ, †m‡nZzAvgiv ej‡Z cwi †h, web¨vmwU cvq mgmPwj|

mvivsk

mPwjZv GKwU MYmsL¨v †iLvi cPzK †_‡K mP¨MZvi gvÎvi eYbv †`q| Gi gva¨‡g Avgiv GKwU web¨v‡mi AvKwZ. m¤ú‡K Rvb‡Z cwi| mPwjZv cwigvcwU n‡jv wØZxq cwiNv‡Zi e‡Mi Dci PZz_ cwiNv‡Zi Abyôv‡Z| mP¨MZvi Dci wbfi K‡i wZb ai‡Yi †iLvi cwiPq Avgiv cvB mgmPwj, AwZmPwj Ges AbwZmPwj| †h †iLvi gvSLvbwU DPy_v‡K Ges cvš `ÿwU Xvjyn‡q Abyfwgk †iLvq †b‡g hvq wKš¿úk K‡i bv Zv‡K mgmPwj †iLv e‡j| AwZ gvÎvq mP¨M †iLv‡K AwZmPwj Ges myiyg †iLvi Zyiybvq Kg mP¨M †iLv‡K AbwZmPwj †iLv e‡j|

c¨vWÜi g‡j¨vqb

ˆbe©¨³K ck

mwVK DËi‡i i c¨vk wUK (√) wPý w`b –

1| mP¨jZv cwigvcwU n‡jv:
   K. c_g cwiNv‡Zi e‡Mi Dci PZz cwiNv‡Zi Abycv Z
   L. wØZxq cwiNv‡Zi e‡Mi Dci PZz cwiNv‡Zi Abycv Z
   M. ZZxq cwiNv‡Zi e‡Mi Dci PZz cwiNv‡Zi Abycv Z
   N. Dc‡i i †KvbwUB bq

2| mP¨jZv mnM wbY‡qi mwVK m~Î ‡KvbwU?

   K. $\beta_2 = \frac{\mu_4}{\mu_2^{\,2}}$

   L. $\beta_2 = \frac{\mu_2^{\,2}}{\mu_4}$

   M. $\beta_2 = \frac{\mu_3}{\mu_2^{\,2}}$

   N. $\beta_2 = \frac{\mu}{\mu_2^{\,2}}$

3| †Kvb cwiNv‡Zi gva¨‡g mP¨jZvi gvÎv cwigvc Kiv nq?
   K. c_g
   L. wØZxq
   M. ZZxq
   N. PZz

msw¶ß ck

1| mP¨jZv wK?
2| wK fv‡e mP¨jZvi cwigvc wbYq Kiv nq?

iPbvg~jK ck

1| mP¨jZv wK? mP¨jZvi cKvi‡f`mgn Av‡jvPbv Kiæb|
2| wKfv‡e Awbe¨¯ DcvË †_‡K mP¨jZvi mnM wbYq Kiv nq D`vniYmn Av‡jvPbv Kiæb|

cvV- 4

¯vfweK webvm
Normal Distribution

GB cvV †k‡l hv Rvbv hv‡e —

- ¯vfweK webvm wK
- ¯vfweK webv‡mi ˆewkó¨
- mmxg ebvg Amxg MYmsL¨v webvm
- ¯vfweK †iLvi Aax‡b AvqZb
- cwigvZ ¯vfweK mvivYi e¨envi

¯vfweK webvm wK (What is Normal Distribution)?

¯vfweK webvm n‡jv cwimsL¨v‡bi me‡P‡q ¸iæZc¥K GKwU m¤vebv webvm (probability distribution)| hLb Avgiv mxwgZ msL¨K ch‡e¶Y wb‡q GKwU AwfÁZvjä webvm Dc¯vcb Kwi, ZLb Zv ¯vfweK webv‡mi i¡c †bq bv| wKš hLb Amxg msL¨K ch‡e¶Y wb‡q GKwU webvm ˆZix Kiv nq, ZLb Zv ¯vfweK webv‡mi AvKvi avivi Y K‡i| AZGe, ejv hvq †h, ¯vfweK webvm n‡jv GKwU we‡kl ai‡Yi MYmsL¨v webvm, hv ev¯‡e A‡bK AwfÁZvjä webv‡mi Lye KvQvKwQ nq| ¯vfweK webvm †m mKj AwfÁZvjä webv‡mi GKwU ÔAv`kÕ wnmv‡e KvR K‡i| GB we‡kl ai‡Yi webvmwU ¯vfweK †iLv bv‡gI cwiwPZ| ¯vfweK †iLv n‡jv GKwU webv‡mi ‡Kejgv Î GKwU AwKvi| GKwU webv‡mi Av‡iv A‡bK ai‡Yi AvKvi i‡q‡Q| cwimsL¨v‡b GwU cvqB e¨envi Kiv n‡q _v‡K| KviY, GwU GKwU ep¸r msL¨K m¤vebv webvm‡K Lye Dc‡hvMx fv‡e eYbv K‡i, hv wm×všÍgj¶K cwimsL¨v‡bi †¶‡Î e¨vcKfv‡e e¨eüZ nq|

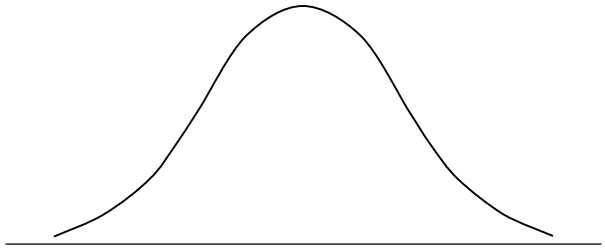
¯vfweK webvm n‡jv GKwU we‡kl ai‡Yi MYmsL¨v webvm, hv ev¯‡e A‡bK AwfÁZvjä webv‡mi Lye KvQvKwQ nq|

Ô¯vfweKÕ kãwU Ôˆbwgwë KÕ (usual) ev Ômvavi YÕ (common) A_ †evSv‡bvi Rb¨ e¨envi Kiv nq bv| cKZc‡¶, GwU ¯vfwe‡Ki Z¡bvq GKwU Kwlg wbgvY (construct) n‡Z cv‡i| hw` I ev¯‡e RM‡Z K`wPr GB webvmwUi †`Lv cvIqv hvq, wKš ZwË K webvm (theoretical distribution) wnmv‡e Gi ¸iæZ Acwimxg| ¯vfweK webvm Aa¨q‡bi ¸iæZ GB gyû‡Z cwZfvZ bv n‡j I cieZx Aa¨vq¸‡jv‡Z Gi e¨envi wU ¯úó n‡q DV‡e| AwfÁZvjä webvm‡K (empirical distributions) cwigvZ webv‡mi wbKUeZx i¡c cvIqv hvq e‡jB ïay †h ¯vfweK webvm Dc‡hvMx Zv bq, we‡kl K‡i Av‡ivnx cwimsL¨v‡b (inductive statistics) Gi ZwË K Zvrc‡hi Kvi‡Y I GwU we‡kl fv‡e ¸iæZc¥K|

