5

we¯wZi cwigvc
Measures of Dispersion

BDwbU 4-G Avgiv DcvËiwki †K›`xq ceYZvi wewfb i¡c Av‡jvPbv Ki‡Z wM‡q †`‡LwQ †h, †K›`xq ceYZvi mvi-ms‡¶cgj-K cwigvc¸‡jv GKwU web¨v‡mi cwZwbwaZkxj Av`k msL¨v gvb‡K wb‡`k K‡i| wKš GKwU web¨vm‡K eYbvi Rb¨ GB cwZwbwaZkxj Av`k gvb¸‡jv h‡_ó bq| KviY, web¨v‡mi Aš—f©³ DcvËiwk GKwU we‡kl ¯v‡b †K›`x fzZ nevi ceYZv c`kb Ki‡j I cwZwU gvb †mB †K›`xq gvbwU †_‡K wKQzdv wfb| GB wfbZvi gvÎv hZ †ekx n‡e †K›`xq Av`k gvb¸‡jvi cwZwbwaZkxjZv ZZ †ekx nvm cv‡e| A_vr, GKwU DcvËiwki web¨v‡mi g‡a¨ mgi¡cZv I wfbZvi `ÿZv Dcv`vbB we`¨gvb _v‡K| AZGe, DcvËiwki we¯wZ. m¤ú‡K Rvbv bv _vK‡j GKwU web¨v‡mi m¤úY© eYbv m¤úb Kiv hvq bv| Dcv‡Ëi cKwZ. I we‡klYi c‡qvRbxqZvi Dci wfwË K‡i we¯wZi wewfb cwigvc wbYq Kiv hvq| we¯wZi cwigvc¸‡jv †K›`xq ceYZvi cwigvc¸‡jvi cwZwbwaZkxjZvi gvÎv m¤ú‡K GKwU m²vi aviYv c`vb K‡i| we¯wZi `ÿZi‡Yi cwigvc i‡q‡Q — wbi¼k I Av‡cw¶K| wbi¼k cwigvc¸‡iv Dcv‡Ëi g‡a¨ wfb wfb cwigvc GK‡Ki gva¨‡g wfbZvi eYbv †`q Ges Av‡cw¶K cwigvc¸‡jv †Kvb GK‡K cKwkZ nq bv e‡j †m¸‡jv GKwU cwigZ gvÎvq wfbZvi Zzjbvi Rb¨ Dc‡hvMx|

GB BDwb‡U Avgiv †h cvV¸‡jv Aa¨qb Ki‡ev †m¸‡jv n‡jv:

- cvV - 1 : we¯wZi msÁv, cKwZ. I ¸iæZ
- cvV - 2 : we¯wZi wbi¼k cwigvc – 1: cwimi I PZz_ e¨eavb
- cvV - 3 : we¯wZi wbi¼k cwigvc – 2: Mo e¨eavb I kZnwiK cwimi
- cvV - 4 : we¯wZi wbi¼k cwigvc – 3: cwigZ e¨eavb
- cvV - 5 : we¯wZi wbi¼k cwigvc – 4: †f`v¼
- cvV - 6 : cwigZ e¨eavb I ‡f`v‡¼i e¨vL¨v Ges e¨envi
- cvV - 7 : we¯wZi Av‡cw¶K cwigvc – e¨eavbv¼ Ges cwigZ gvb
- cvV - 8 : we¯wZi cwigv‡ci wewfb c×wZi g‡a¨ Zzjbv Ges wbevPb

cvV - 1

we¯wZi msÁv, cKwZ. I ¸iæZ
Definition, Nature and Importance of Dispersion

GB cvV †k‡l hv Rvbv hv‡e —

- we¯wZi cwigvc wK
- we¯wZi cKwZ.
- we¯wZi cwigv‡ci ¸iæZ
- we¯wZi GKwU DËg cwigv‡ci ‰ewkó¨
- we¯wZi cwigv‡ci cKvi‡f`

we¯wZi cwigvc wK (What is Measures of Dispersion)?

web¨v‡mi Ašfy³ DcvEiwki we¯wZ. m¤ú‡K bv Rvb‡j GKwU web¨v‡mi m¤úY¨ eYbv m¤úb Kiv hvq bv|

†K›`xq ceYZvi cwigvc¸‡jvi Av‡jvPbvi mgq †`‡L wQ †h, †m¸‡jv GKwU web¨v‡mi Ašfy³ DcvËiwki ¸"Qe×Zv‡K eYbv K‡i, ga¨ gv‡bi Ae¯vb wbYq K‡i Ges AwaK cwigv‡Y NUgvb msL¨v‡K wPwýZ K‡i| GBme mvi-ms‡¶cgj-K cwigv‡ci gva¨‡g Avgiv GKwU web¨v‡mi Av`k gvb¸‡jv‡K cvB hv †mB web¨v‡mi cwZwbwaZkxj ˆewkó¨‡K cKvk K‡i Ges `ßi ev Z‡ZwaK web¨v‡mi g‡a¨ Zzjbv‡K mnRZi K‡i| wKš GKwU web¨vm‡K eYbvi Rb¨ GB cwZwbwaZkxj GKK gvbwU h‡_ó bq| web¨v‡mi Ašfy³ DcvËiwki we¯wZ. (dispersion of data) m¤ú‡K bv Rvb‡j GKwU web¨v‡mi m¤úY¨ eYbv m¤úb Kiv hvq bv| GB cv‡V Avgiv Dcv‡Ëi we¯wZ. m¤ú‡K Av‡jvPbv Ki‡ev|

G wel‡q Av‡jvPbvi c‡e †K›`xq gvb¸‡jvi cwZwbwaZkxjZvi welqwU we‡ePbv Kiv hvK| cwimsL¨vbMZ Dcv‡Ëi Ašwbwnz (inherent) ˆewkó¨wU n‡jv †h, mgi¡cZvi Dcv`vbwU (element of homogeneity) eRvq †i‡LB †m¸‡jv wewfb gvÎvq wfbZv (variability) c`kb K‡i _v‡K| GKwU D`vniY †`qv hvK| aiv hvK, 50 b¤‡ii GKwU cix¶vq 10 Rb Qv‡Î-Qv‡Îx wbgwjwLZ b¤i †c‡q‡Q,

20 23 24 26 27 28 28 29 30 45

GB msL¨v¸‡jvi MvwYwZK Mo n‡jv,

$$\bar{x} = \frac{\sum x_i}{N}$$

$$= \frac{20+23+24+26+27+28+28+29+30+45}{10}$$

$$= \frac{280}{10}$$

$$\therefore \quad \bar{x} = 28$$

wbYxZ Mo msL¨wU GB 10wU iwkgvjvi cwZwbwaZKvix gvb n‡j I j¶¨ Kiv hvq †h, gvÎ `ßiRb Qv‡Î-Qv‡Îx 28 b¤i †c‡q‡Q, 3 Rb †c‡q‡Q Zvi †P‡q †ekx hvi m‡ev"P msL¨wU n‡jv 45 Ges 5

Rb †c‡q‡Q Zvi †P‡q Kg hvi mewbg msL¨wU n‡jv 20| A_vr, cwZwU cwimsL¨wbK Dcv‡Ëi wfbZv GLv‡b j¶¨ Kiv hv‡”Q| ïayZvB bq, †`Lv hv‡”Q †h, DcvË iwkgvjv Mo †_‡K GKwU we¯Z.cwiwa wb‡q Qwo‡q i‡q‡Q| †K›`xq gvb †_‡K DcvËiwki GB Qwo‡q _vKvi ceYZv‡KB we¯wZ.e‡j|

we¯wZi GB cwiwa †h‡nZz20 †_‡K 45 chš Qwo‡q i‡q‡Q, †m‡nZzmn‡RB ck †Zvjv †h‡Z cv‡i ‡h, Mo gvb 25 wK mwZ¨B GB DcvËiwk‡K cwZwbwaZ K‡i? hw` K‡i, Z‡e KZUkzK‡i? GKwU web¨vm‡K mwVKfv‡e eYbvi Rb¨ GB ck `ÿZUi mgvavb AZ¨š Ri¤^ix| ‡h me †KŠkj c‡qvM K‡i G mKj c‡ki mgvavb cvIqv hvq †m¸‡jv we¯wZi cwigvc e‡j cwiwPZ| myweavb`ófv‡e ejv hvq †h, we¯wZi cwigvc n‡jv †m mKj cwigvc hv †K›`xq gvb †_‡K cwZwU gv‡bi Qwo‡q _vKvi Mo gvÎv‡K wbi¤^cY K‡i|

> we¯wZi cwigvc n‡jv †m me cwigvc hv †K›`xq gvb †_‡K cwZwU gv‡bi Qwo‡q _vKvi Mo gvÎv‡K wbi¤^cY K‡i|

we¯wZi cKwZ. (Nature of Dispersion)

DcvËiwki g‡a¨ we¯wZi cKwZ.‡K eÜ‡Z n‡j Avgv‡`i `ÿZU welq m¤ú‡K Rvb‡Z n‡e| c_gwU n‡jv, web¨v‡mi Ašfy³ DcvË iwkgvjvi cwZwU gv‡bi AvKvi KZUkzafb Ges cwZwU gvb Mo †_‡K KZUkzafb| wØZxq welqwU n‡jv, gvb¸‡jv M‡oi Pvicv‡k wKfv‡e Qwo‡q i‡q‡Q? A_vr, Mo gv‡bi Dc‡i †ekx gvb i‡q‡Q, bwK bx‡P †ekx gvb i‡q‡Q? GKwU D`vniY w`‡q welqwU ¯úó K‡i †evSv hvK| aiv hvK, 10 Rb Qv‡Î-Qv‡Îx `ÿZU cix¶vq AskMnY K‡i‡Q| djvdj cKv‡ki ci †`Lv †Mj ‡h, `ÿZU cix¶vq cvß b¤‡ii Mo gvb GKB, wKš cvß b¤i¸‡jvi g‡a¨ cv_K¨ i‡q‡Q| mviwY 5.1.1-G `ÿZU cix¶vi Mo gvbmn cvß b¤‡ii web¨vmwU Dc¯vcb Kiv n‡jv|

> DcvEiwki g‡a¨ we¯wZi cKwZ.‡K eÜ‡Z n‡j Avgv‡`i `ÿZU welq m¤ú‡K Rvb‡Z n‡e| c_gwU n‡jv, web¨v‡mi Ašfy³ DcvË iwkgvjvi cwZwU gv‡bi AvKvi KZUkzafb Ges cwZwU gvb Mo †_‡K KZUkzafb| wØZxq welqwU n‡jv, gvb¸‡jv M‡oi Pvicv‡k wKfv‡e Qwo‡q i‡q‡Q? A_vr, Mo gv‡bi Dc‡i †ekx gvb i‡q‡Q, bwK bx‡P †ekx gvb i‡q‡Q?

mviwY 5.1.1: `ÿZU cix¶vq 10 Rb Qv‡Î-Qv‡Îx cvß b¤‡ii web¨vm

cix¶v	cvß b¤i										Mo
cix¶v 1	25	25	26	27	27	29	29	30	30	32	28
cix¶v 2	20	23	24	26	27	28	28	29	30	45	28

†h‡nZz`ÿZU cix¶vq cvß b¤‡ii Mo gvb 28, †m‡nZz†h †KD GB Dcmsnv‡i DcbxZ n‡Z cv‡ib †h, `ÿZU cix¶vqB Qv‡Î-Qv‡Îx‡`i KZKvhZvi gv‡Îv GKB iKg| wKš `ÿZU DcvË web¨vm wKQyv wbweofv‡e ch‡e¶Y Ki‡j ‡`Lv hvq †h, cix¶v 1-G Qv‡Î-Qv‡Îx‡`i cvß b¤i¸‡jv Mo gv‡bi Lvgy KvQvKwQ| Mo gvb ‡_‡K mewbg gv‡bi cv_K¨ n‡jv 3 Ges m‡ev”P gv‡bi cv_K¨ n‡jv 4| G ‡¶‡Î ejv hvq †h, Mo gvbwU web¨vmwU‡K h_vh_fv‡e cwZwbwaZ K‡i‡Q| wKš cix¶v 2-G cwZwU b¤i Mo gvb †_‡K †ek Qwo‡q wQwU‡q i‡q‡Q| Mo †_‡K mewbg gv‡bi cv_K¨ n‡jv 8 Ges m‡ev”P gv‡bi cv_K¨ n‡jv 17| Mo gvb GB web¨vmwU‡K ‡Zgbfv‡e cwZwbwaZ K‡i bv ej‡jB P‡j| AZGe, Dcmsnv‡i ejv hvq †h, DcvËiwki g‡a¨ we¯wZ. GKwU ¯vfweK e¨vcvi Ges GB we¯wZi gvÎv GKwU web¨vm †_‡K Av‡iKwU web¨v‡m wfb †_‡K wfbZi n‡q _v‡K| Kv‡RB, ïaygvÎ †K›`xq gv‡bi wfwË‡Z DcvË web¨vm‡K eYbv Kiv h_vh_ bq| †K›`xq gv‡bi mv‡_ we¯wZi welqwU‡KI we‡ePbvq Avb‡Z n‡e|

we¯wZi cwigvt ci ¸iæZ (Importance of Measures of Dispersion)

Avgiv †R‡bwQ †h, †K›`xq ceYZvi cwigvc¸‡jv w`‡q Avgiv DcvËiwki †K›`xfe‡bi Mo ceYZv‡K Rvb‡Z cwi Ges we¯wZi cwigvc¸‡jv w`‡q ‡K›`xq gvb †_‡K cwZwU gv‡bi Mo we¯wZ‡K Rvb‡Z cwi| GKwU web¨vm‡K eYbv ev GKwaK web¨vm‡K Zæ‡bvi †¶‡Î †K›`xq gvb h‡_ó bq| we¯wZi cwigvc n‡jv GKwU AZ¨vek¨Kxq cwimsL¨vbK c‡qvRbxqZv| KviY, we¯wZi cwigvc¸‡jv GKwU DcvË iwkgvjvi wfZ‡i cwZwU gv‡bi fwgKwU‡K ¯úó K‡i †Zv‡j; GKwU DcvË iwkgvjvi wfZ‡i mgi¡cZv wbY‡qi Dcvq wnmv‡e e¨en¡Z nq; ‡K›`xq gv‡bi wbfi‡hvM¨Zv wbavi‡Y mvnvh¨ K‡i; DcvËiwki g‡a¨ wfbZv wbqš‡Yi wfwË wnmv‡e KvR K‡i; `ßi ev Z‡ZwaK web¨v‡mi we¯wZ‡K Zæ‡bv Ki‡Z mvnvh¨ K‡i; Ges Ab¨vb¨ D"PZi cwimsL¨vbK †KŠk‡ji e¨envi‡K mnRZi K‡i|

DcvËiwki we¯wZ. cwigv‡ci ci hw` †`Lv hvq †h, DcvËiwki gvb¸‡jv †K›`xq gvb †_‡K Lye †ekx Qwo‡q Av‡Q, ZLb Avgiv ej‡Z cwi †h, web¨vmwUi g‡a¨ mgi¡cZvi Afve i‡q‡Q| Avi hw` gvb¸‡jv Lye KvQvKwQ _v‡K Z‡e ejv hvq †h, web¨vmwU mgi¡c i‡q‡Q Ges ‡K›`xq gvb G ‡¶‡Î h_vh_fv‡e cwZwbwaZkxj fwgKv cvjb K‡i‡Q|

we¯wZi GKwU DËg cwigv‡ci ˆewkó¨ (Properties of a Good Measure of Dispersion)

we¯wZi cwigvc¸‡jvi cwZwUi wKQyKvhvejx i‡q‡Q Ges wKQymwgwaev I mxgve×ZvI i‡q‡Q| we¯wZi GKwU DËg cwigv‡ci wbgwjwLZ ˆewkó¨mgn¨ _vKv c‡qvRb|

c_gZt, we¯wZi cwigvc‡K mnR‡eva¨ n‡Z n‡e|

wØZxqZt, GwU‡K mn‡R wbYq‡hvM¨ n‡Z n‡e|

ZZxqZt, GwU‡K mwbw`ófv‡e msÁwqZ n‡Z n‡e|

PZzZt, we¯wZi cwigvc‡K cieZx exRMwYwZK cwiMYbvi Dc‡hvMx n‡Z n‡e|

cÂgZt, GwUi bgybv w¯wZkxjZv _vK‡Z n‡e| Ges

lôZt, GwU‡K Pig gvb Øviv A‡hŠw³Kfv‡e cfweZ nIqv Pj‡e bv|

we¯wZi cwigv‡ci cKvi‡f` (Types of Measures of Dispersion)

we wZi cwigvc `ßvi‡Yi n‡q _v‡K – wbi¼k I Av‡cw¶K| c`Ë DcvË iwkgvjv †h GK‡K cwigvcKZ.nq

e¨vcK A‡_ we¯wZi cwigvc `ßvi‡Yi n‡q _v‡K – wbi¼k I Av‡cw¶K| c`Ë DcvË iwkgvjv †h GK‡K cwigvcKZ.nq (†hgb, UvKv, eQi, wK‡jvMvg, BZ¨w`) we¯wZi wbi¼k cwigvc¸‡jv †mB GK‡KB cKwkZ nq| GB cwigvc¸‡jv w`‡q `ßi ev Z‡ZwaK web¨v‡mi g‡a¨ Zæ‡bv Kiv m¤¢e nq hw` PjK¸‡jv GKB GK‡K cwigvcKZ.nq Ges cvq GKB †K›`xq gvb _v‡K| hw` PjK¸‡jv wfb GK‡K cwigvcKZ.nq Ges wfb †K›`xq gvb Øviv cwZwbwaZkxj nq †m †¶‡Î we¯wZi wbi¼k cwigvc¸‡jv Zæ‡bv‡hvM¨ _v‡K bv| G ai‡Yi cwiw¯wZ‡Z we¯wZi Av‡cw¶K cwigvc wbYq Ki‡Z nq|

we ¬wZi Av‡cw¶K cwigvc memgqB cwigv‡ci GKK †_‡K ¯vaxb n‡q _v‡K|

we¯wZi Av‡cw¶K cwigvc n‡jv M‡oi †cw¶‡Z we¯wZi wbi¼k cwigv‡ci Abyæwc| we¯wZi Av‡cw¶K cwigvc memgqB cwigv‡ci GKK †_‡K ¯vaxb n‡q _v‡K| G wel‡q cieZx cvV¸‡jv‡Z we¯wiZ Av‡jvPbv Kiv n‡e|

we¯ŵZi wbi¼k cwigvc¸‡jv n‡jv:

1. cwimi (Range)
2. AvšPZzK cwimi ev PZzK e¨eavb (The Interquartile Range or Quartile Deviation)
3. Mo e¨eavb (Mean Deviation)
4. cwigvZ e¨eavb (Standard Deviation)
5. †f`v¼ (Variance)

we¯ŵZi Av‡cw¶K cwigvc¸‡jv n‡jv:

1. PZzK e¨eavbv¼ (Coefficient of Quartile Deviation)
2. Mo e¨eavbv¼ (Coefficient of Mean Deviation)
3. e¨eavbv¼ (Coefficient of Variation)

mvivsk

†K›`xq gvb †_‡K DcvËiwki Qwo‡q _vKvi ceYZv‡KB we¯ŵZ e‡j | we¯ŵZi cwigvc `ßaiYi n‡q _v‡K — wbi¼k I Av‡cw¶K | †h GK‡K DcvË iwkgvjv‡K cwigvc Kiv nq, †mB GK‡KB wbi¼k cwigvc¸‡jv‡K wbYq Kiv nq | Ab¨w`‡K, hw` †Kvb DcvË wfb GK‡K cwigvcKZ.nq Ges wfb †K›`xq gvb Øviv cwZwbwaZkxj nq, †m †¶‡Î we¯ŵZi Av‡cw¶K cwigvc wbYq Ki‡Z nq wbi•Kk cwigv‡ci GKK †_‡K Av‡cw¶KZvq i¡cvši K‡i |

cv‡VwËi gj¨vqb

^be¨w³K ck

mwVK DË‡i i cv‡k wUK (√) wPý w`b –

1| e¨vcK A‡_ we¯wZi cwigvc KZ ai‡Yi?
   K. `ßvi ai‡Yi
   L. wZb ai‡Yi
   M. Pvi ai‡Yi
   N. cvßvi ai‡Yi

2| hw` DcvËi wkí gvb¸‡jv †K›`xq gvb †_‡K Lye †ekx Qwo‡q _v‡K Zvn‡j Avgiv ej‡Z cwi:
   K. web¨vmwUi g‡a¨ mgi¦cZvi Afve bvB|
   L. web¨vmwUi g‡a¨ †K›`xq ceYZv bvB|
   M. web¨vmwUi g‡a¨ mgi¦cZvi Afve i‡q‡Q|
   N. web¨vmwU g‡a¨ †K›`xq ceYZv i‡q‡Q|

3| ‡K›`xq gvb †_‡K cwZwU gv‡bi Qwo‡q _vKvq Mo gvÎv‡K wbi¦cY K‡i:
   K. Mo e¨eavb
   L. e¨eavbv¼
   M. cwigvZ e¨eavb
   N. Dc‡i i me KÛwU

msw¶ß ck

1| we¯wZi cwigvc wK?