¯vfweK webvm hw` I Av‡ivnx ev wm×všÍgj¶K cwimsL¨v‡bi (inductive or inferential statistics) AvËZvfy³, eYbvgj¶K cwimsL¨v‡b GwUi Av‡jvPbv Kiv n‡"Q cavbZt wZbwU Kvi‡Y| c_gZt, GwU GKwU mgmPvj †iLv| wØZxqZt, we¯‹Zi cwigvc wnmv‡e cwigvZ e¨eav‡bi e¨vL¨v †`evi Rb¨ GwU we‡kl fv‡e mvnvh¨ K‡i| ZZxqZt, cwimsL¨vbMZ cix¶vq (statistical tests) e¨envi‡ii j‡¶¨ GKwU ce- avivY †c‡Z mvnvh¨ K‡i| AZGe, ¯vfweK webvm m¤ú‡K Av‡jvPbvwU GLbB Kiv c‡qvRb hv‡Z cieZx‡Z Avgv‡`i Amyeavq co‡Z bv nq|

vfweK †iLv n‡jv GKwU myYg Ges GK cPzK m¤¢jZ web¨vm, hvi Mo, ga¨gv Ges cPzK hyMcrfv‡e GKB ¯v‡b wg‡j hvq|

myYbwU `ófv‡e ejv hvq †h, ¯vfweK †iLv n‡jv we‡klai‡Yi GKwU myYg gm„Y †iLv| †h‡nZz ¯vfweK †iLv wbLyZfv‡e gm„Y Ges Awb`ófv‡e epr msL¨K ch‡e¶‡Yi Dci wfwË K‡i MwVZ, †m‡nZzGwU‡K †Kejgvî cKZ.DcvË m¤¢jZ MYmsL¨v web¨vm w`‡qB mwVKfv‡e wbgvY Kiv m¤¢e| GwU AvKwZ‡Z GKwU N›Uvi (bell-shaped) gZ| †h‡nZzGwU GKwU myYg Ges GK cPzK m¤¢jZ (symmetrical and unimodal) web¨vm, †m‡nZzGi Mo, ga¨gv Ges cPzK hyMcrfv‡e GKB ¯v‡b wg‡j hvq| ¯vfweK web¨v‡mi mvaviY i¡cwU wPÎ 6.4.1-G †`Lv‡bv n‡jv|



wPÎ 6.4.1: ¯vfweK web¨v‡mi mvaviY i¡c

Avgiv c‡ei cvVmg‡n †R‡bwQ †h, GKwU ¯vfweK web¨v‡mi mPvjZvi gvb n‡jv 3 Ges †h‡nZz GwU GKwU myYg †iLv, †m‡nZzGwUi e¼gZvi gvb n‡jv ÔOÕ| GwU GKwU GK cPzK m¤úb web¨vm, hv ‡`L‡Z N›Uv AvKwZi Ges hv‡K m-2 fv‡e wbgwjwLZ m‡Îi gva¨‡g eYbv Kiv hvqt

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\frac{(x_i-\bar{x})^2}{s}}$$

‡hLv‡b, y = ‡h ‡Kvb c`Ë x gv‡bi Rb¨ ¯vfweK †iLvi D”PZv
x = ‡h †Kvb msL¨v gvb
e = natural logarithm-Gi wfwË, hvi gvb n‡jv 2.71828
π = ÔcvBÕ a‡eK, hvi gvb n‡jv 3.14159
σ = cwigvZ e¨eavb
$\bar{x}$ = MwYwZK Mo

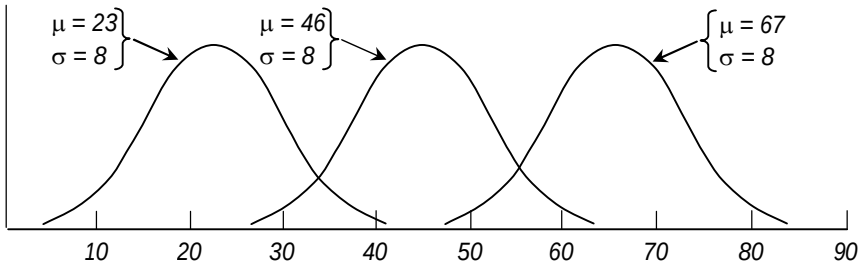
m‡Îi g‡a¨ j¶¨Yxq welq¸‡jv n‡jv †h, y-n‡jv †h †Kvb c`Ë x-Gi gv‡bi Rb¨ ¯vfweK †iLvi D”PZv| e Ges π n‡jv `ywU a‡eK gvb (constant value) hv Avgv‡`i wbYq Ki‡Z n‡e bv| m‡Îi †Kejgvî `ywU msw¶ß cwigv‡ci mv‡_ m¤ú„³ | GKwU n‡jv MwYwZK Mo ($\bar{x}$) Ges Aciwu n‡jv cwigvZ e¨eavb (σ)| AZGe, ¯vfweK †iLvi cKZ.i¡cwU Rvbv hv‡e h` Avgiv MwYwZK Mo I cwigvZ e¨eav‡bi gvb `ywU cvB| Ab¨ K_vq, we‡fb ¯vfweK †iLv cwZwU msw¶k- óweb¨v‡mi MwYwZK Mo I cwigvZ e¨eav‡bi mg¤¢q MwVZ|

¯vfweK †iLvi MwYwZK m‡Îi MwYZkv¯¨e`‡`i Kv‡Q A‡c¶vKZfv‡e mnR n‡j I GB chv‡qi cvVK‡`i Kv‡Q welqwU GZ mnR bq| Z‡e GLv‡b ¯¨i welqwU n‡jv †h, ¯vfweK †iLvi cwZwU we›`y(y-Gi gvb) wbY‡qi Rb¨ GB m‡Îi Avgv‡`i KLbI e¨envi Ki‡Z n‡e bv| KviY, †mB gvb¸‡jv cvevi Rb¨ cwigvZ ¯vfweK mviwY (standard normal table) c¯Z Kiv n‡q‡Q| AZGe, m‡Îi gyL¯ K‡i g‡b ivLvi I †Zgb c‡qvRb †bB| GLv‡b GwU D‡jL Kiv n‡q‡Q GB ZwËK web¨v‡mi wKQyewkó¨‡K †evSv I cix¶v Kivi Rb¨| ¯vfweK †iLv †h GKwU

we‡klai‡Yi †iLv, hv MwYwZK Mo I cwigZ e¨eav‡bi Dci wfwË K‡i iwPZ, Zv †evSv‡bvi Rb¨ GLv‡b GwUi D‡jL Kiv n‡q‡Q|

wZbwU web¨v‡mi wZbwU wfb Mo gvb Ges GKB cwigZ e¨eav‡bi gvb cvIqv hvq, †m ‡¶‡Î ¯vfweK †iLvi AvKwZi †Kvb cwieZb n‡e bv, Z‡e †iLv¸‡jvi Ae¯v‡bi cwieZb NU‡e|

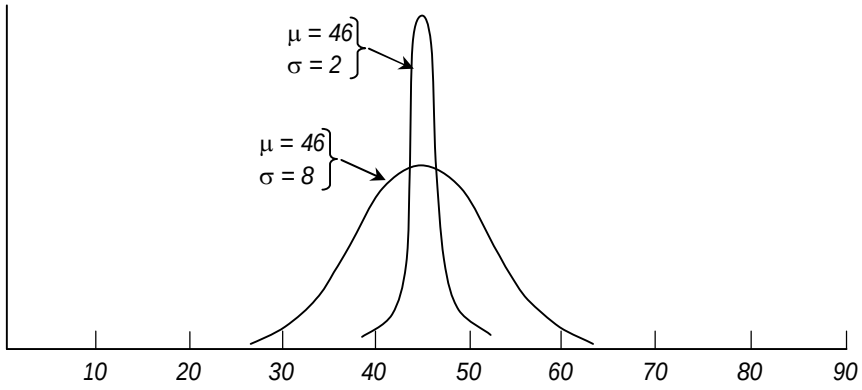
†Kvb ¯vfweK †iLvi myôzb`ó mgxKiYwU cvIqv †h‡Z cv‡i †mB web¨v‡mi MwYwZK Mo I cwigZ e¨eav‡bi h_vh_ gvb e¨envi K‡i| GB `ÿwU cwigv‡ci gv‡bi ZviZg¨ n‡j GKwU web¨v‡mi AvKwZ. I Ae¯v‡bi Dci Zvi cfve co‡e| †hgb, hw` wZbwU web¨v‡mi wZbwU wfb Mo gvb (h_vµ‡g 23, 46 Ges 67) Ges GKB cwigZ e¨eav‡bi gvb cvIqv hvq (†hgb, 8), †m ‡¶‡Î ¯vfweK †iLvi AvKwZi †Kvb cwieZb n‡e bv Z‡e †iLv¸‡jvi Ae¯v‡bi cwieZb NU‡e| wPÎ 6.4.2-G Zv †`Lv‡bv n‡jv|



wPÎ 6.4.2 : wZbwU ¯vfweK †iLvi Ae¯vbMZ wfbZv

hw` `ÿwU web¨v‡mi cwigZ e¨eav‡bi gvb wfb nq (h_vµ‡g 2 I 8) Ges Mo gvb GKB _v‡K (†hgb, 46), †m †¶‡Î †iLvi Ae¯v‡bi †Kvb cwieZb n‡e bv, Z‡e AvKwZi cwieZb n‡e| wPÎ 6.4.3-G Zv †`Lv‡bv n‡jv|

hw` `ÿwU web¨v‡mi cwigZ e¨eav‡bi gvb wfb nq Ges Mo gvb GKB _v‡K, †m †¶‡Î †iLvi Ae¯v‡bi †Kvb cwieZb n‡e bv, Z‡e AvKwZi cwieZb n‡e|



wPÎ 6.4.3 : `ÿwU wfb ¯vfweK †iLv

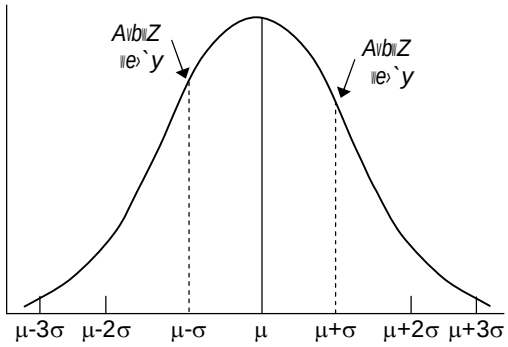
wPÎ `ÿwU chv‡jvPbv Ki‡j GKwU welq ¯úó n‡q D‡V †h, M‡oi cwieZ‡bi d‡j ¯vfweK †iLvi AvKwZi †Kvb cwieZb nq wb, wKš Ae¯v‡bi cwieZb n‡q‡Q Ges cwigZ e¨eav‡bi cwieZ‡bi d‡j †iLvi Ae¯v‡bi †Kvb cwieZb nq wb, wKš AvKwZi cwieZb n‡q‡Q| GKwU welq GL‡b we‡klfv‡e g‡b ivLv c‡qvRb †h, me myôg-N›Uv AvKwZi †iLvB ¯vfweK †iLv bq| GKB cwigZ e¨eavb nIqv m‡ÉI GK cPzK wewkó myôg †iLv ¯vfweK †iLvi †P‡q AwaK mPwj n‡Z cv‡i Avevi Kg mPwj n‡Z cv‡i| Avgiv Rwb †h, ‡h ‡iLv ¯vfweK †iLvi †P‡q †ekx mPwj Zv‡K AwZmPwj †iLv e‡j Ges †h †iLv ¯vfweK †iLvi †P‡q Kg mPwj nq Zv‡K AbwZmPwj ‡iLv e‡j| AwZmPwj I AbwZmPwj †iLvi mgxKiY¸‡jv MwYwZK Mo I cwigZ e¨eavb QvovI Ab¨vb¨ msw¶ß cwigv‡ci mv‡_ m¤ú„³|

¯vfweK web¨v‡mi ˆewkó¨ (Characteristics of Normal Distribution)

GKwU ¯vfweK web¨v‡mi wbgwjwLZ ˆewkó¨¸‡jv i‡q‡Q:

vfweK web¨vm n‡jv GKwU MwYwZK fv‡e msÁwqZ A‡c¶K, hv †Kej gvÎ wKQy MYmsL¨v web¨vm Øviv Avmb nq|