2| we¯wZi cKwZ ej‡Z wK †evSvq?

iPbvgj¨K ck

1| we¯wZi cwigvc wK? we¯wZi GKwU DËg cwigv‡ci ^ewkó¨mgn Av‡jvPbv Kiæb|

2| wewfb ai‡Yi we¯wZi cwigvcmgn Av‡jvPbv Kiæb|

cvV - 2

we¯wZi wbi¼k cwigvc – 1: cwimi I PZzK e¨eavb
Absolute Measures of Dispersion – 1: Range and Quartile Deviation

GB cvV †k‡l hv Rvbv hv‡e —

- cwimi wK
- web¨¯ Dcv Ë †_‡K cwimi wbYq
- cwim‡ii mgvavb I Amgvavb
- Avav-AvšPZzK cwimi ev PZzK e¨eavb
- PZzK e¨eav‡bi mgvavb I Amgvavb

cwimi wK (What is Range)?

we¯wZi cwigv‡ci me‡P‡q mnR, mvaviY Ges ¯‹z cwigvcwU n‡jv cwimi| Dcv Ëiwki m‡ev"P I mewbg gv‡bi ga¨Kvi cv_K¨B n‡jv cwimi| cwimi wbY‡qi m‡¤§wU n‡jv,

we¯wZi cwigv‡ci me‡P‡q mnR, mvaviY Ges ¯‹z cwigvcwU n‡jv cwimi| Dcv Ëiwki m‡ev"P I mewbg gv‡bi ga¨Kvi cv_K¨B n‡jv cwimi|

cwimi = m‡ev"P gvb – mewbg gvb

cZx‡Ki ga¨‡g,

cwimi = L – S

†hLv‡b, L = m‡ev"P gvb

S = mewbg gvb

aiv hvK, 10 Rb e¨w³i IRb (gv‡bi D"Pµg Abyhv‡i)

110 115 120 127 130 132 137 140 142 152 cvDÛ

GLv‡b m‡ev"P gvb n‡jv 152 Ges mewbg gvb n‡jv 110 cvDÛ| m¤§Abyhvqx GB Dcv Ëiwki cwimi n‡jv,

cwimi = L – S
= 152 – 110
= 42

A_vr, 10 Rb e¨w³i IR‡bi e¨eavb n‡jv 42 cvDÛ| wKš Gfv‡e cwimi wbY‡qi GKwU mgm¨v i‡q‡Q| KviY, ewYZ mxgvq (stated limit) m‡ev"P Ges mewbg gvb GKwU cÖ‡msL¨vq cKwkZ nevi d‡j h_v_ mxgvwU (true limit) GLv‡b we‡ewPZ nqwb| Avgiv Rwb †h, hw` I e¨w³i IRb e‡×i ¶ZvZg cwigvY 1 cvDÛ-Gi GK‡K cKwkZ n‡q‡Q, wKš h_v_ mxgvwU n‡jv g‡Zt mewbg mxgvi 0.5 cvDÛ bx‡P Ges m‡ev"P mxgvi 0.5 cvDÛ Dc‡i| A_vr, Avgv‡`i D`vni‡Yi †¶‡Î hw` h_v_ mxgvq m‡ev"P gvb 152.5 Ges mewbg gvb 109.5 nq, Z‡e cwimi n‡e,

cwimi = 152.5 – 109.5 = 43.0

GLv‡b cwim‡ii gvb 1 †e‡o †M‡j I GwUB n‡jv cwim‡ii cKZ.Ges m-2 gvb| G welqwU‡K m‡Îi g‡a¨ Aš—f³ Kiv hvq Gfv‡e,

$$\text{cwimi} = (\text{m‡ev"P gvb} - \text{mewbg gvb}) + \text{cwigvc GK‡Ki ¶ZZg e„×i cwigvY}$$

Avgv‡`i D`vniY Abyhvqx, IRb e„×i ¶ZZg e„×i cwigvY n‡jv 1 cvDÛ (†h‡nZzc¥- msL¨vq cKwkZ n‡q‡Q)| AZGe,

$$\begin{aligned}\text{cwimi} &= (152 - 110) + 1\\ &= 42 + 1\\ &= 43\end{aligned}$$

GLv‡b mvaviY Áv‡bi GKwU welqI PgrKvi fv‡e we‡ewPZ n‡q‡Q hv GKwU åvwš‡K `i- K‡i‡Q Avgv‡`i A‡MvP‡iB| mvaviY fv‡e, Avgiv hLb †Kvb e¨eavb wbYq Kwi ZLb mewbg mxgvi c_g gvbwU‡K ev` w`‡q Kwi| †hgb, cwimi wbY‡qi †¶‡Î 152 †_‡K 110-Gi e¨eavb wbYq Ki‡Z M‡q MYbv ïi“ K‡iwQ 110-†K ev` w`‡q 111 †_‡K, hvi d‡j cvß djvdj n‡q‡Q 42| wKš 110-†K MYbv †_‡K ev` †`evi d‡j Avgiv gj-Zt DcvËiwk †_‡K Avgv‡`i A‡MvP‡iB 110 msL¨v gvbwU‡K ev` w`‡q †d‡jwQ Ges Gi d‡j cKZc‡¶ DcvËiwki †gvU msL¨v wM‡q `wo‡q‡Q 9 Rb, 10 Rb bq| Kv‡RB, cwimi wbY‡qi †¶‡Î MZvbyMwZK m¤ú„wU e¨envi bv K‡i cwikwjZ m¤ú„wU e¨envi Kiv †kq| hw` PjK¸‡jv µgmwPK gvÎvq cwigvcKZ.nq, †m †¶‡Î MZvbyMwZK m¤ú„wUB h‡_ó|

GLb GKwU ck Avm‡Z cv‡i †h, memgqB wK Avgiv m‡ev"P I mewbg gv‡bi e¨eav‡bi mv‡_ 1 †hvM Ki‡ev? DËiwU n‡jv ÔbvÕ| KviY, cwigvc GK‡Ki ¶ZZg e„×i cwigvY memgq GKB iKg _v‡K bv| cwigvc GK‡Ki ¶ZZg e„×i cwigvcwU wbawiZ nq GKwU GKK †_‡K Ab¨ GK‡Ki `i‡Zi Dci| mviwY 5.2.2-G wewfb cwigvc GK‡Ki ¶ZZg e„×i cwigvY-Gi wKQy D`vniY †`qv n‡jv|

mviwY 5.2.2: cwigvc GK‡Ki ¶ZZg e„×i cwigvY Ges cwimi wbY‡qi wKQyD`vniY

ch‡e¶‡Yi bgybv gvb					m‡ev"P gvb	mewbg gvb	¶ZZg e„×i cwigvY	cwimi
5.3	3.7	8.2	4.5	6.1	10.2	2.1	0.1	8.2
5.32	3.75	8.26	4.53	6.18	10.28	2.17	0.01	8.12
3000	5000	4000	2000	7000	10000	2000	1000	9000
450	350	250	750	600	1000	250	50	800
28	21	45	71	38	125	18	1	108
	$70\frac{1}{5}$	$80\frac{4}{5}$	$95\frac{2}{5}$	$62\frac{3}{5}$	$185\frac{3}{5}$	$45\frac{1}{5}$	$\frac{1}{5}$	$140\frac{3}{5}$

wewfbœ – DcvE †_‡K m-2 fv‡e cwimi wbYq Kiv hvq bv| KviY, GKwU MYmsL¨v wb‡ek‡b gj- DcvËiwk‡K Dc¯wcZ bv K‡i †m¸‡jv KZ¸‡jv †kYx‡Z wef³ K‡i MYmsL¨v wb‡ek‡b cKvk Kiv nq|

wewfbœ¯— DcvË †_‡K cwimi wbYq (Computing Range from Grouped Data)

wewfbœ¯— DcvË †_‡K m-2 fv‡e cwimi wbYq Kiv hvq bv| KviY, GKwU MYmsL¨v wb‡ek‡b gj- DcvËiwk‡K Dc¯wcZ bv K‡i †m¸‡jv KZ¸‡jv †kYx‡Z wef³ K‡i MYmsL¨v wb‡ek‡b cKvk Kiv nq| Avi GB MYmsL¨v wb‡ekb †_‡K cwim‡ii m-2 gvb wbYq Kiv bv †M‡j I Gi m¤ú‡K GKwU aviYv cvIqv hvq| aiv hvK, 10 Rb e¨w³ i gwmK Avq wbgi¡c,

4000 4400 4500 5000 5200 6100 6500 7500 7800 8000 UvKv

mviwY 5.2.3-G GB 10 Rb e¨w³i gwmK Av‡qi MYmsL¨v web¨vmwU Dc¯wcZ n‡jv|

mviwY 5.2.3t 12 Rb e¨w³i gwmK Av‡qi web¨vm

‡kYx mxgv (UvKvq)	NUbmsL¨v (e¨w³)
4000 – 5000	3
5000 – 6000	2
6000 – 7000	2
7000 – 8000	3
‡gvU	10

web¨ DcvE †_‡K cwimi wbYxZ nq m‡ev"P †kYxi EaŸ mxgvi gvb †_‡K mewbg †kYxi wbg mxgvi gv‡bi e¨eavb wbY‡qi gva¨‡g|

web¯— DcvË †_‡K cwimi wbYxZ nq m‡ev"P †kYxi Eaÿ mxgvi gvb †_‡K mewbg †kYxi wbg mxgvi gv‡bi e¨eüZ nq web¯— Dcv‡Ëi ‡¶‡Î †h m‡Î wU e¨eüZ nq web¯— Dcv‡Ëi ‡¶‡Î I GKB m‡Î e¨eüZ nq| A_vr,

cwimi = (m‡ev"P gvb – mewbg gvb) + cwigvc GK‡Ki ¶ÿZg e„×i cwigvY

GLv‡b m‡ev"P Av‡qi cwigvY 8000 UvKv Ges mewbg Av‡qi cwigvY 4000 UvKv Ges ¶ÿZg e„×i cwigvY 100 UvKv| AZGe,

cwimi = (8000 – 4000) + 100
= 4000 + 100
= 4100

∴ wbYxZ cwimi n‡jv 4100 UvKv

g‡b ivL‡Z n‡e †h, GwU Avbgwb‡K gvb, cKZ.gvb bq| hw` DcvËi wki gj¬ gvb ¸‡jv Rvbv bv hvq Ges mivmwi MYmsL¨v wb‡ekb †_‡K cwimi wbYq Ki‡Z nq, Z‡e cwigvc GK‡Ki ¶ÿZg e„×i cwigvY wbY‡qi †¶‡Î GKwU mgm¨v ˆZix nq| KviY, cwigvc GK‡Ki ¶ÿZg e„×i cKZ.cwigvYwU Rvbv hvq bv| †m ‡¶‡Î c¬ msL¨vi Rb¨ cwigvc GK‡Ki ¶ÿZg e„×i cwigvY wnmv‡e AšZ 1 †hvM Ki‡j åwši ‡Kvb AeKvk _v‡K bv|

cwim‡ii mvgvb Ges Amvgvb (Advantages & Disadvantages of Range)

we¯wZi cwigvc wnmv‡e cwim‡ii mvij¨ (simplicity) n‡jv Gi cavb mvgvb| Lvgv mn‡R Ges `ªZ Dcv‡Ëi we¯wZ. cwigv‡ci Rb¨ cwimiB me‡P‡q Dc‡hvMx cwigvc| G ‡¶‡Î DcvËi wki mKj gvb Rvbvi c‡qvRb nq bv| ïaygvÎ m‡ev"P I mewbg gvb RvbvB h‡_ó| Dciš, cwimsL¨vb m¤ú‡K hvi¨v †Zgb Ávb i‡Lb bv, Zv‡`i Rb¨ cwimiB we¯wZi me‡P‡q Dc‡hvMx cwigvc| hLb DcvËmgn¬ µgmwP‡K gvÎvq Dc¯wcZ nq ZLb cwimi me‡P‡q mvgvbRbK cwigvc| msL¨v gvb ¸‡jv‡K gv‡bi µg Abyw‡i mwR‡q m‡ev"P I mewbg gv‡bi cv_K¨ wbYq K‡i Avgiv Lvgv mn‡RB cwim‡ii gvb †c‡Z cwi| GLv‡b D‡jL Kiv c‡qvRb †h, bvgwP‡K gvÎvq cwigvcKZ.DcvË †h‡nZzgv‡bi µg Abyw‡i web¯— Kiv hvq bv, †m‡nZz¡mLv‡b cwimi wbYq Kiv m¤e nq bv|

we¯wZi cwigvc wnmv‡e cwimi‡ii cZ¨qMZ mvij¨ †hgb Gi mvgvav †Zgwb GwU Gi Amvgvav I | cwimi †Kvb cwikw¶Z cwigvc bq| GwU gvÎ `ÿvU gv‡bi wfwË‡Z wbYxZ nq, hv gj¨Zt DcvËiwki Pig `ÿvU gvb| †h‡nZz Pig gvb mvaviYZt `j¨f Ges A¯vfweK nq, †m‡nZz Pig gvb Øviv wbYxZ †Kvb cwigvc Av`k n‡Z cv‡i bv| A_vr, cwimi Pig gvb ev Amvavi Y msL¨v gvb Øviv Lvgv †ekx cfweZ nq| aiv hvK, 10 Rb e¨w³i gwmK Avq wbgi¡c,

5000 5700 6000 6300 8000 7800 6700 8500 8700 20000 UvKv

AZGe, cwimi = (20000 – 5000) + 100
= 15000 + 100
= 15100

G †¶‡Î †`Lv hvq †h, GKwU gvÎ Pig gv‡bi Kvi‡Y cwim‡ii gvb m‡eÿvP gvbwU Qvov cwZwU gv‡bi †P‡q A¯vfweKfv‡e eo n‡q‡Q, hv cÖkvmwU m¤ú‡K GKwU åwšg¨K avivYv †`q| †kYx web¯— Dcv‡Ëi †¶‡Î D¤§y³ †kYx e¨wß n‡j cwimi wbYq Kiv m¤e bq| cwimi bgybv wePvi Øviv ‡ekx cfweZ nq| ‡h‡nZz cwimi wbY‡q ïaygvÎ mewbg I m‡ev”P gv‡bi c‡qvRb nq, †m‡nZz ga¨eZx gvb¸‡jv m¤ú‡K cwimi †Kvb avivYv w`‡Z cv‡i bv| myZivs, web¨v‡mi mKj gv‡bi Rb¨ we¯wZi cwigvc wnmv‡e cwimi Av‡`Šwbfikxj bq|

cwimi gvÎ `ÿvU gv‡bi Dci wfwË K‡i wbYxZ Ges DcvËiwki gvb¸‡jv m¤ú‡K †Kvb avivYv w`‡Z m¶g nq bv| cwim‡ii GB mxgve×Zv `j¨Kivi D‡Ï‡k¨ PZzK e¨eavb ev Avav-AvšPZzK cwimi wbYq Kiv nq|

Avav-AvšPZzK cwimi ev PZzK e¨eavb (Semi-Interquartile Range or Quartile Deviation)