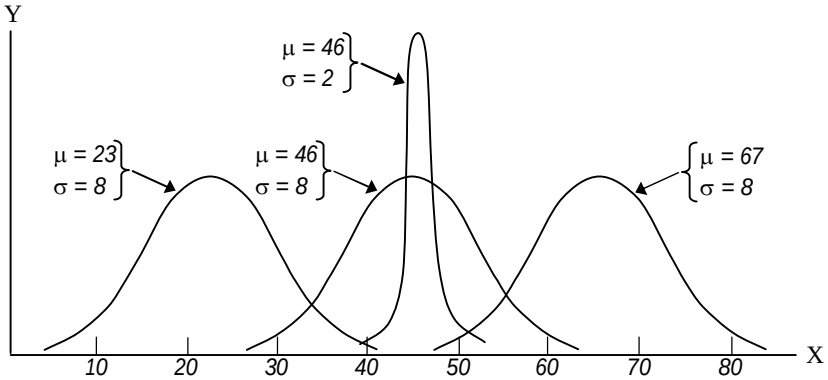
1. ¯vfweK web¨vm n‡jv GKwU MwYwZK fv‡e msÁwqZ A‡c¶K (mathematically defined function), hv †Kej gvÎ wKQyMYmsL¨v web¨vm Øviv Avmb (approximated) nq|
2. ¯vfweK web¨vm mylg Ges mgM‡Ki Mo gv‡bi Dci †K›`xfzZ, hv MxK eYgvjvi †QvU A¶‡ii 'μ' (ÔwgDÕ) cZxK Øviv wPwýZ nq| cwigZ †iLvi `ßi A‡aK mgMK Mo gv‡bi `ßi ‡K `ßwU gyLvgyL cwZwe¤ (mirror image) ˆZix K‡i|
3. ¯vfweK †iLv †h we›`yZ AeZj (concave) †_‡K DË‡j (convex) cwiewZZ nq, hv Avbwe›`y (point of inflection) bv‡g AvL¨wqZ nq, †mB we›`y‡jv Mo †_‡K Dfq w`‡K 1 cwigZ e¨eavb `i‡Z Ae¯vb K‡i| wPÎ 6.1.1-G Zv wPwýZ K‡i ‡`Lv‡bv n‡jv|



wPÎ 6.1.1t GKwU ¯vfweK †iLvi Avbwe›`yPwýZKiY

vfweK †iLvi cvš– `ßwU Abyfwg K †iLvi KvQvKwQ Ae¯vb K‡i, wKš KL‡bvB ¯úk K‡i bv|

4. ¯vfweK †iLvi cvš `ßwU Abyfwg K †iLvi KvQvKwQ Ae¯vb K‡i, wKš KL‡bvB ¯úk K‡i bv| Zvi A_ n‡jv, me‡P‡q D"P Ges wbg x-gv‡bi Rb¨ †Kvb ÔkbÕ y-gvb nq bv| Z‡e GB gvb¸‡jv cvq k‡b¨i KvQvKwQ nq|
5. cKZc‡¶, Amxg msL¨K ¯vfweK †iLv i‡q‡Q| A_vr, Mo (μ) I cwigZ e¨eav‡bi (σ) cwZwU m¤ve¨ gv‡bi Rb¨ GKwU wfb ¯vfweK †iLv i‡q‡Q, hvi wKQybgybv wPÎ 6.1.2-G c`kb Kiv n‡jv|



wPÎ 6.1.2: cwiewZZ Mo I cwigZ e¨eav‡bi gv‡bi Dci wfwË K‡i K‡qK ai‡Yi ¯vfweK †iLv

wPÎwU‡Z j¶¨ Kiv hvq †h, Mo-Gi (μ) gvb cwieZb nevi mv‡_ mv‡_ Abyfwg K †iLvq ¯vfweK †iLvi Ae¯v‡bi cwieZb N‡U‡Q Ges cwiwgZ e¨eav‡bi (σ) gvb cwieZ‡bi mv‡_ mv‡_ web¨v‡mi e¨wßi cwieZ‡bi gva¨‡g ¯vfweK †iLvi i¡‡ci cwieZb N‡U‡Q|

6. ¯vfweK †iLvi GKwU m‡ev"P we›`yi‡q‡Q hv Mo gv‡b N‡U| Avgiv hZB Mo †_‡K `ßr`‡K bvg‡Z _wK ¯vfweK †iLvi D"PZv ZZB Kg‡Z _v‡K| †iLvwU fwgi w`‡K bvg‡Z _v‡K wKš KLbB fwg‡K ¯úk K‡i bv|

GKwU vfweK web¨v‡m Mo, ga¨gv I cPz‡Ki gvb mgvb nq|

7. ‡h‡nZz¯vfweK †iLvq GKwU gvÎ m‡ev"P we›`yi‡q‡Q, †m‡nZz¯vfweK †iLv n‡jv GKwU GK cPzK wewkó †iLv| A_vr, Gi GKwU gvÎ cPzK i‡q‡Q Ges GKwU ¯vfweK web¨v‡mi Mo, ga¨gv I cPz‡Ki gvb mgvb nq|

vfweK †iLv n‡jv GKwU GK cPzK wewkó †iLv|

8. ¯vfweK †iLvq c_g Ges ZZxq PZzK `ßvU ga¨gv †_‡K mgvb `i‡Z Ae¯vb K‡i|

9. GKwU ¯vfweK web¨v‡m $\beta_1 = 0$ Ges $\beta_2 = 3$ nq|

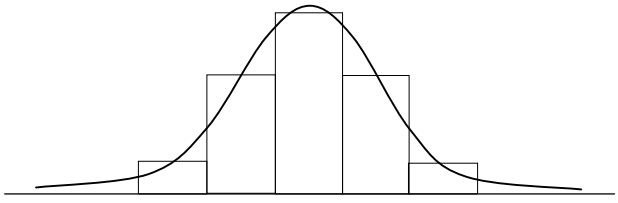
10. GKwU ¯vfweK †iLvq Mo †_‡K ±1, ±2 Ges ±3 cwiwgZ e¨eavb h_vμ‡g 68.27%, 95.45% Ges 99.74% AvqZb `Lj K‡i _v‡K|

mmxg ebvg Amxg MYmsL¨v web¨vm (Finite Versus Infinite Frequency Distributions)