Avgiv Rwb ‡h, cwimi gvÎ `ÿvU gv‡bi Dci wfwË K‡i wbYxZ Ges DcvËiwki gvb¸‡jv m¤ú‡K †Kvb avivYv w`‡Z m¶g nq bv| cwim‡ii GB mxgve×Zv `j¨Kivi D‡Ï‡k¨ PZzK e¨eavb ev Avav-AvšPZzK cwimi wbYq Kiv nq| GwU GK cKv‡ii cwimi, Z‡e DcvËiwki D”PZg I wbgZg gv‡bi cv_‡K¨i bq| GLv‡b DcvËiwki c_g I ZZxq PZz‡Ki ga¨Kvi cv_K¨ we‡ePbv Kiv nq| DcvËiwk‡K gv‡bi D”P µg Abyfv‡i mwR‡q ‡gvU gvb¸‡jv‡K mgvb Pvi fv‡M fvM K‡i wb‡Z nq| Gi G‡KKwU fvM‡K PZzK e‡j | AvšPZzK cwimi n‡jv ZZxq PZzK gvb †_‡K c_g PZzK gv‡bi e¨eavb| cZx‡Ki gva¨‡g mÂwU n‡jv,

$$Avš PZzK\ cwimi = Q_3 - Q_1$$

AvšPZzK cwimi‡K 2 w`‡q fvM K‡i Avav-AvšPZzK cwimi ev PZzK e¨eav‡b cwiYZ Kiv nq| A_vr, PZzK e¨eavb n‡jv c_g I ZZxq PZz‡Ki ga¨Kvi e¨eav‡bi A‡aK| cZx‡Ki gva¨‡g mÂwU n‡jv,

$$PZzK\ e¨eavb = \frac{Q_3 - Q_1}{2}$$

GKwU D`vniY †`qv hvK| aiv hvK, 12 Rb e¨w³i gwmK Avq (gv‡bi µg Abyfv‡i) wbgi¡c,

3900 4000 4200 5300 5700 6000
6200 6200 6700 7200 8000 8200 UvKv

GB iwkgvjvi ZZxq PZzK (Q_3) n‡e 6700 Ges c_g PZzK (Q_1) n‡e 4200| AZGe,

$$\text{PZz_K e¨eavb} = \frac{Q_3 - Q_1}{2}$$

$$= \frac{6700 - 4200}{2}$$

$$= \frac{2500}{2}$$

$$= 1250$$

∴ wbYxZ PZz_K e¨eavb n‡j v 1250 UvKv

PZz_K e¨eav‡bi mvgvaweK I Amvgvaweav (Advantages and Disadvantages of Quartile Deviation)

ga¨gv †hgb †K›`xq ceYZvi GKwU Ae¯vbg~jK cwigvc (positional measure), PZz_K e¨eavb †Zgwb we¯wZi cwigv‡ci GKwU Ae¯vbg~jK cwigvc| PZz_K e¨eavb Pig gvb Øviv Kg cfweZ nq e‡j GwU Zvi bvg~jK fv‡e AwaKZi w¯wZkxj cwigvc| GwU µgmPK gvÎvq cwigvcKZ.Dcv‡Ëi †¶‡Î we‡klfv‡e c‡hvR¨| PZz_K e¨eavb gviv‡Z¡Kfv‡e e¼g web¨v‡mi †¶‡Î AwaKZi Dc‡hvMx cwigvc| Db¥y³ †kYx web¨v‡mi †¶‡Î we¯wZi cwigv‡ci Rb¨ GwUi we‡kl Dc‡hvMxZv i‡q‡Q| PZz_K e¨eavb A‡bK †¶‡Î cwimi A‡c¶v †ekx DbZZi GKwU cwigvc e‡j we‡ewPZ nq|

we¯wZi cwigvc wnmv‡e PZz_K e¨eav‡bi wKQyAmvgvaweav I i‡q‡Q| GwU GKwU web¨v‡mi 50 fvM Dcv Ëiwk‡K AeÁv K‡i| A_vr, c_g 25 kZvsk Ges †kl 25 kZvsk Dcv Ëiwk‡K KLbB we‡ePbvq Av‡b bv| PZz_K e¨eavb †h‡nZzwb¨v‡mi cwZwU msL¨v gvb Øviv wbYxZ nq bv, †m‡nZzGwU‡K we¯wZi GKwU DËg cwigvc wnmv‡e we‡ePbv Kiv hvq bv| PZz_K e¨eavb exRMwYwZK cwiMYbvi Dc‡hvMx bq| bgybv wePvi Øviv GwU Lvb †ekx cfweZ nq| Ae¯vbg~jK cwigvc nevi Kvi‡Y A‡bK cwimsL¨vbwe` PZz_K e¨eavb‡K we¯wZi cwigvc wnmv‡e ¯xKvi Ki‡Z Pvb bv| GwU‡K we¯wZi cwigvc bv e‡j Ae¯v‡bi cwigvc e‡j AwfwnZ K‡i _v‡Kb| KviY, GwU Mo †_‡K cwZwU gv‡bi Qwo‡q _vKvi welqwU c`kb bv K‡i GKwU gvb †_‡K Av‡iKwU gv‡bi `i‡Z¡K wb‡`k K‡i|

mvivsk

we¯wZi cwigv‡ci me‡P‡q mnR cwigvcwU n‡jv cwimi| cwimi Rvbvi Rb¨ Dcv Ëiwki mKj gvb Rvbvi c‡qvRb nq bv| ïaygvÎv m‡ev"P I mewbg gvb RvbvB h‡_ó| †h‡nZzcwimi gvÎ `ÿwU gv‡bi Dci wfwË K‡i wbYxZ, †m‡nZzDcv Ëiwki Ab¨vb¨ gvb¸‡jv m¤ú‡K †Kvb avivYv w`‡Z m¶g nq bv| GB mxgve×Zv `i~ Kivi Rb¨ PZz_K e¨eavb ev Avav-AvšPZz_K cwimi wbYq Kiv nq|

cv‡WvËi gj¨vqb

^be¨vw³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| cwimi wbY‡qi m¤ú~n‡jv:

K. m‡ev”P gvb – ga¨g gvb
L. m‡ev”P gvb – mewbg gvb
M. mewbg gvb + m‡ev”P gvb
N. ga¨g gvb × wbg gvb

2| GK‡Ki ¶ZvZg e„w×i cwigvcwU wbawiZ nq:

K. GKwU GK‡Ki mv‡_ Ab¨ GK‡Ki †hvMd‡ji Dci
L. GKwU GK‡Ki mv‡_ Ab¨ GK‡Ki ¸Yd‡ji Dci
M. GKwU GK‡Ki mv‡_ Ab¨ GK‡Ki cv_‡K¨i eMd‡ji Dci
N. GKwU GKK †_‡K Ab¨ GK‡Ki `i‡Zi Dci

3| cwimsL¨v‡bi mxgve×Zv `i~Kivi Rb¨:

K. DcvËiwki Mo wbYq Ki‡Z nq
L. DcvËiwki c_g I ZZxq PZz_‡Ki †hvMdj wbYq Ki‡Z nq
M. DcvËiwki c_g I ZZxq PZz_‡Ki ga¨Kvi cv_K¨ we‡ePbv Kiv nq
N. Dc‡ii me KÛwU

msw¶ß ck

1| cwim‡ii msÁv w`b |

2| D`vniYmn Avš-PZzK wbYq c×wZ Av‡jvPbv Ki¤b|

iPbvgj¨K ck

1| cwim‡ii mvgvav Ges Amvgvav¸‡jv Av‡jvPbv Ki¤b|

2| PZzK e¨eav‡bi mvgvav I Amvgvav¸‡jv Av‡jvPbv Ki¤b|

cvV - 3

we¯wZi wbi¼k cwigvc – 2: Mo e¨eavb I kZnwiK cwimi
Absolute Measures of Dispersion – 2: Mean Deviation and Percentile Range

GB cvV †k‡l hv Rvbv hv‡e —

- Mo e¨eavb wK
- Aweb¨¯ DcvË †_‡K Mo e¨eavb wbYq
- web¨¯ DcvË †_‡K Mo e¨eavb wbYq
- Mo e¨eav‡bi myeavi I Amyeavi
- kZnwiK cwimi

Mo e¨eavb wK (What is Mean Deviation)?

c‡ei cv‡V we¯wZi †h `ÿvU wbi¼k cwigvc (cwimi I PZz_K e¨eavb) m¤ú‡K Av‡jvPbv Kiv n‡q‡Q Zvi †KvbwUB DcvËiwki me¸‡jv gvb‡K we‡ePbvq G‡b we¯wZi cwigvc K‡i bv| A_P Avgiv Rwb †h, we¯wZi GKwU DËg cwigvc wnmv‡e we‡ewPZ n‡Z n‡j DcvËiwki me¸‡jv msL¨v gvb‡K we‡ePbvq Avbv c‡qvRb| hw` Avgiv me gvb‡K e¨envi Ki‡Z PvB, Zvn‡j mvaviY Ávb Avgv‡`i‡K GwUB ej‡e †h, `ÿvU cwiwš‘K we›`ÿzaKvi e¨eav‡bi w`‡K `wó wbe× bv K‡i Avgiv GKwU cwigv‡ci gvÎvi (measurement scale) †Kvb GKwU wbw`ó we›`y‡_‡K cwZwU gvb KZUkye¨eav‡b Qwo‡q i‡q‡Q Zvi wfwË‡Z we¯„wZi welqwU †evSvi †Póv Ki‡Z cwi| DcvËiwki gvb¸‡jv hZ ‡ekx Qwo‡q wQwU‡q _vK‡e †mB wbw`ó we›`y‡_‡K gvb¸‡jvi e¨eavb ZZ eo‡e| wKš †mB wbw`ó we›`ÿvi wK n‡e Ges gvb¸‡jv wKfv‡e Qwo‡q i‡q‡Q Zv Avgiv wKfv‡e wbaviY Ki‡ev? mgvavb wnmv‡e ejv hvq †h, †K›`xq ceYZvi †h †Kvb GKwU cwigvc †_‡K DcvËiwki cwZwU gv‡bi Ab‡c¶ wePvi (absolute difference) mgwó wbYq K‡i Zvi GKwU Mo wbYq Ki‡Z cwi|

†K›`xq ceYZvi cwigvc wnmv‡e cPwjK I ga¨gv e¨envi Kiv †M‡j I Avgiv mvaviYZt MwYwZK Mo‡KB Ab‡c¶ wePvi wbY‡qi Rb¨ e¨envi K‡i _wK| KviY, c_gZt, AwaKvsk †¶‡Î MwYwZK MoB n‡jv †K›`xq ceYZvi me‡P‡q DËg cwigvc| wØZxqZt, Mo †_‡K cwZwU gv‡bi e¨eav‡bi †hvM dj ÔOÕ nevi Kvi‡Y MwYwZK Mo‡K DcvËiwki gva¨vKl‡Yi †K›` (center of gravity) ejv nq| Ges ZZxqZt, MwYwZK Mo †_‡K cwZwU gv‡bi eMKZ. e¨eav‡bi †hvMdj memgqB Ab¨ †h †Kvb gvb †_‡K cwZwU gv‡bi eMKZ.e¨eav‡bi †hvMd‡j i Zzjbvq ¶zZi nq| we¯wZi cwigvc wbY‡q GB ‰ewkó¨¸‡jv ¸iæZc¥K|

> †K›`xq ceYZvi cwigvc wnmv‡e cPwjK I ga¨gv e¨envi Kiv †M‡j I Avgiv mvaviYZt MwYwZK Mo‡KB Ab‡c¶ wePvi wbY‡qi Rb¨ e¨envi K‡i _wK| KviY, AwaKvsk †¶‡Î MwYwZK MoB n‡jv †K›`xq ceYZvi me‡P‡q DËg cwigvc|

GLv‡b ck Avm‡Z cv‡i †h, Ab‡c¶ wePvi †bqv nq ‡Kb? Gi DË‡i ejv hvq †h, †h‡nZz Mo †_‡K cwZwU gv‡bi abvZ¥K Ges FYvZ¥K wePvi¸‡jvi †hvMdj ÔOÕ nq, †m‡nZz ÔOÕ †hvMdj‡K Gov‡Z n‡j FYvZ¥K wPý‡K ‡Kvb bv †Kvbfv‡e D‡c¶v Ki‡Z n‡e| FYvZ¥K wPý D‡c¶v Kivi GKwU c×wZ n‡jv Ab‡c¶ wePvi wbYq Kiv| AZGe, Mo e¨eavb n‡jv MwYwZK Mo †_‡K cwZwU gv‡bi Ab‡c¶ wePvi Mo|

Aweb¨ DcvË ‡_‡K Mo e¨eavb wbYq (Computing Mean Deviation from Ungrouped Data)

msÁv Abyôvqx Mo e¨eavb wbY‡qi mÁ‡wU‡K cZx‡Ki gva¨‡g wbgwjwLZ Dcv‡q cKvk Kiv hvq,

$$\text{M.D.} = \frac{\sum\left|\left(x_i - \bar{x}\right)\right|}{N}$$

‡hLv‡b, M.D. = Mo e¨eavb

x_i = DcvËiwki cwZwU gvb

$\bar{x}$ = MwYwZK Mo

N = DcvËiwki †gvU msL¨v

$\left|\ \right|$ = Ab‡c¶ `Û

$\left|\left(x_i - \bar{x}\right)\right|$ = MwYwZK Mo †_‡K cwZwU gv‡bi Ab‡c¶ wePvšÍZ

$\sum$ = mgwó wPý

GKwU D`vniYi gva¨‡g mÁ‡wU c‡qvM K‡i Aweb¨ DcvË †_‡K Mo e¨eavb wbYq K‡i †`Lv hvK| aiv hvK, 10 Rb e¨w³i eqm

18 21 23 27 30 36 40 43 49 50 eQi

Avgiv Rwb †h, Mo e¨eavb wbYq Ki‡Z n‡j c_‡g DcvËiwki MwYwZK Mo wbYq K‡i wb‡Z nq| AZGe, 10 Rb e¨w³i eq‡mi Mo n‡jv,

$$\bar{x} = \frac{\sum x_i}{N}$$

$$= \frac{18+21+23+27+30+36+40+43+49+50}{10}$$

$$= \frac{337}{10}$$

$$= 33.7$$

$\therefore \quad \bar{x} = 33.7$

GLb wbYxZ MwYwZK Mo †_‡K cwZwU gv‡bi Ab‡c¶ wePvšÍZ wbYq K‡i †mB Ab‡c¶ wePvšÍZ¸‡jvi mgwó wbYq Ki‡Z n‡e, hv mviwY 5.3.1-G †`Lv‡bv n‡jv|

mviwY 5.3.1: 10 Rb e¨w³i eq‡mi Mo e¨eavb wbYq

µwgK bs	x_i	$\left\|(x_i - \bar{x})\right\|$
1	18	15.7
2	21	1.7
3	23	10.7
4	27	6.7
5	30	3.7
6	36	2.3
7	40	6.3
8	43	9.3
9	49	15.3
10	50	16.3
‡gvU	337	99.0

GLb Ab‡c¶ wePvi mgwó gvbwU‡K m‡Î c‡qvM K‡i Avgiv Mo e¨eavb wbYq Ki‡Z cwi|

$$\text{M.D.} = \frac{\sum\left|(x_i - \bar{x})\right|}{N}$$

$$= \frac{99}{10}$$

$$= 9.9$$

∴ wbYxZ Mo e¨eavb n‡jv 9.9 eQi

AZGe, Avgiv ej‡Z cwi †h, 10 Rb e¨w³i eq‡mi cwZwU gvb Mo †_‡K Ab‡c¶fv‡e M‡o 9.9 eQi Qwo‡q i‡q‡Q|

web¨¯— DcvË †_‡K Mo e¨eavb wbYq (Computing Mean Deviation from Grouped Data)

‡kYx web¨¯— Dcv‡Ëi †¶‡Î Dcv‡Ëi g‡a¨ iwkgvjv Avgv‡`i nv‡Z _v‡K bv e‡j Ab‡c¶ wePvi wbY‡qi Rb¨ cwZwbwaZg‡a¨K gvb cvevi †¶‡Î mgm¨vi mgvb nq| GKwU cwZwbwaZkxj gvb cvevi Rb¨ cwZwU †kYx e¨wßi AšÍfyl³ gvb¸‡jv mswkó †kYx e¨wßi g‡a¨ mgvbfv‡e web¨¯— a‡i wb‡q cwZwU †kYx e¨wßi ga¨-we›`ywbYq Ki‡Z nq| AZGe, c_‡g cwZwU †kYx e¨wßi ga¨-we›`ywbY‡qi ci †mB MYmsL¨v wb‡ek‡bi Mo wbYq Ki‡Z nq| ZvicI, wbYxZ Mo †_‡K cwZwU †kYxi ga¨-we›`y‡jvi Ab‡c¶ wePvi wbYq K‡i cwZwU †kYxi Ab‡c¶ wePvi‡K †mB †kYxi NUbmsL¨v w`‡q ¸Y K‡i Ab‡c¶ wePvi mgwó wbYq Ki‡Z nq| me†k‡l, cvß gvbwU‡K †gvU NUbmsL¨v w`‡q fvM Ki‡Z nq| †kYx web¨¯— Dcv‡Ëi †¶‡Î Mo e¨eavb wbY‡qi m‡Î wU n‡jv,

$$\text{M.D.} = \frac{\sum\left|f_i(x_i - \bar{x})\right|}{N}$$

GLv‡b, f_i = cwZ †kYxi MYmsL¨v

x_i = ‡kYx ga¨-we› `y

$\bar{x}$ = MwYwZK Mo

N = ‡gvU NUbmsL¨v

$|(x_i - \bar{x})|$ = ‡kYx ga¨-we› `y‡_‡K MwYwZK M‡oi Ab‡c¶ wePvi Z

Mo e¨eavb wbY‡qi GKwU D`vniY mviwY 5.3.2-G Dc¯vcb Kiv nj|

mviwY 5.3.2: 20 Rb e¨w³i gwmK Av‡qi DcvË e¨envi K‡i Mo e¨eavb wbYq

gwmK Avq (UvKv)	‡jvK msL¨v (f_i)	ga¨-we› `y (x_i)	f_ix_i	$\sum\|(x_i - \bar{x})\|$	$\sum\|f_i(x_i - \bar{x})\|$
5000 - 6000	1	5500	5500	2800	2800
6000 - 7000	3	6500	19500	1800	5400
7000 - 8000	6	7500	45000	800	4800
8000 - 9000	3	8500	25500	200	600
9000 - 10000	4	9500	38000	1200	4800
10000 - 11000	2	10500	21000	2200	4400
11000 - 12000	1	11500	11500	3200	3200
‡gvU	**20**		**166000**		**26000**

c`Ë Dcv‡Ëi MwYwZK Mo n‡jv,

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$= \frac{166000}{20}$$

$$= 8300$$

$\therefore$ $\bar{x}$ = 8300 UvKv

Ges $\sum f_i |x_i - \bar{x}|$ = 26000

GLb cvß gvbwU‡K m‡Ĥ c‡qvM K‡i Avgiv Mo e¨eavb wbYq Ki‡Z cwi|AZGe,

$$\text{M.D.} = \frac{\sum f_i |x_i - \bar{x}|}{N}$$

$$= \frac{26000}{20}$$

$$= 1300$$

$\therefore$ wbYxZ Mo e¨eavb n‡jv 1300 UvKv

Mo e¨eav‡bi mycav I Amycav

Mo e¨eav‡bi cavb mycawU n‡jv GwU DcvËiwki me¸‡jv gvb‡K we‡ePbvq Av‡b| Zv QvovGwU h‡_ó mnR‡eva¨ GKwU cwigvc| Mo m¤ú‡K avi Yv Av‡Q Ggb e¨w³ i c‡¶ AwZ mn‡RB Mo e¨eavb‡K †evSv m¤e| Z‡e Mo †_‡K cwZwU msL¨vi wePvi wbY‡q Avgiv †h Ab‡c¶ Zv e¨envi Kwi Zv Mo e¨eavb‡K cieZx exRMwYwZK cwiMYbvi Rb¨ Aby‡hvMx K‡i ‡Zv‡j| Db³ †kYx e¨wß m¤wjZ Dcv‡Ëi †¶‡Î Mo e¨eavb wbYq Kiv hvq bv| †h‡nZzDcvËiwki me¸‡jv gvbB GLv‡b e¨eüZ nq, †m‡nZz‡Kvb GKwU gvb Rvbv bv _vK‡j Mo e¨eavb wbYq Kiv m¤e bq| Mo e¨eavb‡K ZËMZfv‡e e¨vL¨v Kiv hvq bv e‡j mvgwRK M‡elYvq GwUi e¨envi Lvb GKUv j¶¨ Kiv hvq bv|