Dcv‡Ëi msw¶ßKiY I Dc¯vcb m¤ú‡K Av‡jvPbv Ki‡Z wM‡q Avgiv BDwbU 3-G MYmsL¨v wb‡ekb m¤ú‡K wel`fv‡e †R‡bwQ| †mLv‡b Avgiv ch‡e¶Yjä DcvËiwk‡K †kYxe× K‡i KZ¸‡jv †kYx e¨wßi g‡a¨ NUbmsL¨v web¨vm ^Zix K‡iwQ, hv MYmsL¨v wb‡ekb wnmv‡e cwiwPZ n‡q‡Q| A‡bK ep,r msL¨vi DcvËiwk wb‡q MwVZ n‡j I cKZc‡¶ GB MYmsL¨v wb‡ekb¸‡jv mmxg msL¨K ch‡e¶Y Øviv c¯Z Kiv n‡q _v‡K|

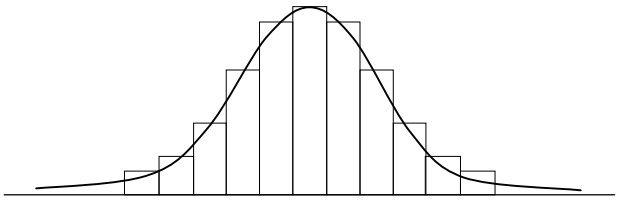
MwYZkv¯e`MY wKš cvqB Amxg msL¨K DcvËiwki wfwË‡Z MwVZ web¨vm‡K Dc‡hvMx g‡b K‡ib| AvqZ‡jL ev MYmsL¨v eûfyRi gva¨‡g cKwkZ LvBR KvUv ch‡e¶Yjä web¨v‡mi cwie‡Z Amxg msL¨K ch‡e¶‡Yi wfwË‡Z MwVZ GKwU gmwY †iLv Abygvb Kiv hvq, hv Zzjbvg¡‡Kfv‡e mij MwYwZK mgxKi‡Yi gva¨‡g cKwkZ nq| ¯vfweK †iLv n‡jv Ggb GKwU †iLv| wKfv‡e GB gmwY †iLvwU weKwkZ nq Zv †`Lv hvK|

aiv hvK, GKwU AvqZ‡j‡Li cvBwU †kYx e¨wß i‡q‡Q Ges Abygvb Kiv hvK †h, web¨vmwU myg| GB cvBwU †kYx e¨wß m¤úb KwíZ web¨vmwU‡K AvqZ‡j‡Li gva¨‡g Dc¯vcb Ki‡j †kYx e¨wß¸‡jv ck¯ n‡e Ges wPÎwU n‡e wbgi¡c,



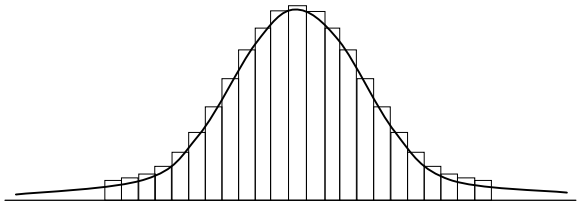
wPÎ 6.4.3: cvBu †kYx e¨wß m¤úb GKwU web¨vm AvqZ‡jL

wKš hw` ch‡e¶‡Yi msL¨v e„ô× Kiv hvq, Z‡e †kYx e¨wß¸‡jv msKxY n‡e Ges †mB †kYx e¨wß¸‡jv‡K AvqZ‡j‡L Dc¯¿cb Ki‡j wPÎwU n‡e wbgi¡c,



wPÎ 6.4.4 : AwaK msL¨K †kYx e¨wß m¤úb GKwU web¨v‡mi AvqZ‡j L

hw` ch‡e¶Y msL¨v Av‡iv e„ô× Kiv hvq, Z‡e †kYx e¨wßi msL¨v Av‡iv e„ô× cv‡e Ges †m¸‡jv Av‡iv msKxY n‡e| †mB †kYx e¨wß m¤wjZ web¨vmwU‡K AvqZ‡j‡L Dc¯¿cb Ki‡j wPÎ 6.4.5-Gi i¡c †b‡e|



wPÎ 6.4.5: Av‡iv AwaK msL¨K †kYx e¨wßm¤úb web¨v‡mi AvqZ‡j L

cwZwU PZ‡fR-Gi Aax‡b †h AvqZb i‡q‡Q Zv cwZwU †kYx e¨wßi Ašfy³ ch‡e¶‡Yi mgvbgvZ‡K cwZwbwaZ K‡i|

wPÎ¸‡jv‡K chv‡jvPbv Ki‡j †`Lv hvq †h, gm„Y †iLwU ˆZix n‡q‡Q cwZwU mg‡KvYx PZ‡fR ev AvqZ‡¶‡Îi (rectangles) Dc‡ii ga¨‑we›`y‡K †hvM K‡i| wPÎ¸‡jv †_‡K GwU ¯úó †h, PZ‡fR msL¨v hZ e„ô× †c‡q‡Q, †iLwU ZZ †ekx gm„Y n‡q‡Q| Ab¨ K_vq, cwZwU †kYx e¨wß hZ msKxY n‡q‡Q, †iLwU ZZ gm„Y n‡q‡Q| GB f‡e DË‡ivËi ch‡e¶Y msL¨v e„ô× Ges †kYx e¨wßi msKxYKi‡Yi d‡j GK mgq †`Lv hv‡e †h, †kYx e¨wß¸‡jv‡i g‡a¨ Avi †Kvb cv_K¨ _vK‡Q bv| d‡j, †iLwU gm„Y †_‡K gm„YZi n‡e| Avgiv Amxg msL¨K ch‡e¶Y wb‡q ev¯‡e KvR Ki‡Z bv cvi‡j I Kw•LZ gvÎvq ï×Zvi (accuracy) g‡a¨ e„nr msL¨K PZ‡fR‡K Abygvb Ki‡Z cwi, hv gm„Y †iLvq Avmb (approximate) n‡e| GwU g‡b ivLv c‡qvRb †h, cwZwU PZ‡fR-Gi Aax‡b †h AvqZb i‡q‡Q Zv cwZwU †kYx e¨wßi Ašfy³ ch‡e¶‡Yi mgvbgvZ‡K cwZwbwaZ K‡i|