Mo e¨eav‡bi cavb mycawU n‡jv GwU DcvËiwki me¸‡jv gvb‡K we‡ePbvq Av‡b|

kZnwiK cwimi (Percentile Range)

we¯wZ. cwigv‡ci Aci GKwU Ae¯vbwfwËK cwigvc n‡jv kZnwiK cwimi (pecrentile range)| KLbI KLbI GKwU web¨v‡mi wK cwigvY DcvËiwk GKwU wbw`ó cwim‡ii g‡a¨ Ae¯vb K‡i Zv Avgiv Rvb‡Z AvMnx nB| G ai‡Yi GKwU cwigvc n‡jv kZnwiK cwimi, hv mvaviYZt wk¶vgj¨K cwigv‡ci †¶‡Î e¨eüZ nq|

GB cwigvcwU web¨v‡mi mewbg 10 kZvsk Ges m‡ev"P 10 kZvsk DcvË‡K ev` w`‡q ga¨eZx 80 kZvsk DcvËiwk wb‡q KvR K‡i| kZnwiK-Gi mv‡_ PZz_K, ga¨gv Ges `knwiK (decile)-Gi GKwU m¤úK i‡q‡Q| †hgb,

D_1 = c_g `knwiK = 10g kZnwiK

D_2 = wØZxq `knwiK = 20Zg kZnwiK

Q_1 = c_g PZz_K = 25Zg kZnwiK

D_3 = ZZxq `knwiK = 30Zg kZnwiK

D_4 = PZz_ `knwiK = 40Zg kZnwiK

Q_2 = D_5 = wØZxq PZz_K = cÂg `knwiK = ga¨gv = 50Zg kZnwiK

D_6 = lô `knwiK = 60Zg kZnwiK

D_7 = mßg `knwiK = 70Zg kZnwiK

Q_3 = ZZxq PZz_K = 75Zg kZnwiK

D_8 = Aóg `knwiK = 80Zg kZnwiK

D_9 = beg `knwiK = 90Zg kZnwiK

GLv‡b j¶¨Yxq †h, 10g kZnwiK gvbwU n‡jv c_g `knwiK Ges 90Zg kZnwiK n‡jv beg `knwiK| †m Kvi‡Y, GB cwigvcwU‡K 10-90 kZnwiK cwimi e‡j| GwU mvaviY cwim‡ii gZ Pig gvb Øviv cfweZ nq bv| KviY, Pig gvb¸‡jv c_g I †kl 10 kZvs‡ki g‡a¨ _v‡K Ges kZnwiK cwimi †m¸‡jv we‡ePbvq Av‡b bv| hv †nvK, †mB mycawU GB cwigvcwUi GKwU gvivZ‡K mxgve×Zv ˆZix K‡i| †h‡nZzGwU web¨v‡mi me gvb‡K e¨envi K‡i bv, †m‡nZz 10g kZnwi‡Ki bx‡P ev 90Zg kZnwi‡Ki Dc‡ii msL¨v gvb¸‡jv Nb‡Zi mv‡_ i‡q‡Q bwK Qwo‡q wQwU‡q i‡q‡Q Zv Avgiv Rvb‡Z cwi bv| d‡j, we¯wZi Dci DcvËiwki cÖfve wU‡K Rvbv hvq bv|

mvivsk

GKwU DËg cwigvc wnmv‡e we‡ewPZ n‡Z n‡j DcvËiwki me¸‡jv msL¨v gvb‡K we‡ePbvq Avbv c‡qvRb| DcvËiwki gvb¸‡jv hZ †ekx Qwo‡q wQwU‡q _vK‡e, wbw`ó we›`y†_‡K gvb¸‡jvi e¨eavb ZZ evo‡e| gvb¸‡jv wKfv‡e Qwo‡q i‡q‡Q Zv wbavi‡Yi Rb¨ †K›`xq ceYZvi †h †Kvb GKwU cwigvc †_‡K DcvËiwki cwZwU gv‡bi Ab‡c¶ wePvi Zvi mgwó wbYq K‡i Zvi GKwU Mo wbYq K‡i †`L‡Z nq| G ‡¶‡Î mvaviYZt MwYwZK Mo‡KB mewaK e¨envi Kiv n‡q _v‡K| we¯wZi GB cwigvcwU‡K Mo e¨eavb e‡j| we¯wZ. cwigv‡ci Aci GKwU cwigvc n‡jv kZnwiK cwimi| kZnwiK cwimi †h‡nZzme gvb‡K e¨envi K‡i bv, †m‡nZz10g kZnwi‡Ki bx‡P Ges 90Zg kZnwi‡Ki Dc‡ii msL¨ gvb¸‡jv Nb‡Zi mv‡_ i‡q‡Q, bwK Qwo‡q wQwU‡q i‡q‡Q Zv Avgiv Rvb‡Z cwi bv|

cv‡VwËi gj¨vqb

^be¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| MwYwZK Mo †_‡K cwZwU gv‡bi Ab‡c¶ wePvi Mo‡K:
   K. ‡f`v¼ e‡j
   L. cwigvZ e¨eavb e‡j
   M. Mo e¨eavb e‡j
   N. cwimi e‡j

2| web¨¯— DcvË †_‡K Mo e¨eavb wbY‡qi m¤úwU n‡jv:
   K. $\text{M.D.} = \frac{\sum f_i \left| x_i - \bar{x} \right|}{N}$
   L. $\text{M.D.} = \frac{\sum \left(x_i - \bar{x} \right)^2}{N}$
   M. $\text{M.D.} = \frac{\sum \left(x_i - \bar{x} \right)^2}{N-1}$
   N. $\text{M.D.} = \frac{\sum \left(x_i^{\,2} - \bar{x} \right)}{N}$

3| we¯wZi cwigv‡ci Ae¯vb wfwËK cwigvc n‡jv:
   K. kZnvwiK cwimi
   L. c_g PZzK
   M. ZZxq PZzK
   N. Dc‡ii me KÛwU

msw¶ß ck

1| msÁv w`b I ^ewkó¨ Av‡jvPbv Kiæb:
   K. kZnvwiK cwimi
   L. web¨¯— DcvË †_‡K Mo e¨eavb wbYq

iPbvgj¨K ck

1| Mo e¨eavb wK? mgvR M‡elYvq Gi e¨envi Av‡jvPbv Kiæb|

2| Mo e¨eav‡bi mvgvb I Amvgvb Av‡jvPbv Kiæb|

cwV - 4

we¯wZi wbi¼k cwigvc – 3: cwiwgZ e¨eavb
Absolute Measures of Dispersion – 3: Standard Deviation

GB cwV †k‡l hv Rvbv hv‡e —

- cwiwgZ e¨eavb wK
- cwiwgZ e¨eav‡bi mÂ‡
- Awe‡b¨¯ DcvË †_‡K msÁvMZ m‡Îi gva¨‡g cwiwgZ e¨eavb wbYq
- Awe‡b¨¯ DcvË †_‡K msw¶ß c×wZ‡Z cwiwgZ e¨eavb wbYq
- we‡b¨¯ DcvË †_‡K msÁvMZ m‡Îi gva¨‡g cwiwgZ e¨eavb wbYq
- we‡b¨¯ DcvË †_‡K msw¶ß c×wZ‡Z cwiwgZ e¨eavb wbYq
- cwiwgZ e¨eav‡bi mvgvav I Amvgvav
- cwiwgZ e¨eav‡bi e¨envi

cwiwgZ e¨eavb wK (What is Standard Deviation)?

Avgiv †R‡bwQ †h, Mo e¨eavb DcvËiwki me gvb‡K we‡ePbvq G‡b we¯wZi eYbv Ki‡j I GwU wbY‡q FYvZ¥K wPý D‡c¶v Kivi Rb¨ Ab‡c¶ wePvi Z e¨eüZ nq e‡j GwU‡K ZËMZfv‡e e¨vL¨v Kiv hvq bv Ges cieZx exRMwYwZK cwiMYbvq e¨envi Kiv hvq bv| Avgiv GI †R‡bwQ †h, Mo gvb †_‡K cwZwU gv‡bi wePvi ÔOÕ †hvMdj‡K D‡c¶v Kivi Rb¨B Mo e¨eav‡b Ab‡c¶ wePvi wbYq Kiv nq| wKš FYvZ¥K wPý‡K D‡c¶v Kivi Av‡iKwU c×wZ i‡q‡Q hvi gva¨‡g wbYxZ gvbwU we¯wZi cwigv‡ci me‡P‡q ¸iæZc¥ ZËMZ e¨vL¨v c`vb K‡i| Ab‡c¶ wePvi wbYq bv K‡i wePvi¸‡jvi eM K‡i wb‡jB FYvZ¥K wPý¸‡jv mn‡RB AeÁv Kiv hvq Ges eMKZ.wePvi Zvi †hvMd‡j‡ji Mo wbYq K‡i Zvi eMg‡j wb‡j ZËMZfv‡e e¨vL¨v‡hvM¨ GKwU cwigvc wbYq Kiv hvq| ‡mB cwigvcwU n‡jv cwiwgZ e¨eavb|

cwiwgZ e¨eavb n‡jv MwYwZK Mo †_‡K cwZwU gv‡bi eMKZ.wePvi Zvi M‡oi eMg‡j|

myb`ófv‡e ejv hvq †h, cwiwgZ e¨eavb n‡jv MwYwZK Mo †_‡K cwZwU gv‡bi eMKZ.wePvi Zvi M‡oi eMg‡j| GwU wewfb cZx‡Ki gva¨‡g Dc¯vcb Kiv nq| GB cZxK¸‡jv Lye B¸iæZc¥ n‡q D‡V hLb Avgiv eYbvg‡j‡K cwimsL¨vb‡K AwZµg K‡i wmÜvšg‡j‡K cwimsL¨v‡b Aš—f³ bgybv I mgMK wb‡q Av‡jvPbv ïiæ Kwi| eYbvg‡j‡K cwi¯wZi ZvI hLb ïaygvÎ GKwU web¨v‡mi we¯wZi eYbv Ki‡Z nq, ZLb Bs‡iRx eYgvjvi †QvU A¶‡ii ÔsÕ-†K cwiwgZ e¨eav‡bi cZxK wnmv‡e e¨envi Kiv nq| hLb bgybv DcvË †_‡K mgM‡Ki ˆewkó¨‡K cv°jb Kiv nq ZLb wm×všg‡j‡K cwimsL¨v‡b bgybvi cwiwgZ e¨eavb‡K wPwýZ Kivi †¶‡Î cwiwgZ e¨eav‡bi cZxK wnmv‡e Ô§Õ-‡K e¨envi Kiv nq| GwU n‡jv cv°wjZ cwiwgZ e¨eavb| Ô§Õ cZxKwU‡K 's-hat' e‡j| wm×všg‡j‡K cwimsL¨v‡b mgM‡Ki cwiwgZ e¨eavb‡K wPwýZ Ki‡Z wM‡q MxK eYgvjvi †QvU A¶‡ii σ (wmMgv)-‡K e¨envi Kiv nq|

cwiwgZ e¨eavb wbY‡qi mÂ‡ (Formula for Computing Standard Deviation)

cwiwgZ e¨eav‡bi msÁwU‡K we‡klY Ki‡j Gi mÂwU Lye mn‡RB `k¨gvb n‡q D‡V| Avgiv Rwb †h, MwYwZK Mo †_‡K cwZwU gv‡bi eMKZ.wePvi Zvi M‡oi eMg‡j n‡jv cwiwgZ e¨eavb|

GB msÁwUi cwZwU Ask‡K e¨vß‡Z cvi‡jB m¤ú‡Îi Dcv`vb¸‡jv cvIqv hv‡e| †hgb,

$$\text{eMgj} = \sqrt{\quad}$$

$$\text{M‡oi} = \frac{\sum}{N}$$

MwYwZK Mo †_‡K cwZwU gv‡bi wePvi eM $= (x_i - \bar{x})^2$

cwigvZ e¨eavb $= s$

GB UK‡iv Ask¸‡jv msÁv Abyhvqx hvi hvi ¯v‡b ¯vcb Ki‡j Avgiv cÖ_g m¤úwU cvB|

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}}$$

‡hLv‡b, s = cwigvZ e¨eavb

x_i = Dcv‡Ëiwk‡i cwZwU gvb

$\bar{x}$ = MwYwZK Mo

N = †gvU NUbmsL¨v

$\sqrt{\quad}$ = eMgj

$(x_i - \bar{x})^2$ = Mo †_‡K cwZwU gv‡bi wePvi eM

$\sum$ = mgwó wPý

GB m¤úwU n‡jv msÁvMZ m¤ú| msÁvMZ m¤Îi gva¨‡g cwimsL¨vb wbYq c×wZ‡K mivmwi c×wZ e‡j|

Aweb¨¯— Dcv‡Ë †_‡K msÁvMZ m¤Îi gva¨‡g cwigvZ e¨eavb wbYq (Computing Standard Deviation from Ungrouped Data Using Definitional Formula)

GKwU D`vniYi gva¨‡g msÁvMZ m¤ú e¨envi K‡i Aweb¨¯— Dcv‡Ë †\_‡K cwigvZ e¨eavb wbYq K‡i †`Lv hvK| aiv hvK, 5 R‡bi GKwU `‡ji m`m¨‡`i IRb

182 200 203 205 210 cvDÛ

Ges 5 R‡bi Ab¨ GKwU `‡ji m`m¨‡`i IRb

140 168 190 200 302 cvDÛ

GB `ßiv `‡ji cwigvZ e¨eavb wbYq c×wZ mviwY 5.4.1-G †`Lv‡bv n‡jv|

mvivwY 5.4.1: msÁvwMZ m‡Îi gva¨‡g 5 m`m¨ wewkó `jvU `‡ji cwigvZ e¨eavb wbYq

c_g `j				wØZxq `j			
x_i	$\bar{x}$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	x_i	$\bar{x}$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
182	200	–18	324	140	200	–60	3600
200	200	0	0	168	200	–32	1024
203	200	3	9	190	200	–10	100
205	200	5	25	200	200	0	0
210	200	10	100	302	200	102	10404
1000			**458**	**1000**			**15128**

Avgv‡`i `jvU `‡ji m`m¨‡`i IR‡bi Mo wbYq K‡i wb‡Z n‡e| AZGe,

$$\bar{x}_1 = \frac{\sum x_i}{N} = \frac{1000}{5} = 200$$

$$\bar{x}_2 = \frac{\sum x_i}{N} = \frac{1000}{5} = 200$$

∴ wbYxZ MwYwZK Mo $\bar{x}_1 = 200$ ∴ wbYxZ MwYwZK Mo $\bar{x}_2 = 200$

GLb wbYxZ MwYwZK Mo e¨envi K‡i `jvU `‡ji cwigvZ e¨eavb wbYq Kiv hvq|

$$s_1 = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}} = \sqrt{\frac{458}{5}} = \sqrt{91.6} = 9.57$$

$$s_2 = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N}} = \sqrt{\frac{15128}{5}} = \sqrt{3025.6} = 55.01$$

∴ wbYxZ cwigvZ e¨eavb $s_1 = 9.57$ ∴ wbYxZ cwigvZ e¨eavb $s_2 = 55.01$

Aweb¨ DcvË †_‡K msw¶ß c×wZ‡Z cwigvZ e¨eavb wbYq (Computing Standard Deviation from Ungrouped Data Using Short-Cut Method)

Gi c‡e †h m‡Îwe¨envi K‡i cwigvZ e¨eavb wbYq Kiv n‡q‡Q †mwU n‡jv msÁvwMZ m‡Î| msÁvwMZ m‡Îwe cwigvZ e¨eavb wbY‡qi †h c×wZ wb‡`k K‡i Zv‡K cZ¨¶ c×wZ e‡j| msÁvwMZ m‡Îi mvaviY n‡jv, GKwU cwimsL¨v‡bi Rb¨ ‡hfv‡e nq Zv GB m‡Îi gva¨‡g †`Lv hvq| wKš Amyeavwe n‡jv, GB c×wZ‡Z cwigvZ e¨eavb wbYq AwaKZi kgmva¨| hLb DcvËiwki msL¨v A‡bK †ekx nq ZLb GwU mgqmva¨I e‡U| †m Kvi‡Y cwimsL¨vbwe`MY cwigvZ e¨eavb wbY‡qi msw¶ß c×wZ D™veb K‡i‡Qb| msw¶ß c×wZ‡Z cwZwU gv‡bi wePvi wbYq Ki‡Z nq bv| DcvËiwki mgvó Ges cwZwU gv‡bi e‡Mi mgvó wbYq Ki‡jB P‡j|

msw¶ß c×wZ‡Z cwigvZ e¨eavb wbY‡qi m¤úwU n‡jv,

$$s = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2}$$

‡hLv‡b, x_i = DcvËiwk‡i cwZwU gvb

$\sum x_i$ = DcvËiwk‡i mgwó

$\sum x_i^2$ = cwZwU gv‡bi e‡Mi mgwó

s = cwigvZ e¨eavb

mviwY 5.4.1-G c`Ë c_g `‡ji m`m¨‡`i IR‡bi DcvËiwk e¨envi K‡i mviwY 5.4.2-G msw¶ß c×wZ‡Z cwigvZ e¨eavb wbYq K‡i †`Lv‡bv n‡jv|

mviwY 5.4.2: Awe‡`¨ DcvË †_‡K msw¶ß c×wZ‡Z cwigvZ e¨eavb wbYq

x_i	x_i^2
182	33124
200	40000
203	41209
205	42025
210	44100
$\sum x_i$ = **1000**	$\sum x_i^2$ = **200458**

cvß gvb¸‡jv m¤ú‡K© c‡qvM K‡i Avgiv cwigvZ e¨eavb wbYq Ki‡Z cwi|

$$s = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2}$$

$$= \sqrt{\frac{200458}{5} - \left(\frac{1000}{5}\right)^2}$$

$$= \sqrt{40091.6 - (200)^2}$$

$$= \sqrt{40091.6 - 40000}$$

$$= \sqrt{91.6}$$

$$= 9.57$$

∴ wbYxZ cwigvZ e¨eavb n‡jv 9.57

web¨— DcvË †_‡K msÁvMZ m‡Îi gva¨‡g cwigZ e¨eavb wbYq (Computing Standard Deviation from Grouped Data Using Definitional Formula)

Avgiv Rwb †h, msÁvMZ m‡Îi gva¨‡g cwimsL¨vb wbY‡qi c×wZ‡K cZ¨¶ c×wZ e‡j | weB¨— DcvË †_‡K cZ¨¶ c×wZ‡Z cwigZ e¨eavb wbY‡qi m‡Iwu n‡jv,

$$s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$$

‡hLv‡b, s = cwigZ e¨eavb

f_i = cwZwU †kYxi NUbmsL¨v

x_i = ‡kYx e¨wßi ga¨-we›`y

$\bar{x}$ = MwYwZK Mo

N = ‡gvU NUbmsL¨v

$(x_i - \bar{x})^2$ = Mo †_‡K cwZwU †kYx ga¨-we›`y wePvi eM

m‡Îi c‡qvRbxq gvb¸‡jv †c‡Z n‡j c_‡g DcvËi wbYq Mo wbY‡q Ki‡Z n‡e| wØZxq av‡c, wbYxZ Mo †_‡K cwZwU †kYx e¨wßi ga¨-we›`y wePvi wbY‡q Ki‡Z n‡e| Gici, wePvi¸‡jvi eM K‡i wb‡q cwZwU †kYxi NUbmsL¨v w`‡q mswkó wePvi eM¸‡jv‡K ¸Y K‡i cvß ¸Yd‡ji mgwó wbYq Ki‡Z n‡e| mvwY 5.4.3-G †m¸‡jv av‡c av‡c wbYq K‡i †`Lv‡bv n‡jv|

mvwY 5.4.3: weB¨— DcvË †_‡K msÁvMZ m‡Îi gva¨‡g cwigZ e¨eavb wbYq

‡kYx e¨wß (C.I.)	†kYx e¨wßi ga¨-we›`y (x_i)	NUbmsL¨v (f_i)	($f_i x_i$)	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
1-9	5	400	2000	–12.1	146.41	58564.00
10-18	14	500	7000	–3.1	9.61	4805.00
19-27	23	300	6900	5.9	34.81	10443.00
28-36	32	150	4800	14.9	222.01	33301.50
37-45	41	100	4100	23.9	571.21	57121.00
‡gvU		**1450**	**24800**			**164234.50**

$$\therefore \quad \bar{x} = \frac{\sum f_i x_i}{N}$$

$$= \frac{24800}{1450}$$

$$= 17.10$$

$$\therefore \quad \bar{x} = 17.10$$

cvß gvb¸‡jv m‡Î c‡qvM K‡i Avgiv cwigZ e¨eav‡bi gvbwU cvB|

$$AZGe,\ s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{N}}$$

$$= \sqrt{\frac{164234.5}{1450}}$$

$$= \sqrt{113.27}$$

$$= 10.64$$

$\therefore$ wbYxZ cwigZ e¨eavb n‡jv 10.64

web¨¯— DcvË †_‡K msw¶ß c×wZ‡Z cwigZ e¨eavb wbYq (Computing Standard Deviation from Grouped Data Using Short-Cut Method)

‡kYx web¨¯— DcvË †_‡K msw¶ß c×wZ‡Z cwigZ e¨eavb wbY‡qi †¶‡Î c_‡g †kYx e¨wßi ga¨-we›`ywbY‡qi ci Zvi †h †Kvb GKwU‡K Abygv‡bi Mo a‡i wb‡q †mB Abygv‡bi Mo †_‡K cwZwU ga¨-we›`y¯‹i wePvi wbYq Ki‡Z nq| wØZxq av‡c, ¯‹i wePvi‡K cwZwU †kYxi NUbmsL¨v w`‡q ¸Y Ki‡Z n‡e| ZZxq av‡c, ¯‹i wePvi¸‡jvi eM K‡i wb‡Z nq Ges me †k‡l, eMKZ.wePvi¸‡jv‡K cwZwU †kYxi NUbmsL¨v w`‡q ¸Y K‡i cvß ¸Ydj¸‡jvi mgwó wbYq Ki‡Z nq| msw¶ß c×wZ‡Z cwigZ e¨eavb wbY‡qi exRMwYwZK mÂwU n‡jv,

$$s = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \times C.I.$$

‡hLv‡b, s = cwigZ e¨eavb

f_i = †kYxf³ NUbmsL¨v

$d = \frac{x_i - A}{C.I.}$ = ¯‹i wePvi‡Z

x_i = †kYx e¨wßi ga¨-we›`y

A = Abygv‡bi Mo

N = †gvU NUbmsL¨v

C.I. = †kYx e¨wß

mviwY 5.4.3-G c`Ë DcvË e¨envi K‡i mviwY 5.4.4-G msw¶ß c×wZ‡Z cwigZ e¨eavb wbYq K‡i †`Lv‡bv n‡jv|

mviwY 5.4.4: web¨ DcvË †_‡K msw¶ß c×wZ‡Z cwigZ e¨eavb wbYq

‡kYx e¨wß (C.I.)	‡kYx e¨wßi ga¨-we› `y (x_i)	NUbmsL¨v (f_i)	¯i wePvi $d = \frac{x_i - A}{C.I.}$	fd	d^2	fd^2
1 - 9	5	400	–2	–800	4	1600
10 - 18	14	500	–1	–500	1	500
19 - 27	23 = A	300	0	0	0	0
28 - 36	32	150	1	150	1	150
37 - 45	41	100	2	200	4	400
‡gvU		1450		–950		2650

cwigZ e¨eavb wbY‡qi m‡Î e¨envi‡i Rb¨ c‡qvRbxq gvb¸‡jv Avgiv †c‡q wM‡qwQ| Gevi ‡m¸‡jv m‡Î c‡qvM Ki‡j Avgiv cwigZ e¨eav‡bi gvbwU †c‡Z cwi| AZGe,

$$s = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \times C.I.$$

$$= \sqrt{\frac{2650}{1450} - \left(\frac{-950}{1450}\right)^2} \times 9$$

$$= \sqrt{1.83 - (-0.66)^2} \times 9$$

$$= \sqrt{1.83 - 0.44} \times 9$$

$$= \sqrt{1.39} \times 9$$

$$= 1.18 \times 9$$

$$= 10.62$$

∴ wbYxZ cwigZ e¨eavb n‡jv 10.62

`ÿvU c×wZ‡ZB GKB gvb cvIqv hvq, Z‡e `kwg‡Ki c‡i `ÿvU K‡i msL¨v †bIqv‡Z GB c×wZ‡Z cwigZ e¨eav‡bi gvb .02 Kg n‡q‡Q|

cwigZ e¨eav‡bi myweavi Ges Amyweavi (Advantages and Disadvantages of Standard Deviation)

DcvËiwki mKj gvb‡K Ab‡c¶fv‡e bv †f‡e cKZfv‡e we‡ePbv KivB n‡jv cwigZ e¨eav‡bi cavb myweav| A_vr, Mo e¨eavb wbY‡qi mgq FYvZ‡K wPý‡K D‡c¶v Kivi Rb¨ †h Ab‡c¶ wePvi wbYq Kiv nq Zv GKwU Kwlg Dcvq| ZËMZfv‡e Mo e¨eav‡bi †Kvb e¨vL¨v ‡`qv hvq bv Ges cieZx exRMwYZK cwiMYbvqI c‡qvM Kiv hvq bv| FbvZ‡K wPý D‡c¶v Kivi GB Kwlg c×wZ‡Z mó mxgve×Zv¸‡jv GKgvÎ cwigZ e¨eav‡bi gva¨‡gB `i- Kiv m¤e| cwigZ e¨eavb‡K Lye mn‡RB cieZx exRMwYZK cwiMYbvi †¶‡Î c‡qvM Kiv hvq| Ab¨vb¨ cwigv‡ci Zzjbvq cwigZ e¨eavb bgybv wePvi Øviv Kg cfweZ nq e‡j GwU A‡bK

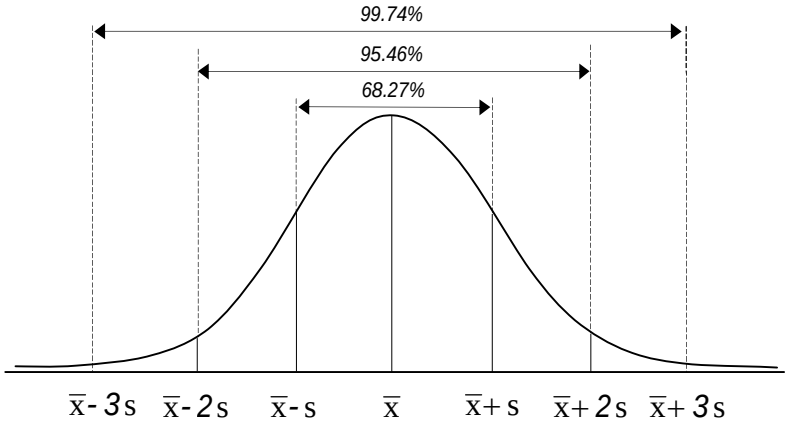
†ekx ¯xwZkxj GKwU cwigvc| `ßi ev Z‡ZwaK DcvËiwki cwiwgZ e¨eavb †_‡K Avgiv mwnsjZ cwiwgZ e¨eavb wbYq Ki‡Z cwi|

Z‡e †h‡nZzcwiwgZ e¨eavb wbY‡q DcvËiwki mKj gvb‡K we‡ePbvq Avbv nq, ‡m‡nZzGwU Pig ev Amvavi Y gvb Øviv Lye mn‡RB cfweZ nq| cwiwgZ e¨eavb wbY‡qi †¶‡Î MwYwZK Mo‡K e¨envi Kiv nq e‡j MwYwZK M‡oi mKj mxgve×Zv cwiwgZ e¨eav‡bi g‡a¨ _v‡K| Mo e¨eav‡bi g‡Zv GwU Dbœ³ †kYx mxgv m¤ijZ Dcv‡Ëi †¶‡Î c‡qvM Kiv hvq bv| cwiwgZ e¨eavb‡K eø‡Z Ges wbYq Ki‡Z n‡j h‡_ó MwYwZK Áv‡bi c‡qvRb nq|

cwiwgZ e¨eav‡bi e¨envi (Use of the Standard Deviation)

we¯wZi cwigvc¸‡jvi g‡a¨ cwiwgZ e¨eavb me‡P‡q Dc‡hvMx Ges D”PZi cwimsL¨wbK we‡kl‡Y GwU‡K e¨vcKfv‡e e¨envi Kiv nq| bgybvqb Ges mn-m¤úK we‡kl‡Yi wfwËwU `wo‡q Av‡Q cwiwgZ e¨eav‡bi Dci| bgybvi AvKvi wbav‡iY cwiwgZ e¨eavb GKwU wbqvgK fwgKv cvjb K‡i| cwiwgZ †iLvi we‡kl‡Y cwiwgZ e¨eav‡bi cwi‡cw¶‡Z n‡q _v‡K| †hgb, GKwU web¨vm hw` myg nq, Z‡e Zv gmwY ¯vfweK †iLvi (smooth normal curve) i¡c avi Y K‡i Ges GKwU myg web¨v‡mi Aax‡b QqwU (M‡oi Wvb w`‡K wZbwU Ges evg w`‡K wZbwU) cwiwgZ e¨eavb i‡q‡Q| GB QqwU cwiwgZ e¨eav‡bi †gvU AvqZb n‡jv 99.74 kZvsk| Ab¨ K_vq, M‡o wZb cwiwgZ e¨eav‡bi Dc‡i Ges bx‡P 99.74 kZvsk NUbmsL¨v Ašfy³; `ßi cwiwgZ e¨eav‡bi Dc‡i Ges bx‡P 95.45 kZvsk; Ges GK cwiwgZ e¨eav‡bi Dc‡i Ges bx‡P 68.27 kZvsk NUbmsL¨v Ašfy³ _v‡K (wPÎ 5.4.1 `óe¨)|

bgybvi AvKvi wbav‡iY cwiwgZ e¨eavb GKwU wbqvgK fwgKv cvjb K‡i|



wPÎ 5.4.1: ¯vfweK †iLvi Aax‡b AvqZb

cwiwgZ e¨eav‡bi Ab¨Zg cavb e¨envi wU n‡jv GKwU web¨v‡mi g‡a¨ cwZwU gv‡bi Av‡cw¶K Ae¯vb wK Zv wbYq Kiv| `ßwU DcvËiwki g‡a¨ mgi¡cZv ev wfbZvi Zzjbv Kivi Rb¨ cwiwgZ e¨eavb GKwU ev¯e wfwË c`vb K‡i _v‡K|

mvivsk

cwiwgZ e¨eavb n‡jv MwYwZK Mo †_‡K cwZwU gv‡bi eMKZ.wePvizZi M‡oi eMg|| cwiwgZ e¨eavb Aweb¨¯ Ges web¨¯ Dfq cKvi DcvË †_‡K wbYq Kiv m¤e| Dcv‡Ëi eYbv Ges we‡kl‡Y cwiwgZ e¨eavb‡K e¨vcKfv‡e e¨envi Kiv nq| bgybvi AvKvi wbav‡iY cwiwgZ e¨eavb GKwU wbqvgK fwgKv cvjb K‡i| Gi Ab¨Zg e¨envi n‡jv GKwU web¨v‡mi g‡a¨ cwZwU gv‡bi Av‡cw¶K Ae¯v‡bi wPÎ wbYq Kiv|

cv‡VwËi gj¨vqb

^be¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| MwYwZK Mo †_‡K cwZwU gv‡bi eMKZ.weP¨vZi M‡oi eMgj¨ nj:

K. cwigZ Mo
L. ¯¿j¨ weP¨vZ
M. cwigZ e¨eavb
N. Ab‡c¶ weP¨vZ

2| ..... wbY‡q cwZwU gv‡bi e‡Mi mgwói c‡qvM nq|

K. Mo
L. cwigZ e¨eavb
M. cPwK
N. ga¨gv

3| Aweb¨¯¿ DcvË †_‡K msw¶ß c×wZ‡Z cwigZ e¨eavb wbY‡qi m¤¢ejv:

K. $s = \sqrt{\frac{\sum x_i}{N} - \left(\frac{\sum x_i}{N}\right)^2}$

L. $s = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2}$

M. $s = \sqrt{\frac{\sum x_i^2}{N-1} - \left(\frac{\sum x_i}{N-1}\right)^2}$

N. $s = \sqrt{\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N-1}\right)^2}$

msw¶ß ck

1| cwigZ e¨eavb wK?

2| Aweb¨¯¿ DcvË †_‡K msÁvMZ m¤¢ii gva¨‡g cwigZ e¨eavb wbY‡qi myweav¸‡jv ms‡¶‡c Av‡jvPbv Kiæb|

iPbvgj¨K ck

1| cwigZ e¨eav‡bi e¨envi D`vniYmn Av‡jvPbv Kiæb|

2| cwigZ e¨eavb e¨env‡ii myweav Ges Amyweav Av‡jvPbv Kiæb|

cvV - 5

we¯wZi wbi¼k cwigvc – 4: †f`v¼
Absolute Measures of Dispersion – 4: Variance

GB cvV †k‡l hv Rvbv hv‡e —

- †f`v¼ wK
- Awfeb¨¯— DcvË †_‡K msÁvMZ m‡Îi gva¨‡g †f`v¼ wbYq
- Awfeb¨¯— DcvË †_‡K msw¶ß c×wZ‡Z †f`v¼ wbYq
- wfeb¨¯— DcvË †_‡K msÁvMZ m‡Îi gva¨‡g †f`v¼ wbYq
- wfeb¨¯— DcvË †_‡K msw¶ß c×wZ‡Z †f`v¼ wbYq

‡f`v¼ wK (What is Variance)?

eMg‡j bv wb‡q eMKZ. wePvi Mo wbYq Kiv nq, hv †f`v¼ b¤^g cwiwPZ| A_©r, DcvËiwki Mo †_‡K msL¨vgvbmg‡ni wePvwiZi e‡Mi MoB n‡jv †f`v¼|

Lye mn‡RB Avgv‡`i g‡b ck Avm‡Z cv‡i †h, we¯wZi cwigvc wbY‡qi †¶‡Î eMg‡j †bevi †Kvb c‡qvRbxqZv i‡q‡Q wK? Gi mnR DËiwU n‡Z cv‡i †h, cwiwgZ e¨eavb‡K Gfv‡eB msÁvwqZ Kiv n‡q‡Q e‡j eMg‡j wb‡Z nq| wKš GwU Av‡`Š ‡Kvb m‡šÍvlRbK DËi bq| eis ejv hvq, Ab‡c¶Zvi mgm¨v `i‡Kivi Rb¨ †h‡nZzAvgiv MwYwZK Mo †_‡K cwZwU gv‡bi wePvwiZi eM Kwi, †m‡nZz†mB e‡Mi ¶wZci‡Y wnmv‡e eMKZ.wePvwiZi M‡oi eMg‡j wb‡q _wK| Z‡e D"PZi cwimsL¨wbK Kv‡R e¨env‡ii †¶‡Î eMg‡j bv wb‡q eMKZ.wePvwiZi Mo wbYq Kiv nq, hv †f`v¼ bv‡g cwiwPZ| A_¨vr, DcvËiwki Mo †_‡K msL¨v gvbmg‡ni wePvwiZi e‡Mi MoB n‡jv †f`v¼| cwiwgZ e¨eav‡bi mv‡_ †f`v‡¼i cv_K¨ n‡jv cwiwgZ e¨eav‡b wePvwiZi M‡oi eMg‡j wbY‡qi c‡qvRb nq, ‡f`v‡¼ Zvi c‡qvRb nq bv| †f`v¼‡K Mo eMI (mean square) ejv n‡q _v‡K| †f`v¼‡K Bs‡iRx eYgvjvi †QvU A¶‡ii $s^2$ cZx‡Ki gva¨‡g wPwýZ Kiv nq| mgM‡Ki †f`v¼‡K MxK eYgvjvi †QvU A¶‡ii σ^2 cZxK w`‡q wPwýZ Kiv nq|

‡f`v¼ `ßfv‡e wbYq Kiv hvq| c_gZt, wbYxZ cwiwgZ e¨eav‡bi eM K‡i| ‡hgb, cwiwgZ e¨eavb wbY‡qi D`vni‡Y cwiwgZ e¨eav‡bi cvß gvbwU wQj 10.64| GB gvbwU‡K eM K‡i wb‡jB Avgiv †f`v‡¼i gvb cv‡ev| AZGe,

$$s^2 = (10.64)^2$$

$$= 113.21$$

GKwU we‡q g‡b ivLv c‡qvRb †h, wbY‡qi mgq rounding-Gi Kvi‡Y `kwg‡Ki c‡i fMvs‡ki †¶‡Î gv‡bi mvgvb¨ ZviZg¨ NU‡Z cv‡i| Z‡e K¤úDUvi ev D"P ¶gZvm¤úb K¨vjK‡j‡Ui e¨envi Ki‡j †mB ZviZg¨wU Avi _v‡K bv|

wØZxq c×wZwU n‡jv, †f`v¼ wbY‡qi msÁvMZ m‡Î e¨envi K‡i| GLb Avgiv †f`v¼ wbY‡qi msÁvMZ m‡Î e¨envi K‡i Gi wbYq c×wZ Av‡jvPbv Ki‡ev|