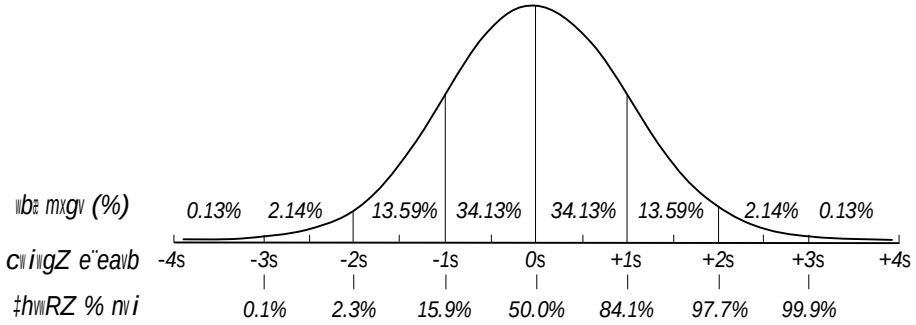
¯vfweK †iLvi Aax‡b AvqZb (Areas under Normal Curve)

GKwU vfweK †iLvi Mo I cwigvZ e¨eavb hvB †nvK bv †Kb Mo Ges †h †Kvb gv‡bi A‡¶i g‡a¨ GKwU wbw`ó AvqZb ev wbw`ó cwigvY ch‡e¶Y msL¨v _vK‡e, hv cwigvZ e¨eavb GK‡Ki †cw¶‡Z Mo †_‡K GKwU mybw`ó `i‡Z Øviv wbawiZ nq|

GKwU wbw`ó †kYx e¨wßi Aax‡b wK cwigvY ch‡e¶Y i‡q‡Q Zv wbi¡cY Kiv Avgv‡`i cvqB c‡qvRb n‡q c‡o| †mŠfvM¨ekZt, ¯vfweK †iLvi GKwU ¸i‡Zc¥ˆ ewkó¨ i‡q‡Q †hwU †mB KvR‡K Zzjbvgj¨Kfv‡e mnR K‡i †Zv‡j| GKwU ¯vfweK †iLvi Mo I cwigvZ e¨eavb hvB †nvK bv †Kb Mo Ges †h †Kvb gv‡bi A‡¶i (ordinate) g‡a¨ GKwU wbw`ó (constant) AvqZb ev wbw`ó cwigvY ch‡e¶Y msL¨v _vK‡e, hv cwigvZ e¨eavb GK‡Ki †cw¶‡Z Mo †_‡K GKwU mybw`ó `i‡Z Øviv wbawiZ nq|

Avgiv Rwb †h, GKwU ¯vfweK †iLvq Mo †_‡K ±1, ±2 Ges ±3 cwigvZ e¨eavb h_vµ‡g 68.27, 95.45, Ges 99.74 kZvsk AvqZb `Lj K‡i _v‡K| AZGe, Avgiv hw` Mo †_‡K

Wvb w`‡K GK cwiwgZ e¨eavb hvB, Z‡e Avgiv wbwðZfv‡e ej‡Z cwi †h, Mo †_‡K †mB we›`ªZ Aw¼Z A‡¶i AvqZb 34.13 kZvsk| AZGe, M‡oi `ßvi GK cwiwgZ e¨eavb `i‡Z `ßvU A‡¶i ga¨ wØ„Y AvqZb 68.27 kZvsk ch‡e¶Y‡K Aš—f©³ K‡i| Ab¨ K_vq, Mo †_‡K GK cwiwgZ e¨eav‡bi g‡a¨ memgq `ßvi ZZxq‡ki wKQy‡ekx cwigvY ch‡e¶Y _vK‡e| GKBfv‡e, Mo Ges `ßvi cwiwgZ e¨eavb `i‡Zi GKwU A‡¶i g‡a¨Kvi AvqZb memgqB 47.73 kZvsk n‡e Ges Mo †_‡K `ßvi cwiwgZ e¨eavb `i‡Zi GK †Rvov A‡¶i g‡a¨ 95 kZvs‡ki wKQy†ekx ch‡e¶Y _v‡K| hw` I ZËMZfv‡e GKwU ¯'vfweK †iLv `ßv`‡KB Amxgfv‡e we¯Z._v‡K, wKš cKZc‡¶ Mo †_‡K wZb cwiwgZ e¨eavb `i‡Zi g‡a¨ cvq me„‡jv (99.74 kZvsk) ch‡e¶Y Aš—f©³ _v‡K|



wPÎ 6.4.6: ¯'vfweK †iLvi Aax‡b AvqZb

Z‡e Mo †_‡K `i‡Z wbY‡qi Rb¨ memgq cwiwgZ e¨eav‡bi h_vh_ „wYZK (exact multiples of standard deviations) wb‡Z n‡e Ggb bq| GKwU ¯'vfweK †iLvi Aax‡b †h †Kvb `ßvU A‡¶i g‡a¨Kvi AvqZb wbYq Kiv m¤¢e| †hgb, Avgiv hw` M‡oi `ßvi cv‡k 1.96 cwiwgZ e¨eavb `i‡Z hvB, Z‡e Avgiv 95 kZvsk AvqZb cv‡ev Ges Mo †_‡K 2.58 cwiwgZ e¨eavb `i‡Z 99 kZvsk AvqZb Aš—f©³ n‡e| ¯'vfweK †iLvi GB ˆewkó¨wU cwiwgZ e¨eav‡bi e¨vL¨vq we‡klfv‡e mnvqK nq| Z‡e GwU g‡b ivLv c‡qvRb †h, ¯'vfweK †iLv cwiwgZ e¨eav‡bi e¨vL¨v c`v‡b mnvqK n‡j I cwiwgZ e¨eav‡bi msÁv c`v‡b GB ˆewkó¨wU‡K e¨envi Kiv hvq bv| ‡ek wKQyAwfÁZvjä MYmsL¨v web¨vm chvßi¡‡c ¯'vfweK web¨v‡mi gZ e‡j AvqZb Ges cwiwgZ e¨eav‡bi g‡a¨Kvi GB m¤úK„‡jv †hŠw³Kfv‡e c‡hvR¨ nq|

¯'vfweK †iLv cwiwgZ e¨eav‡bi e¨vL¨v c`v‡b mnvqK n‡j I cwiwgZ e¨eav‡bi msÁv c`v‡b GB ˆewkó¨wU‡K e¨envi Kiv hvq bv|