Awe¨¯— DcvË †_‡K msÁvMZ m‡Îi gva¨‡g ‡f`v¼ wbYq (Computing Variance from Ungrouped Data Using Definitional Formula)

Awe¨¯— DcvË †_‡K †f`v¼ wbY‡qi Rb¨ wbgwjwLZ msÁvMZ m‡Îwe¨envi Kiv nq|

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{N}$$

‡hLv‡b, s^2 = ‡f`v¼

x_i = Dcv˵i wwki cwZwU gvb

$\bar{x}$ = MwYwZK Mo

N = Dcv˵i wwki †gvU msL¨v

$(x_i - \bar{x})^2$ = Mo †_‡K cwZwU gv‡bi wePvi eM

mvi wY 5.4.5-G Awe¨¯— DcvË †_‡K msÁvMZ m‡Î e¨envi K‡i †f`v¼ wbYq c×wZ †`Lv‡bv n‡jv|

mvi wY 5.4.5: Awe¨¯— DcvË †_‡K msÁvMZ m‡Îi gva¨‡g †f`v¼ wbYq

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
6	– 17	289
10	– 13	169
15	– 8	64
27	4	16
31	8	64
32	9	81
40	17	289
$\sum x_i = 161$		$\sum (x_i - \bar{x})^2 = 972$

c_‡g Avgv‡`i Dcv˵i wwki MwYwZK Mo wbYq K‡i wb‡Z n‡e| AZGe,

$$\bar{x} = \frac{\sum x_i}{N} = \frac{161}{7} = 23$$

∴ wbYxZ Mo n‡jv 23

Gevi Avgv‡`i wbYxZ Mo †_‡K cwZwU gv‡bi wePvi eM K‡i †m¸‡jvi mgwó wbYq Ki‡Z n‡e| cvß wePvi e‡Mi mgwói gvbwU m‡Îi c‡qvM K‡i Dcv˵i wwki †gvU msL¨v w`‡q fvM Ki‡j Avgiv †f`v‡¼i gvb cvB| AZGe,

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{N} = \frac{972}{7} = 138.86$$

∴ wbYxZ †f`v¼ n‡jv 138.86

Awe¨¯— DcvË †_‡K msw¶ß c×wZ‡Z †f`v¼ wbYq (Computing Variance from Ungrouped Data using Short-Cut Method)

msÁvwMZ mÎ e¨envi K‡i †f`v¼ wbYq GKUz Kómva¨ Ges mgq mv‡c¶| ZvB cwimsL¨vbwe`MY †f`v¼ wbY‡qi Rb¨ GKwU mnR mÎ D™veb K‡i‡Qb| †mwU n‡jv,

$$s^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$$

†hLv‡b, s^2 = ‡f`v¼

x_i = DcvËiwki cwZwU gvb

x_i^2 = cwZwU gv‡bi eM

N = DcvËiwki †gvU msL¨v

mviwY 5.4.5-G c`Ë DcvË e¨envi K‡i mviwY 5.4.6-G msw¶ß c×wZ‡Z †f`v¼ wbYq K‡i †`Lv‡bv n‡jv|

mviwY 5.4.6: Awe¨¯— DcvË †_‡K msw¶ß c×wZ‡Z †f`v¼ wbYq

x_i	x_i^2
6	36
10	100
15	225
27	729
31	961
32	1024
40	1600
$\sum x_i = 161$	$\sum x_i^2 = 4675$

cvß gvb¸‡jv m‡Î c‡qvM K‡i Avgiv †f`v‡¼i gvb cvB|

$$s^2 = \frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$$

$$= \frac{4675}{7} - \left(\frac{161}{7}\right)^2$$

$$= 667.86 - 529$$

$$= 138.86$$

∴ wbYxZ †f`v¼ n‡jv 138.86

web¨¯ DcvË †_‡K msÁvMZ m‡Îi gva¨‡g †f`v¼ wbYq (Computing Variance from Grouped Data Using Definitional Formula)

web¨¯ DcvË †_‡K †f`v¼ wbY‡qi msÁvMZ m‡ÎwU n‡jv,

$$s^2 = \frac{\sum f_i (x_i - \bar{x})^2}{N}$$

‡hLv‡b, s^2 = ‡f`v¼

f_i = cwZwU †kYxi NUbmsL¨v

x_i = cwZwU †kYxi ga¨-we›`y

$\bar{x}$ = MwYwZK Mo

N = †gvU NUbmsL¨v

$(x_i - \bar{x})^2$ = Mo †_‡K cwZwU gv‡bi wePvi eM

$\sum$ = mgwó wPý

mvaviY 5.4.7-G Dc¯wcZ 100 Rb Qv‡Îi e¨‡qi DcvË web¨vm‡K e¨envi K‡i Dc‡iv³ m‡Îi gva¨‡g †f`v¼ wbYq K‡i †`Lv‡bv n‡jv|

mvaviY 5.4.7: web¨¯ DcvË †_‡K msÁvMZ m‡Îi gva¨‡g cwZ‡Z †f`v¼ wbYq

wk¶veQi mgvß	NUb msL¨v f_i	‡kYx e¨vwßi ga¨-we›`y x_i	$f_i x_i$	Mo †_‡K cwZwU ga¨-we›`y wePvi $(x_i - \bar{x})$	wePvi eM $(x_i - \bar{x})^2$	$f_i (x_i - \bar{x})^2$
0 - 2	20	1	20	–5.23	27.35	547.00
2 - 4	28	3	84	–3.23	10.43	292.04
6 - 8	19	5	95	–1.23	1.51	28.69
8 - 10	9	7	63	0.77	0.59	5.31
10 - 12	35	9	315	2.77	7.67	268.45
12 - 14	10	11	110	4.77	22.75	227.50
14 - 16	6	13	78	6.77	45.83	274.98
16 - 18	3	15	45	8.77	76.91	230.73
‡gvU	**130**		**810**			**1874.70**

†f`v¼ wbY‡qi ce-kZ wnmv‡e Avgv‡`i MwYwZK Mo wbYq K‡i wb‡Z n‡e| AZGe,

$$\bar{x} = \frac{\sum f_i x_i}{N}$$

$$= \frac{810}{130}$$

$$= 6.23$$

∴ wbYxZ Mo n‡jv 6.23

GLb Mo †_‡K cwZwU †kYx e¨wßi ga¨-we›`y wePvwiZ wbYq Ki‡Z n‡e Ges wePvwiZ ¸‡jvi eM K‡i wb‡q †mB eMKZ.wePvwiZK NUbmsL¨v w`‡q ¸Y K‡i Zvi mgwó wbYq Ki‡Z n‡e| cvß gvb ¸‡jv m‡Î c‡qvM Ki‡j Avgiv ‡f`v‡¼i gvb cvB| AZGe,

$$s^2 = \frac{\sum f_i (x_i - \bar{x})^2}{N} = \frac{1874.7}{130} = 14.42$$

∴ wbYxZ †f`v¼ n‡jv 14.42

web¨vq DcvË †_‡K msw¶ß c×wZ‡Z †f`v¼ wbYq (Computing Variance from Grouped Data Using Short-Cut Method)

web¨vq Dcv‡Ëi †¶‡Î msw¶ß c×wZ‡Z †f`v¼ wbY‡qi Rb¨ mvaviYZt wb‡gi m‡ÐwU e¨eüZ nq|

$$s^2 = \left\{ \frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N} \right)^2 \right\} \times C.I.$$

‡hLv‡b, s^2 = †f`v¼

C.I. = †kYx e¨wß

d = ¯i wePvwiZ = $\frac{x_i - A}{C.I.}$

x_i = †kYxi e¨wßi ga¨-we›`y

A = Abyg Mo

f_i = NUbmsL¨v

N = †gvU NUbmsL¨v

mviwY 5.4.7-G c`Ë DcvË e¨envi K‡i mviwY 5.4.8-G msw¶ß c×wZ‡Z †f`v¼ wbYq K‡i ‡`Lv‡bv n‡jv|

mviwY 5.4.8: web¨vq DcvË †_‡K msw¶ß c×wZ‡Z †f`v¼ wbYq

wk¶vi eQi	NUbmsL¨v f_i	‡kYx e¨wßi ga¨-we›`y (x_i)	$d = \frac{x_i - A}{C.I.}$	d^2	fd^2	fd
0 - 2	20	1	–3	9	180	–60
2 - 4	28	3	–2	4	112	–56
4 - 6	19	5	–1	1	19	–19
6 - 8	9	7 = A	0	0	0	0
8 - 10	35	9	1	1	35	35
10 - 12	10	11	2	4	40	20
12 - 14	6	13	3	9	54	18
14 - 16	3	15	4	16	48	12
†gvU	**130**				**488**	**–50**

cvß gvb¸‡jv m‡Ši c‡qvM K‡i †f`v‡¼i gvb wbYq Ki‡Z cwi|kZGe,

$$
\begin{aligned}
s^2 &= c^2 \times \left\{ \frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N} \right)^2 \right\} \\
&= 2^2 \left\{ \frac{488}{130} - \left(\frac{-50}{130} \right)^2 \right\} \\
&= 4\left\{ 3.754 - (-0.385)^2 \right\} \\
&= 4\left\{ 3.754 - (0.148) \right\} \\
&= 4\{3.606\} \\
&= 14.42
\end{aligned}
$$

∴ wbYxZ †f`v¼ n‡jv 14.42

mvivsk

D"PZi cwimsL¨wbK Kv‡R e¨envi‡i ‡¶‡Î MwYwZK Mo †_‡K cwZwU gv‡bi eMKZ.wePvizZi eMgƒ- bv wb‡q eMKZ.wePvyZi Mo wbYq Kiv nq hv †f`v¼ bv‡g cwiPZ| A_¨vr, Dcv Ëiwki Mo †_‡K msL¨v gvbmg‡ni wePvyZi e‡Mi MoB n‡jv †f`v¼| cwigZ e¨eav‡bi mv‡_ †f`v‡¼i cv_K¨ n‡jv cwigZ e¨eav‡b wePvizZi M‡oi eMgƒ- wbY‡qi c‡qvRb nq, ‡f`v‡¼ Zvi c‡qvRb nq bv| †f`v¼‡K Mo eM I (mean square) ejv n‡q _v‡K| ‡f`v¼ `ywfv‡e wbYq Kiv hvq| c_gZt, wbYxZ cwigZ e¨eav‡bi eM K‡i| wØZxq c×wZwU n‡jv, †f`v¼ wbY‡qi msÁvMZ m‡Ši e¨envi K‡i|

cv‡VvËi gj¨vqb

^be¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| bx‡Pi †Kvbw`U‡K Mo eM e‡j?
- K. cwigvZ e¨eavb
- L. Mo e¨eavb
- M. ‡f`v¼
- N. kZnvwiK cwimi

2| †f`v¼ wbY‡qi Ab¨Zg c×wZ n‡jv:
- K. cwigvZ e¨eav‡bi eM K‡i
- L. Mo e¨eav‡bi eM K‡i
- M. cwim‡ii eM K‡i
- N. Dc‡ii †Kvbw`UB bq

3| web¨¯— DcvË †_‡K ‡f`v¼ wbY‡qi mÂ¨ n‡jv:

K. $s^2 = \frac{\sum f_i (x_i - \overline{x})}{N}$

L. $s^2 = \frac{\sum f_i (x_i - \overline{x})^2}{N}$

M. $s^2 = \frac{\sum (x_i - \overline{x})^2}{N}$

N. $s^2 = \sum f_i (x_i - \overline{x})^2$

msw¶ß ck

1| ‡f`v¼ wK?

2| ‡f`v¼ wbY‡qi c×wZ `ÿwU wK?

iPbvgj¨K ck

1| Aweb¨¯— DcvË †_‡K msw¶ß c×wZ‡Z wKfv‡e †f`v¼ wbYq Kiv nq Zv D`vniYmn Av‡jvPbv Kiæb|

2| web¨¯— DcvË †_‡K msÁvMZ m‡Îi gva¨‡g wKfv‡e †f`v¼ wbYq Kiv nq Zv D`vniYmn Av‡jvPbv Kiæb|

cvV - 6

cwiwgZ e¨eavb I ‡f`v‡¼i e¨vL¨v Ges e¨envi
Interpretation and Use of Standard Deviation and Variance

GB cvV †k‡l hv Rvbv hv‡e —

- cwiwgZ e¨eavb I †f`v‡¼i g‡a¨ Zzjbv
- cwiwgZ e¨eavb I †f`v‡¼i e¨vL¨v
- cwiwgZ e¨eav‡bi e¨envi

cwiwgZ e¨eavb I †f`v‡¼i g‡a¨ Zzjbv (Comparison between Standard Deviation and Variance)

Avgiv Rwb †h, MwYwZK M‡oi gZ cwiwgZ e¨eavb I †f`v¼ wbY‡qi †¶‡Î I DcvËiwki cwZwU gvb e¨eüZ nq| AZGe, DcvËiwki †h †Kvb GKwU gvb cwiewZZ n‡j GB cwigvc `ÿvUi gv‡bi cwieZb NU‡e| GB cwieZb bgybv wePvi Zzjbvq GB cwigvc `ÿvUi †¶‡Î G‡KKwU bgybvi †¶‡Î G‡KKwU cwiwgZ e¨eavb I †f`v‡¼i gvb cvIqv hvq| Z‡e cwimi ev PZzK e¨eav‡bi Zzjbvq GB cwigvc `ÿvUi †¶‡Î bgybv wePvi Kg N‡U| †h‡nZz msL¨v gvb¸‡jv eM K‡i mgwó wbYq Kiv nq, †m‡nZz GB cwigvc `ÿvUi gvb KL‡bvB FYvZ¥K n‡e bv| hw` web¨v‡mi me gvb¸‡jv GKB nq ‡m †¶‡Î cwigvc `ÿvUi gvb Lye Kg n‡j ÔOÕ n‡Z cv‡i| KviY, me gvb¸‡jv ÔOÕ n‡j me wePvi ZzB n‡e ÔOÕ| AZGe, wePvwZi eM¸‡jv Ges Zv‡`i mgwó I ÔOÕ n‡e| wKš ev¯‡e Zv N‡U bv| Avi N‡U bv e‡jB cwimsL¨vb wej wmZv bv n‡q Zv M‡elYv Kv‡h c‡qvRbxqZv wnmv‡e †`Lv †`q|

cwiwgZ e¨eavb‡K ^jwLKfv‡e Dc¯wcZ Kiv hvq, wKš †f`v¼‡K Kiv hvq bv|

Z‡e `ÿvU cwigv‡ci g‡a¨ cv_K¨ i‡q‡Q| †hgb, cwiwgZ e¨eavb gyj- A‡kwaZ gv‡bi GK‡K (original raw score units) _v‡K, A_P †f`v¼ _v‡K eMKZ.GK‡K (squared units)| cwiwgZ e¨eavb‡K ^jwLKfv‡e Dc¯wcZ Kiv hvq, wKš †f`v¼‡K Kiv hvq bv| cwiwgZ e¨eavb n‡jv gv‡bi gvÎvq GKwU `i-Z wb‡`k K‡i, Avi †f`v¼ n‡jv eMKZ.`i-Z| cwiwgZ e¨eavb cwimi, MwYwZK Mo, ga¨gv, BZ¨vw`i gZ GKB GK‡K cKwkZ nq, A_P †f`v¼ Zv K‡i bv| †m Kvi‡Y, cwiwgZ e¨eavb we¯wZi cwigvc wnmv‡e AwaK e¨eüZ nq| Z‡e cwiwgZ e¨eav‡bi †P‡q †f`v‡¼i AwaKZi ZwËK gyj¨ i‡q‡Q| KviY, cwiwgZ e¨eav‡bi lÁvMZ A_ (intuitive meaning) ZZ¶Y cwZfvZ nq bv hZ¶Y bv Zv cwiwgZ †iLvi Aax‡b AvqZb (area under normal curve) wbY‡q e¨envi Kwi| Ab¨ K_vq, cwiwgZ e¨eavb n‡jv GKwU wegvZ msL¨v hvi cwicv¤ e¨envi I e¨vL¨v cieZx cv‡V cwiwgZ gvb wbY‡qi Av‡jvPbvq ¯úó n‡q DV‡e|

cwiwgZ e¨eavb I †f`v‡¼i e¨vLv (Interpretation of Standard Deviation and Variance)

†Kvb cwimsL¨vbMZ cZ¨‡qi KvhKi e¨envi Ki‡Z n‡j Avgv‡`i ïaywbmB cZ¨q¸‡jvi wbYq c×wZ Rvb‡jB Pj‡e bv, †m¸‡jv‡K Ab¨vb¨ cZ¨‡qi mv‡_ m¤úwKZ Ki‡Z n‡e|

GB chv‡q G‡m Avgiv †f`v¼ I cwiwgZ e¨eavb wbY‡qi mvÎ¸‡jv e¨envi K‡i GKwU web¨v‡mi Rb¨ msL¨vgyj-K gvb wbYq Ki‡Z cwi Ges cwiwgZ e¨eavb I ‡f`v‡¼i wbYxZ gvb¸‡jv‡K cvqkt hv`ªxi msL¨v wn‡m‡e g‡b Kiv nq| wKš GB msL¨v gvb¸‡jvi wbYqB h_ó bq| KviY, †Kvb cwimsL¨vbMZ cZ¨‡qi KvhKi e¨envi Ki‡Z n‡j Avgv‡`i ïaywbmB cZ¨q¸‡jvi wbYq c×wZ Rvb‡jB Pj‡e bv, †m¸‡jv‡K Ab¨vb¨ cZ¨‡qi mv‡_ m¤úwKZ Ki‡Z n‡e| myjZvi `óv‡e

Avgv‡`i Rvb‡Z n‡e, †Kb Avgiv cwiwgZ e¨eavb wbYq Ki‡ev? †Kb cvq me M‡elYv cwZ‡e`‡b GwU AšfyÇ Kiv n‡q _v‡K? Ges KLb GwU wbYq Ki‡Z n‡e Ges KLb Ki‡Z n‡e bv? Gi Rb¨ Avgv‡`i cwiwgZ e¨eavb I ‡f`v‡¼i A_en e¨vL¨v Rvbv c‡qvRb|

c_g †h e¨vL¨wU Avgiv w`‡q _wK †mwU n‡jv, hw` cwiwgZ e¨eavb I †f`v‡¼i gvbwU †QvU nq Z‡e Avgiv ewj †h web¨vmwU mgi¡c, Avi hw` eo nq Z‡e Avgiv ewj web¨vmwUi we¯wZ. A‡bK †ekx| GB e¨vL¨wU mivmwi MwYwZK M‡oi mv‡_ m¤úwKZ| KviY, cwiwgZ e¨eav‡bi msÁvi g‡a¨ MwYwZK M‡oi wØZxq ˆewkó¨wU (property of least squares) Ašwbnz i‡q‡Q| web¨v‡mi gvb¸‡jv Mo †_‡K †ekx `i‡Z Ae¯vb Ki‡j wePvwaZ¸‡jv eo n‡e, wePvwaZi eM¸‡jv eo n‡e Ges cwiwgZ e¨eavb I †f`v‡¼i gvb eo n‡e| web¨v‡mi gvb¸‡jv hw` Mo †_‡K Kg `i‡Z Ae¯vb K‡i Z‡e wePvwaZ¸‡jv †QvU n‡e, Zvi eM¸‡jv †QvU n‡e Ges cwiwgZ e¨eavb I †f`v‡¼i gvb †QvU n‡e| mviwY 5.5.1-G c`Ë `ÎvU iwkgvjv‡K ch‡e¶Y Ki‡jB welqwU ¯úó n‡q DV‡e|

mviwY 5.5.1: `ÎvU web¨v‡mi we¯wZi wfbZv

web¨vm ÔKÕ						web¨vm ÔLÕ					
msL¨v gvb =	21	22	22	23	23	msL¨v gvb =	0	3	6	8	9
	23	24	24	25			10	12	15	18	
`i‡Z =	–2	–1	–1	0	0	`i‡Z =	–9	–6	–3	–1	0
	0	1	1	2			1	3	6	9	
wePvwaZi eM =	4	1	1	0	0	wePvwaZi eM =	81	36	9	1	0
	0	1	1	4			1	9	36	81	

$$\text{‡f`v¼}_K = \frac{\text{wePvwaZi e‡Mi mgwó}}{\text{†gvU msL¨v}} = \frac{12}{9} = 1.33$$

$$\text{cwiwgZ e¨eavb}_K = \sqrt{1.33} = 1.15$$

$$\text{‡f`v¼}_L = \frac{\text{wePvwaZi e‡Mi mgwó}}{\text{†gvU msL¨v}} = \frac{254}{9} = 28.22$$

$$\text{cwiwgZ e¨eavb}_L = \sqrt{28.22} = 5.31$$

Dc‡ii mviwY‡Z †`Lv hv‡”Q †h, web¨vm ÔKÕ-Gi msL¨v gvb¸‡jvi g‡a¨ `i‡Z Kg e‡j wePvwaZ¸‡jv ¶z` Ges wePvwaZi eM¸‡jvI ¶z`| d‡j, web¨vm ÔKÕ-Gi †f`v¼ I cwiwgZ e¨eav‡bi gvb ¶y` n‡q‡Q| Ab¨vb¨‡K, web¨vm ÔLÕ-Gi msL¨v gvb¸‡jv †ek `i‡Zi mv‡_ Qwo‡q i‡q‡Q e‡j wePvwaZi gvb¸‡jv eo n‡q‡Q Ges wePvwaZi eM¸‡jvI eo n‡q‡Q| d‡j, †f`v¼ I cwiwgZ e¨eav‡bi gvb ÎvUI eo n‡q‡Q|

mviwY 5.5.1-Gi ch‡e¶Y Avi I GKwU welq‡K ¯úó K‡i †Zv‡j| †mwU n‡jv, †K›`xq ceYZvi cwigvc¸‡jv †hgb we¯wZi gvb m¤ú‡K Avgv‡`i †Kvb avivYv w`‡Z cv‡i bv, †Zgwb cwiwgZ e¨eavb ev †f`v¼I †K›`xq ceYZvi gvb m¤ú‡K †Kvb avivYv w`‡Z cv‡i bv| †hgb, web¨vm ÔKÕ- Gi Mo gvb (23) eo n‡j I †f`v¼ I cwiwgZ e¨eav‡bi gvb (h_vµ‡g 1.33 Ges 1.15) †QvU n‡q‡Q, A_P web¨vm ÔLÕ-Gi Mo gvb (9) †QvU n‡qI †f`v¼ I cwiwgZ e¨eav‡bi gvb (h_vµ‡g 28.22 I 5.31) eo n‡q‡Q| web¨vm ÔKÕ-Gi eo Mo gvb †_‡K GwU g‡b n‡Z

> †K›`xq ceYZvi cwigvc¸‡jv †hgb we¯wZi gvb m¤ú‡K Avgv‡`i †Kvb avivYv w`‡Z cv‡i bv, †Zgwb cwiwgZ e¨eavb ev †f`v¼I †K›`xq ceYZvi gvb m¤ú‡K †Kvb avivYv w`‡Z cv‡i bv|

‡f`v¼ I cwiwgZ e¨eavb ev †f`v‡¼i gvbI eo n‡e Ges we¯wZI eo n‡e| web¨vm ÔLÕ †_‡K wVK Gi wecixZ avivYwU n‡Z cv‡i| A_vr, ‡f`v¼ I cwiwgZ e¨eav‡bi eo gvb †`‡L g‡b n‡Z cv‡i †h, web¨vmwUi Mo gvb eo n‡e| GLv‡b `ÿvU ¸i“cY¨ welq ¯úó n‡q D‡V‡Q| c_gZt, Mo gvbwU eo ev †QvU n‡Z cv‡i †Kejgv‡Î msL¨vi gvb¸‡jvi eo ev †QvU AvKv‡ii Kvi‡Y| wØZxqZt, ‡f`v¼ I cwiwgZ e¨eav‡bi eo ev †QvU gvb msL¨v gvb¸‡jvi AvKv‡ii Dci wbfi K‡i bv, K‡i msL¨v gvb¸‡jv GKwU Av‡iKwU †_‡K KZ `i‡Z Qwo‡q i‡q‡Q Zvi Dci| cwiwgZ web¨vm I †f`v¼ e¨vL¨vi †¶‡Î GB `ÿvU welq ¸i“Zi mv‡_ g‡b ivLv c‡qvRb|

GKwU web¨v‡mi g‡a¨ DcvËiwki †gvU msL¨v e„× †c‡j †f`v¼ I cwiwgZ e¨eav‡bi gvb e„× ev nvm †c‡Z cv‡i| hZevi GKwU K‡i msL¨v web¨v‡mi g‡a¨ hy³ n‡e ZZevi m¤Îi ÔniÕ-Gi (denomentor) gvb e„× cv‡e Ges GKB mv‡_, ÔjeÕ-GI (numerator) K‡qKwU bZzb wePvi eM hy³ n‡e| DcvËiwki †gvU msL¨v evo‡j we¯wZi cwigvc `ÿvUi gvb KZUkze„× ev nvm cv‡e †mwU wbfi Ki‡e web¨v‡m Ašfy³ c‡ei msL¨v gvb¸‡jv †_‡K bZzb msL¨v gvbwU KZUkzafb Zvi Dci| hw` msL¨vwU Mo gv‡bi mgvb ev Lye KvQvKwQ nq, Z‡e wePvi eMwU ÔOÕ ev Zvi KvQvKwQ n‡e Ges wePvi e‡Mi Mo gvbwU wKQyUv n‡j I nvm cv‡e Ges wbYxZ cwiwgZ e¨eavb I †f`v‡¼i gvbI nvm cv‡e| Avi hw` bZzb gvbwU Mo †_‡K A‡bK `ÿ‡Z Ae¯vb K‡i Z‡e wePvi eMwU A¯vfweKfv‡e eo n‡e Ges wePvi e‡Mi Mo e„× cv‡e| d‡j, cwiwgZ e¨eavb I †f`v‡¼i gvb e„× cv‡e|

aiv hvK, web¨vm ÔKÕ-G `ÿvU msL¨v †hvM Kiv n‡q‡Q: 20 Ges 26| Gi d‡j Mo Ges †f`v‡¼i g‡a¨ wK cwieZb n‡”Q Zv †`Lv hvK|

$$\bar{x} = \frac{20+21+22+22+23+23+23+24+24+25+26}{11}$$

$$= \frac{253}{11}$$

$$= 23$$

$$s^2 = \frac{9+4+1+1+0+0+0+1+1+4+9}{11}$$

$$= \frac{30}{11}$$

$$= 2.73$$

20 I 26 msL¨v `ÿvU †hvM Kivi d‡j Mo gv‡bi †Kvb cwieZb bv n‡j I ‡f`v‡¼i gvb 1.33 †_‡K †e‡o wM‡q 2.73 n‡q‡Q| Gi KviYwU n‡jv, msL¨v `ÿvU Ôniô Ges Ôjeô `ÿvUiB gvb ewo‡q w`‡q‡Q Ges msL¨v `ÿvU cwšKfv‡e hy³ nevi d‡j wePvi eM¸‡jv eo AvKvi aviY K‡i wePvi mgvó‡K †ek wKQyUv ewo‡q w`‡q‡Q| wKš hw` web¨vmwU‡Z Av‡iv wZbwU msL¨v (†hgb, 22, 23 I 24) hy³ Kiv nq ZLb wK n‡e? Mo gv‡bi †Kvb cwieZb NU‡e bv Ges †f`v‡¼i gv‡bi cwieZb NU‡j I Zv Lye †ekx evo‡e bv| KviY, bZzb msL¨v¸‡jv Mo gv‡bi KvQvKwQ nIqv‡Z wePvi eM¸‡jv †QvU n‡e Ges †f`v‡¼i gvb wKQyUv e„× cv‡e| Gi A_ GB bq †h, bZzb gvb hy³ Ki‡j Mo gv‡bi †Kvb cwieZb n‡e bv| †ek eo ev †QvU GKwU msL¨v hy³ Ki‡j Mo gv‡bi I cwieZb NU‡e|

†K›`xq ceYZvi †¶‡Î †hgb MwYwZK Mo n‡jv GKwU fvimv‡g¨i we›`y we¯wZi cwigvc‡i †¶‡Î cwiwgZ e¨eavb ‡Zgwb GKwU fvimv‡g¨i we›`y

GLv‡b GKwU welq D‡jL Kiv c‡qvRb †h, G ai‡Yi GKwU †QvU web¨v‡mi †¶‡Î wePvwiZi e‡Mi welqwU Zvr¶wYKfv‡e cgvY Kiv †M‡j I e„r DcvËiwk mxgjZ web¨v‡mi †¶‡Î Zv mxgbq| †m †¶‡Î GKwU cwiwgZ mPK c‡qvRb hvi wecix‡Z bZzb msL¨v gvb hy³ Kivi cfvewU †`‡L web¨v‡mi we¯wZi cKvwU †evSv †h‡Z cv‡i| cwiwgZ e¨eavb GB cwiwgZ mP‡Ki KvRwU K‡i| †K›`xq ceYZvi †¶‡Î †hgb MwYwZK Mo n‡jv GKwU fvimv‡g¨i we›`y(point of balance), we¯wZi cwigvc‡i †¶‡Î cwiwgZ e¨eavb ‡Zgwb GKwU fvimv‡g¨i we›`y hLb †Kvb web¨v‡m bZzb msL¨v gvb hy³ Kiv nq Ges Zv hw` GK cwiwgZ e¨eav‡bi bx‡P nq, Z‡e †f`v‡¼i gvb Kg‡e| KviY, Avgiv Rwb †h wePvwiZi eM¨‡jv †QvU nq e‡j Zvi MowU †QvU nq| welqwU GKwU D`vniY w`‡q †evSv hvK| aiv hvK, 100wU msL¨v iwki Mo 50 Ges cwiwgZ e¨eavb 10| hw` GB 100wU msL¨v gv‡bi mv‡_ Av‡iv 5wU 48 gv‡bi Ges 5wU 52 gv‡bi msL¨v †hvM Kwi, Zvn‡j djvdjwU wK `vovq †`Lv hvK| †h‡nZzMo gvb n‡jv 50, †m‡nZz48 msL¨vgv‡bi wePvwiZi n‡e –2 Ges 52 msL¨v gv‡bi wePvwiZi n‡e +2| GB wePvwiZ¸‡jvi eM Ki‡j n‡e 4 Ges 4| AZGe, GB bZzb 10wU msL¨vi wePvwiZi mgvó n‡e (5×4)+(5×4)=20+20=40|

‡h‡nZzcwiwgZ e¨eavb n‡jv 10, †m‡nZz†f`v¼ Aek¨B n‡e 100 (KviY, †f`v¼ =cwiwgZ e¨eav‡bi eM)| †f`v‡¼i Dci GB bZzb gvb¸‡jvi cfve wK n‡e? DË‡i ejv hvq †h, †f`v‡¼i gvb Kg‡e| KviY, †h gvb¸‡jv hy³ n‡q‡Q (48 Ges 52) ‡m¸‡jv cwiwgZ e¨eav‡bi gv‡bi bx‡P i‡q‡Q| A_vr, cwiwgZ e¨eavb = 10-Gi A_ n‡jv web¨v‡mi gvb¸‡jv Mo †_‡K M‡o GK cwiwgZ e¨eavb Qwo‡q i‡q‡Q| Avgv‡`i gj¨ web¨v‡mi †f`v¼ n‡jv 100| Gevi †`Lv hvK, bZzb gvb¸‡jv hy³ Ki‡j †f`v‡¼i gvb wK `vovq?

gj¨ web¨v‡mi wePvwiZi e‡Mi mgvó n‡jv,

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{100} = 100$$

AZGe, $\sum (x_i - \bar{x})^2 = 100 \times 100$

$= 10000$

‡hLv‡b, $N = 100$

$s^2 = 100$

bZzb web¨v‡mi wePvwiZi e‡Mi mgvó n‡jv,

$$s^2 = \frac{gj¨ e‡Mi mgvó + bZzb e‡Mi mgvó}{gj¨ †gvU msL¨v + bZzb msL¨v}$$

$$= \frac{10000 + 40}{100 + 10}$$

$$= \frac{10040}{110}$$

$$= 91.3$$

$\therefore \quad s^2 = 91.3$

AZGe, cgwYZ n‡jv †h, 10wU bZzb msL¨vi gvb hy³ nIqv‡Z gj¨ †f`v‡¼i gvb 100 nvm †c‡q 91.3 n‡q‡Q| Gevi hw` 5wU 35 msL¨v gvb Ges 5wU 65 msL¨v gvb hy³ Kwi Z‡e †`L‡ev †h, †f`v‡¼i gvb e„× †c‡q‡Q| KviY, bZzb msL¨v¸‡jv Mo †_‡K 15 Kg Ges 15 †ekx nIqv‡Z cwZwU wePvi Zi eM n‡q‡Q 225| 10wU gv‡bi wePvi Zi e‡Mi mgwó n‡e 10 × 225 =2250| GB bZzb wePvi Zi e‡Mi mgwówU gj¨ web¨v‡mi wePvi Zi e‡Mi mgwói mv‡_ †hvM Ki‡j †`Lv hv‡e †h †f`v‡¼i gvb e„× †c‡q‡Q| †hgb, bZzb web¨v‡mi †f`v¼ n‡jv,

$$
\begin{aligned}
s^2 &= \frac{gj¨\ e‡Mi\ mgwó + bZzb\ e‡Mi\ mgwó}{gj¨\ †gvU\ msL¨v + bZzb\ msL¨v} \\
&= \frac{10000 + 2250}{100 + 10} \\
&= \frac{12250}{110} \\
&= 111.4 \\
\therefore \quad s^2 &= 111.4
\end{aligned}
$$

AZGe, †`Lv hv‡”Q †h, Mo †_‡K GK cwigvgZ e¨eav‡bi †P‡q eo ev †QvU msL¨v †hvM Ki‡j †f`v‡¼i gvb e„× cvq| Avi †f`v‡¼i gvb e„× cvevi A_ n‡jv cwigvgZ e¨eav‡bi gvb e„× cvIqv| Z‡e GwU g‡b ivLv c‡qvRb †h, cwigvgZ e¨eavb‡K cwigvgZ gv‡b (standard score) cKwkZ Kivi c‡e GwU‡K h_vh_ fv‡e e¨vL¨v Kiv hvq bv Ges Gi lÁvMZ A_wU Abyôvb Kiv hvq bv|

mvivsk

cwigvgZ e¨eavb Ges ‡f`v¼ n‡jv Nwbófv‡e m¤úwKZ we¯wZi `ÿwU cwigvc| GB `ÿwU cwigvc‡K we¯wZi cavb cwigvc wnmv‡e AwaKvsk M‡elYvq e¨envi Kiv nq| ïaywe¯wZi cwigvc wnmv‡e bq, Ab¨vb¨ D”PZi cwimsL¨wbK I exRMwYwZK cwiMYbvi †¶‡Î GB `ÿwU cwigv‡ci AšÍbwnZ ¶ZZg e‡Mi %ewkó¨wU ¸iæZc¥K fwgKv cvjb K‡i| cwigvgZ e¨eav‡bi mv‡_ †f`v‡¼i cv_K¨ n‡jv cwigvgZ e¨eavb wePvi Zi M‡oi eMgj¨ wbY‡qi c‡qvRb nq, †f`v‡¼ Zvi c‡qvRb nq bv|

cv‡VwËi gj¨vqb

^be¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| ..... ^jwLKfv‡e Dc¯vcb Kiv hvq bv|

K. cwiwgZ e¨eavb
L. Mo e¨eavb
M. cwgZ e¨eavb
N. ‡f`v¼

2| GKwU web¨vm‡K mgi¡c ejv hvq hLb:

K. cwiwgZ e¨eavb †QvU Ges †f`v¼ eo nq
L. cwiwgZ e¨eavb eo Ges †f`v¼ †QvU nq
M. cwiwgZ e¨eavb I ‡f`v‡¼i gvb †QvU nq
N. cwiwgZ e¨eavb I †f`v‡¼i gvb eo nq

3| ‡f`v‡¼i gvb e„w× ev nvm †c‡Z cv‡i hw`:

K. DcvËiwki †gvU msL¨v e„w× cvq
L. DcvËiwki †gvU msL¨v nvm cvq
M. DcvËiwki †gvU msL¨v nvm ev e„w× cvq
N. DcvËiwki †gvU msL¨v mgvb _v‡K

msw¶ß ck

1| cwiwgZ e¨eavb I †f`v‡¼i g‡a¨ cv_K¨ wK?