GKwU D`vniY w`‡q Mo †_‡K †h †Kvb msL¨v gv‡bi g‡a¨Kvi AvqZb wbYq K‡i †`Lv hvK| aiv hvK, GKwU ¯'vfweK †iLvi Mo n‡jv 50 Ges cwiwgZ e¨eavb n‡jv 10| GLb 50 †_‡K 66 A‡¶i g‡a¨ wK cwigvY ch‡e¶Y i‡q‡Q ev KZUkzAvqZb i‡q‡Q Zv †`Lv hvK| c_‡g Avgv‡`i †`L‡Z n‡e †h, 50 †_‡K 66 KZ cwiwgZ e¨eavb `i‡Z i‡q‡Q| GwU Ki‡Z wM‡q Avgv‡`i 50 I 66-Gi e¨eavb wbYq Ki‡Z n‡e Ges cvß djvdj‡K cwiwgZ e¨eav‡bi gvb w`‡q fvM Ki‡Z n‡e| G †¶‡Î wb‡gv³ m‡PwU e¨envi Kiv nq,

$$z = \frac{x_i - \bar{x}}{\sigma}$$

AZGe, $z = \frac{66 - 50}{10}$

$$= \frac{16}{10}$$
$$= 1.6$$

∴ wbYxZ z-gvb n‡j v 1.6

cvß gvbwU‡K Avgiv cwigZ gvb ev z-gvb e‡j _vwK| AZGe, GB cwigZ gvbwUB n‡j v Mo t_‡K †Kvb gv‡bi `i‡Zi gvb`Û| Avgv‡`i D`vni‡Yi †¶‡Î cvß cwigZ gvb 1.6-Gi A_ n‡j v 50 t_‡K 66 gv‡bi `i‡Z n‡j v 1.6 cwigZ e¨eavb| ¯vfweK †iLvi Aax‡b 1.6 cwigZ e¨eav‡bi AvqZb n‡j v 0.4452| A_vr, 50 t_‡K 66 gv‡bi g‡a¨ 44.52 kZvsk ch‡e¶Y Aš—f³ i‡q‡Q| †h‡nZz50 t_‡K 66 gvbwU Wvb w`‡K 1.6 cwigZ e¨eavb `i‡Z i‡q‡Q, †m‡nZz gvbwU abvZ¥K n‡q‡Q| wKš hw` 50 t_‡K 34 gv‡bi g‡a¨Kvi AvqZb wbYq Ki‡Z nq, †m †¶‡Î cvß cwigZ e¨eav‡bi gvbwU FYvZ¥K n‡e, A_vr, –1.6| Ab¨ K_vq, 50 t_‡K 66 gvbwU Mo t_‡K 1.6 cwigZ e¨eavb Dc‡i Ges 50 t_‡K 34 gvbwU –1.6 cwigZ e¨eavb bx‡P Ae¯vb Ki‡Q|

cwigZ ¯vfweK mviwYi e¨envi (Use of Standard Normal Table)

me ¯vfweK web¨v‡mi we‡k- l‡Y †h ZËxq †iLwU e¨envi Kiv nq Zv‡K cwigZ ¯vfweK †iLv e‡j | GB ¯vfweK †iLvi GKwU ÔOÕ Mo Ges GKwU Ô1Õ cwigZ e¨eavb i‡q‡Q|

me ¯vfweK web¨v‡mi we‡k- l‡Y †h ZËxq †iLwU e¨envi Kiv nq Zv‡K cwigZ ¯vfweK †iLv e‡j | GB ¯vfweK †iLvi GKwU ÔOÕ Mo Ges GKwU Ô1Õ cwigZ e¨eavb i‡q‡Q| c_vMZfv‡e, ¯vfweK †iLvi Aax‡b cÖviv AvqZb‡K Ô1Õ e‡j ai v nq| GKwU D`vniY w`‡q wel qwU‡K †evSv hvK| ai v hvK, wPÎ 6.4.6-G c`Ë ¯vfweK †iLvi Abyfwg K A¶ (horizontal axis) GKwU Pj‡Ki gvb Ges Dj¤ A¶ (vertical axis) wewfb gv‡bi NUbmsL¨v‡K cwZwbwaZ K‡i | GB ¯vfweK †iLvi Aax‡b Aš—f³ AvqZb‡K e¨envi K‡i KZ„‡j v c‡ki Reve †c‡Z cwi | †hgb, wK cwigvY ch‡e¶‡Yi gvb ÔKÕ t_‡K ep„Ë i‡q‡Q? wK cwigvY ch‡e¶‡Yi gvb ÔKÕ ‡_‡K ¶zZi i‡q‡Q? Ges wK cwigvY ch‡e¶‡Yi gvb ÔKÕ I ÔLÕ-Gi g‡a¨ i‡q‡Q? ÔKÕ I ÔLÕ GKwU Pj‡Ki †h †Kvb gvb n‡Z cv‡i | wKš †h wel qwU j ¶¨Yxq Zv n‡j v, cwZwU c‡ki mv‡_ GKwU e¨wßi wel q RwoZ, hv gj¬Zt Dj¤ A‡¶i G‡KKwU Ask| cwigZ ¯vfweK mviwY t_‡K h_vh_ AvqZb wbY‡qi gva¨‡g Avgiv G me c‡ki Reve w`‡Z cwi |

GKwU m¤úv¶ ¯vfweK web¨v‡mi Rb¨ cwigZ ¯vfweK mviwY e¨envi Ki‡Z n‡j Avgv‡`i g‡b ivL‡Z n‡e †h, web¨vmwU myfg Ges ÔOÕ M‡oi Dc‡i 50 kZvsk I bx‡P 50 kZvsk AvqZb i‡q‡Q|

cwigZ ¯vfweK mviwY ÔOÕ M‡oi Dc‡i wbw`ó e¨wßi Rb¨ ¯vfweK †iLvi Aax‡b wbw`ó AvqZb‡K wb‡`k K‡i | GKwU m¤úv¶ ¯vfweK web¨v‡mi Rb¨ cwigZ ¯vfweK mviwY e¨envi Ki‡Z n‡j Avgv‡`i g‡b ivL‡Z n‡e †h, web¨vmwU myfg Ges ÔOÕ M‡oi Dc‡i 50 kZvsk I bx‡P 50 kZvsk AvqZb i‡q‡Q| GKwU cwigZ ¯vfweK mviwYi wZbwU ¯¤ i‡q‡Q| †m„‡j v n‡j v:

- z — cwigZ ¯vfweK †iLvi Dj¤ A‡¶i Dci GKwU we›`y
- z Ges $\bar{x}$-Gi g‡a¨ AvqZb — Mo $(\bar{x})$ t_‡K GKwU wbw`ó z-gvb chš we¯Z.GKwU e¨wßi ga¨eZx AvqZb|
- z-Gi evB‡i i gvb — z-gvb t_‡K Amxg chš we¯Z.GKwU e¨wßi ga¨eZx AvqZb|

mviwY 6.4.1-G cwigZ ¯vfweK mviwYi GKwU ms¶ß fvl¨ t`Lv‡bv n‡j v| m¤úv¶ mviwYwU cwik‡ói mviwY cwi-2 -G Dc¯wcZ n‡q‡Q|

mviwY 6.4.1: cwigvY ¯'vfweK mviwYi GKwU msw¶ß fvl¨

z	z Ges $\bar{x}$ -Gi gaeZx AvqZb	z-Gi evB‡ii AvqZb
0	0.0000	0.5000
0.25	0.0987	0.4013
0.50	0.1915	0.3085
0.75	0.2734	0.2266
1.00	0.3413	0.1587
1.25	0.3944	0.1056
1.50	0.4332	0.0668
1.75	0.4599	0.0401
2.00	0.4772	0.0228
2.25	0.4878	0.0122
2.50	0.4938	0.0062
2.75	0.4970	0.0030
3.00	0.4987	0.0013
3.50	0.4998	0.0002
4.00	0.49997	0.00003

c‡ei wegvb ck¸‡jvi Av‡jv‡K Gevi Avgiv ev¯'eagx ck Ki‡Z cwi| †hgb, ÔOÕ Ges Ô1Õ Gi g‡a¨ wK cwigvY ch‡e¶Y i‡q‡Q? †h‡nZzGB web¨v‡mi Mo ÔOÕ Ges †h‡nZzPj‡Ki gvb¸‡jv

¯'‡¤ †`qv Av‡Q, †m‡nZz, Ges $\bar{x}$ -Gi gaeZx AvqZ‡bi mgvbycvZ¸‡jv wØZxq ¯'‡¤ †`qv Av‡Q| G †¶‡Î ÔOÕ †_‡K 1.00-Gi g‡a¨ 34.13 kZvsk ch‡e¶Y i‡q‡Q| GLb Avgiv wØZxq ckwU Ki‡Z cwi| †hgb, 1.00 Gi †ekx KZ¸‡jv ch‡e¶Y i‡q‡Q? GB c‡ki DËiwU cvIqv hv‡e mviwYi ZZxq ¯'‡¤ †_‡K| z-gvb 1.00-Gi wecix‡Z c`Ë gvbwU n‡jv 0.1587| A_vr, 1.00-Gi †ekx ch‡e¶Y i‡q‡Q 15.87 kZvsk| ZZxq ckwU n‡jv, 1.00-Gi Kg wK cwigvY ch‡e¶Y i‡q‡Q? GB c‡ki Rev‡e ¯'vfweK †iLvi evg cv‡ki A‡aK AvqZ‡bi (0.5000) mv‡_ Mo Ges 1.00 Gi gaeZx AvqZb‡K (0.3413) †hvM Ki‡Z n‡e| AZGe, 1.00-Gi evg cv‡ki †gvU AvqZb n‡jv (0.5000+0.3413)=0.8413| A_vr, 1.00-Gi bx‡P GB web¨v‡mi 84 kZvs‡ki wKQyek‡x ch‡e¶Y i‡q‡Q|

GKwU ¸iæZc¥wIq G ‡¶‡Î g‡b ivLv c‡qvRb †h, mviwY‡Z †Kvb FYvZ¥K gv‡bi ¯¤ †bB| KviY, †h‡nZz ¯'vfweK †iLv myg nq, †m‡nZzÔOÕ †_‡K +1.00-Gi Aax‡b cvß AvqZb ÔOÕ †_‡K-1.00-Gi Aax‡b cvß AvqZ‡bi mgvb n‡e| wKš Gfv‡e AvqZb wbY‡qi cwigvY †ekx n‡j †h †Kvb mgq fy wbY‡qi m¤vebv †_‡K hvq| fy Kivi GB m¤vebv‡K Gov‡Z n‡j memgqB wPÎ A¼b K‡i AvqZb‡K wPwýZ K‡i wb‡Z n‡e|

mvivsk

cwimsL¨v‡bi me‡P‡q ¸iæZc¥GKwU web¨vm nj ¯'vfweK web¨vm| ¯'vfweK web¨vm n‡jv GKwU we‡kl ai‡Yi MYmsL¨v web¨vm hv ev¯'‡e AwfÁZvjä web¨v‡mi KvQvKwQ| ¯'vfweK web¨vm gj¨Z: wm×všÍgj¨K cwimsL¨v‡bi AvIZvf³| eYbvgj¨K cwimsL¨v‡b ¯'vfweK web¨v‡mi Av‡jvPbv ¯'vb cvq wZbwU Kvi‡Y| c_gZt, GwU GKwU mgmPwj †iLv, wØZxqZt, we¯'wZi cwigvc wnmv‡e cwigvY e¨eav‡bi e¨vL¨v †`evi Rb¨ GwU we‡klfv‡e mvnv‡h¨ K‡i, Ges ZZxqZt, cwimsL¨vbMZ cix¶vq e¨envi‡¶¨ GKwU ce-avi Yv cvevi Rb¨|

cv‡VvËi gj¨vqb

^be¨w³K ck

mwVK DË‡i i cv‡k wUK (√) wPý w`b –

1| ‡h †iLv¯fweK †iLvi †P‡q †ekx mPwj Zv‡K:
K. AbwZmPwj e‡j
L. AwZmPwj e‡j
M. mvavi YmPwj e‡j
N. Dc‡i i me¸‡jv

2| ¯vfweK †iLvi cKZ.i¡cwU Rvbvi Rb¨ c‡qvRb:
K. MvwYwZK Mo
L. cwigvZ e¨eavb
M. MvwYwZK Mo I cwigvZ e¨eavb
N. MvwYwZK Mo I †f`v¼

3| GKwU¯vfweK †iLvi Mo †_‡K ±3 cwigvZ e¨eavb ..... AvqZb `Lj K‡i _v‡K|
K. 68.27%
L. 77.62%
M. 95.45%
N. 99.74%

msw¶ß ck

1| ¯vfweK web¨v‡mi ^ewkó¨¸‡jv wK?

2| ¯vfweK †iLvi Aax‡b AvqZb ej‡Z wK †evSv‡bv nq?

iPbvgj¨K ck

1| ¯vfweK web¨v‡mi ¸iæZ Av‡jvPbv Kiæb|

2| ¯vfweK web¨vm wK? mmxg ebvg Amxg MYmsL¨vi web¨vm ej‡Z wK †evSvq Zv Av‡jvPbv Kiæb|