2| gv‡bi Av‡cw¶K Ae¯vb wbY‡q cwiwgZ e¨eav‡bi e¨envi Av‡jvPbv Kiæb|

iPbvgj¨K ck

1| DcvË eYbvq cwiwgZ e¨e¨avb I †f`v‡¼i g‡a¨ †KvbwU‡K Avcwb AwaK hy³ hy³ e‡j g‡b K‡ib? Avcbvi DË‡ii ¯c‡¶ hy³ w`b|

2| Dchy³ D`vniYmn cwiwgZ e¨eavb I †f`v‡¼I e¨vL¨v w`b|

cvV - 7

we¯wZi Av‡cw¶K cwigvc – e¨eavbv¼ Ges cwiwgZ gvb
Relative Measures of Dispersion – Coefficient of Variation and Standard Score

GB cvV †k‡l hv Rvbv hv‡e —

- we¯wZi Av‡cw¶K cwigvc
- e¨eavbv¼
- e¨eavbv¼ wbYq
- cwiwgZ gvb
- cwiwgZ gv‡bi ˆewkó¨
- cwiwgZ gvb wbYq
- cwiwgZ gv‡bi myeav

we¯wZi Av‡cw¶K cwigvc (Relative Measures of Dispersion)

GKwU wb‡ek‡bi Mo ‡_‡K msL¨v gvb¸‡jvi we¯wZi cwigv‡ci Rb¨ wbi¼k we¯vi cwigvc (absolute measures of dispersion) Dc‡hvMx n‡j l `ßi ev Z‡ZwaK DcvËmwii g‡a¨ Zviv Z‡g¨i wfwË‡Z Zzjbv Kivi Rb¨ Dc‡hvMx bq| wbi¼k we¯vi cwigvc¸‡jv Dcv‡Ëi g‡a¨ wfb wfb cwigvc GK‡Ki (†hgb, UvKv, eQi, wK‡jvMvg, BZ¨vw`) gva¨‡g wfbZvi eYbv †`q| `ßvU DcvË web¨v‡mi cwigvc GKK hw` wfb wfb nq Z‡e wbi¼k cwigv‡ci gva¨‡g wfbZvi Zzjbv Kiv hvq bv| AZGe, wfb cwigvc GKK m¤¢jZ `ßvU DcvËiwki g‡a¨ wfbZvi Zzjbvi Rb¨ wbi¼k e¨eavb¸‡jv‡K Av‡cw¶K c‡` i¡cvšwiZ Kivi c‡qvRb nq| Av‡cw¶K we¯vi cwigvc¸‡jv we¯wZi wbi¼k cwigvc¸‡jvi cwiewZZ i¡c gvÎ| we¯wZi Av‡cw¶K cwigvc¸‡jv †Kvb GK‡K cKwkZ nq bv| †m¸‡jv GKK wenxb e‡j Zzjbvi †¶‡Î we‡klfv‡e Dc‡hvMx| Av‡cw¶K we¯vi cwigv‡ci GKwaK c×wZ i‡q‡Q| †hgb,

1. PZzK e¨eavbv¼ (Coefficient of Quartile Deviation)
2. Mo e¨eavbv¼ (Coefficient of Mean Deviation)
3. e¨eavbv¼ (Coefficient of Variation)

Gi g‡a¨ me‡P‡q ¸i“Zc¥ Ges mewaK e¨eüZ c×wZ n‡jv e¨eavbv¼| †mwU Avgiv GLb Av‡jvPbv Ki‡ev|

e¨eavbv¼ (Coefficient of Variation)

‡Kvb DcvËiwki Mo Ges cwiwgZ e¨eav‡bi kZKiv Abyôv‡Z †mB DcvËiwki e¨eavbv¼ ejv nq| A_vr,

$$\text{C.V.} = \frac{\text{S}}{\overline{\text{X}}} \times 100$$

‡hLv‡b, C.V. = e¨eavbv¼

s = cwigvZ e¨eavb

$\bar{x}$ = MwYwZK Mo

hLb `ßvU DcvËiwki g‡a¨ we¯vi cwigv‡ci Rb¨ e¨eavbv¼ e¨eüZ nq, ZLb †h DcvËiwki e¨eavbv‡¼i gvb eo nq, †mB DcvËiwki web¨vmwU‡K AwaK we¯Z.ev Amgi¡c e‡j MY¨ Kiv nq| Ab¨vb¨ ‡K, †h DcvËiwki e¨eavbv‡¼i gvb Kg n‡e †mB DcvËiwki web¨vmwU‡K Kg we¯Z.Ges AwaK mgi¡c e‡j MY¨ Kiv nq| we¯wZi Av‡cw¶K cwigvc wbY‡qi Rb¨ we¯wZi cwigv‡ci †h †Kvb wbi¼k c×wZ (cwimi, PZzK e¨eavb, Mo e¨eavb I cwigvZ e¨eavb) mswkó †K›`xq ceYZvi cwigv‡ci (ga¨gv, MwYwZK Mo) mvnv‡h¨ e¨envi Kiv hvq| wKš cwimsL¨vbwe`MY we¯wZi Av‡cw¶K cwigv‡ci †¶‡Î cvq memgqB cwigvZ e¨eavb I MwYwZK Mo e¨envi K‡i _v‡Kb|

e¨eavbv¼ wbYq (Computing Coefficient of Variation)

Avgiv Rwb †h, †Kvb DcvËiwki Mo I cwigvZ e¨eav‡bi kZKiv Abyôv‡K e¨eavbv¼ e‡j| mZ¨vi‡s, e¨eavbv¼ wbY‡qi c‡e DcvËiwki Mo I cwigvZ e¨eavb wbYq Kiv Avek¨K| mviwY 5.6.1-G c`Ë DcvË e¨envi K‡i e¨eavbv¼ wbY‡qi gva¨‡g `ßvU DcvËiwki g‡a¨ we`¨gvb we¯wZi Zzjbv Kiv n‡jv|

mviwY 5.6.1: `ßvU Mv‡gi gwnjv‡`i gwmK Av‡qi web¨vm

gwmK Avq (UvKvq)	Mvg ÔKÕ (gwnjvi msL¨v)	Mvg ÔLÕ (gwnjvi msL¨v)
0 - 2000	42	22
2000 - 4000	23	56
4000 - 6000	15	12
6000 - 8000	10	4
8000 - 10000	8	6
10000 - 12000	1	0
12000 - 14000	1	0
‡gvU	100	100

mÎ Abyôvqx, c_‡g Mvg ÔKÕ-Gi e¨veavbv¼ wbY‡qi Rb¨ c‡qvRbxq gvb¸‡jv (A_vr, MwYwZK Mo I cwigvZ e¨eavb) mviwY 5.6.2-G wbYq K‡i †`Lv‡bv n‡jv|

mviwY 5.6.2: Mvg ÔKÔ-Gi gwnjv‡`i gwmK Av‡qi Mo I cwigvZ e¨eavb wbYq

gwmK Avq (UvKv)	gwnjvi msL¨v f_i	‡kYx ga¨-we› `y x_i	$d = \frac{x_i - A}{C.I.}$	d^2	fd	fd^2
0 - 2000	42	1000	–3	9	–126	378
2000 - 4000	23	3000	–2	4	–46	92
4000 - 6000	15	5000	–1	1	–15	15
6000 - 8000	10	7000 = A	0	0	0	0
8000 - 10000	8	9000	1	1	8	8
10000 - 12000	1	11000	2	4	2	4
12000 - 14000	1	13000	3	9	3	9
‡gvU	100				–174	506

$$\bar{x}_K = A + \frac{\sum fd}{N} \times C.I.$$

$$= 7000 + \left(\frac{-174}{100}\right) \times 2000$$

$$= 7000 + (-1.74) \times 2000$$

$$= 7000 - 3480$$

$$= 3520 \text{ UvKv}$$

A_vr, Mvg ÔKÔ-Gi gwnjv‡`i gwmK Av‡qi Mo n‡jv 3520 UvKv|

Gevi Avgv‡`i Mvg ÔKÔ-Gi gwnjv‡`i Av‡qi cwigvZ e¨eavb wbYq Ki‡Z n‡e|

$$\therefore \quad s_K = C.I. \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= 2000 \sqrt{\frac{506}{100} - \left(\frac{-174}{100}\right)^2}$$

$$= 2000 \sqrt{5.06 - (-1.74)^2}$$

$$= 2000 \sqrt{5.06 - 3.03}$$

$$= 2000 \times \sqrt{2.03}$$

$$= 2000 \times 1.42$$

$$= 2840$$

A_vr, Mvg ÔKÔ-Gi gwnjv‡`i Av‡qi cwigvZ e¨eavb 2800 UvKv

AZGe, Mvg ÔKÕ-Gi gwnjv‡`i Av‡qi e¨eavbv¼ n‡jv,

$$C.V_K = \frac{s}{\bar{x}} \times 100$$

$$= \frac{2840}{3520} \times 100$$

$$= 80.68$$

A_vr, Mvg ÔKÕ-Gi gwnjv‡`i Av‡qi e¨eavbv¼ n‡jv 80.68|

mviwY 5.6.3-G Mvg ÔLÕ-Gi gwnjv‡`i Av‡qi web¨v‡mi Mo I cwigvZ e¨eavb wbYq K‡i †`Lv‡bv n‡jv|

mviwY 5.6.3: Mvg ÔLÕ-Gi gwnjv‡`i Av‡qi web¨v‡mi Mo I cwigvZ e¨eavb wbYq

gwmK Avq (UvKv)	gwnjvi msL¨v	‡kYx ga¨-we› `y x_i	$d = \frac{x_i - A}{C.I.}$	d^2	fd	fd^2
0 - 2000	22	1000	–3	9	–66	198
2000 - 4000	56	3000	–2	4	–112	224
4000 - 6000	12	5000	–1	1	–12	12
6000 - 8000	4	7000 = A	0	0	0	0
8000 - 10000	6	9000	1	1	6	6
10000 - 12000	0	11000	2	4	0	0
12000 - 14000	0	13000	3	9	0	0
‡gvU	100				–184	440

$$\therefore \quad \bar{x}_L = A + \frac{\sum fd}{N} \times C.I.$$

$$= 7000 + \frac{-184}{100} \times 2000$$

$$= 7000 + (-1.84) \times 2000$$

$$= 7000 - 3680$$

$$= 3320$$

A_vr Mvg ÔLÕ Gi gwnjv‡`i Av‡qi Mo n‡jv 3320 UvKv|

Gevi Avgv‡`i Mvg ÔLÕ-Gi gwnjv‡`i Av‡qi cwigvZ e¨eavb wbYq Ki‡Z n‡e|

$$\therefore \quad s_L = C.I.\sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$= 2000\sqrt{\frac{440}{100}-\left(\frac{-184}{100}\right)^2}$$

$$= 2000\sqrt{4.4-(-1.84)^2}$$

$$= 2000\sqrt{4.4-3.39}$$

$$= 2000\times\sqrt{1.01}$$

$$= 2000\times 1.005$$

$$= 2010$$

A_vr, Mvg ÔLÔ-Gi gwnjv‡`i Av‡qi cwiwgZ e¨eavb n‡jv 2010 UvKv|

AZGe, Mvg ÔLÔ Gi gwnjv‡`i Av‡qi e¨eavbv¼ n‡jv,

$$\text{C.V}_L = \frac{\text{S}}{\overline{\text{x}}}\times 100$$

$$= \frac{2010}{3320}\times 100$$

$$= 60.54$$

A_vr, Mvg ÔLÔ-Gi gwnjv‡`i Av‡qi e¨eavbv¼ n‡jv 60.54|

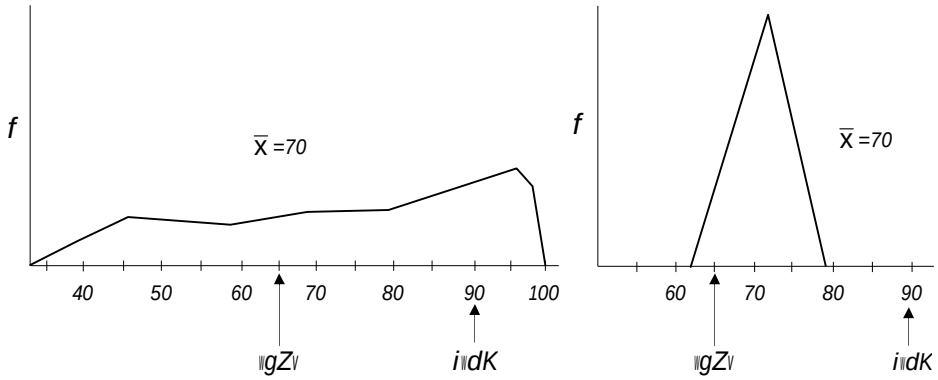
‡h‡nZz Mvg ÔKÔ-Gi gwnjv‡`i gwmK Av‡qi e¨eavbv¼ (80.68) Mvg ÔLÔ-Gi gwnjv‡`i gwmK Av‡qi e¨eavbv¼ (60.54) A‡c¶v †ekx, †m‡nZz Dcmsnv‡i Avgiv ej‡Z cwi †h, Mvg ÔKÔ-Gi gwnjv‡`i gwmK Av‡qi wëb¨vm Mvg ÔLÔ-Gi gwnjv‡`i gwmK Av‡qi wëb¨vm A‡c¶v ‡ekx wew¶ß ev we¯Z|

cwiwgZ gvb (Standard Score)

cwiwgZ e¨eavb Avgv‡`i‡K Mo †_‡K msL¨v gvb¸‡jvi Mo we¯wZ. m¤ú‡K avivYv †`q| wKš GQvovI †Kvb DcvËiwki †Kvb we‡kl GKwU gv‡bi Ae¯vb Avgv‡`i Rvbvi c‡qvRb n‡Z cv‡i| cwiwgZ gvb wbY‡qi gva¨‡g Avgiv Zv Rvb‡Z cwi| kZnwi‡Ki mxgve×ZwU n‡jv †h, †m¸‡jv‡K †hvM Kiv hvq bv, ¸Y Kiv hvq bv, Mo Kiv hvq bv, Ges RwUj cwimYbvi Rb¨ mw¤§wjZ Kiv hvq bv| GK ¸"Q ch‡e¶‡Yi g‡a¨ GKwU ch‡e¶Y wKfv‡e Ges KZUkZa‡fb Zv wb‡`‡ki Rb¨ cwiwgZ gvb GKwU weKí c×wZ c`vb K‡i| cwiwgZ gvb‡K Mo Kiv hvq Ges RwUj cwimsL¨vbK cwimYbvqI e¨envi Kiv hvq| cwiwgZ gvb‡K z-gvbI ejv n‡q _v‡K|

GKwU D`vniY †`qv hvK| aiv hvK, †Kvb GKwU ‡kYx‡Z `ÿwU cix¶v †bqv n‡jv| GKwU ga¨eZx cix¶v Ges Aciwu Pevš cix¶v| `ÿwU cix¶vi djvdjB QvÎ-QvÎx‡`i †MW wbavi‡Y mgvb ¸iæZ enb Ki‡e| †`Lv ‡Mj, Dfq cix¶vq QvÎ-QvÎx‡`i cvß b¤i‡ii Mo 60| ga¨eZx cix¶vq wgZv 80 b¤i †c‡q‡Q Ges Pevš cix¶vq †m †c‡q‡Q 55 b¤i| Ab¨`‡K, iwdK ga¨eZx cix¶vq 55 Ges Pevš cix¶vq 80 b¤i †c‡q‡Q| Avgiv wK ej‡Z cwi †h, iwdK I wgZv me‡k‡l GKB iKg djvdj K‡i‡Q? Zviv `ÿR‡bB wK GKB †MW cv‡e?

hw` wbi¼k msL¨v gv‡bi Dci wfwË K‡i cix¶vi djvdj we‡klY Kiv nq, Zvn‡j DËi n‡e Ônüô| KviY, Zviv `ÿZv cix¶vq mgvb b¤i †c‡q‡Q| A_vr, Df‡qB †gvU 135 b¤i †c‡q‡Q Ges Df‡qB `ÿZv cix¶vq M‡o 67.5 b¤i †c‡q‡Q| wKš, hw` cÖKvi †kYxi †cw¶‡Z Zv‡`i Av‡cw¶K KwZZ‡K we‡ePbvq Avbv nq, Zvn‡j DËiwU n‡e Ôbvô| †`Lv hv‡e †h, cix¶vq Zv‡`i `ÿZv‡bi ARb mgvb bq| wb‡gi †jLwP‡Îi gva¨‡g `ÿZv cix¶vq QvÎ-QvÎx‡`i cwß b¤‡ii



ga¨eZx cix¶vq cwß b¤i Pewš– cix¶vq cwß b¤i

MYmsL¨v webv‡mi w`‡K j¶¨ Kiv hvK|

wPÎ 5.6.1: `ÿZv cix¶vq QvÎ-QvÎx‡`i cwß b¤‡ii webvm

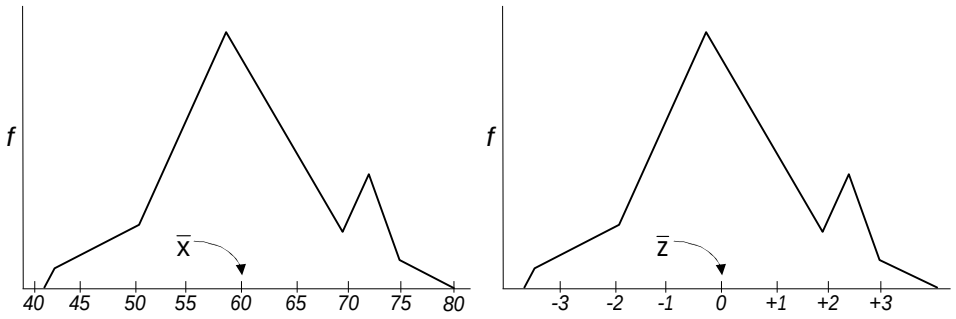
Dc‡ii wP‡Î †`Lv hvq ‡h, Df‡qB GKwU cix¶vq Mo †_‡K 20 b¤i †ekx Ges Aci cix¶vq Mo †_‡K 5 b¤i Kg †c‡q‡Q| wKš cÖKvi †kYxi †cw¶‡Z †`Lv hvq †h, ga¨eZx cix¶vq wgZvi cwß 80 b¤‡ii Zzjbvq iwd‡Ki cix¶vi 80 b¤i fv‡jv Ae¯vb c`kb K‡i| KviYwU n‡jv, ga¨eZx cix¶vi b¤i¸‡jv A‡bK †ekx we¯Zfv‡e Qwo‡q i‡q‡Q| A_vr, `ÿZv webvmi we¯wZ. `ÿZvi Kg Ges d‡j `ÿZv webvmi g‡a¨ cwiwgZ e¨eavb wfb n‡q‡Q| ga¨eZx cix¶vi cwiwgZ e¨eavb 20 Ges Pewš cix¶vi cwiwgZ e¨eavb 5| we¯wZi GB wfbZwUB cwZwU gv‡bi Av‡cw¶K Ae¯vb wbY‡qi Rb¨ baviK wnmv‡e KvR K‡i|

G iKg cwi¯wZ‡Z Pewš cix¶vq Mo †_‡K 20 b¤i †ekx cvIqwU †h ga¨eZx cix¶vi Mo †_‡K 20 b¤i cvIqvi †P‡q AwaKZi †kq Zv wbYq Kivi Rb¨ Avgv‡`i GKwU Dcvq ‡ei Kiv c‡qvRb| cwZwU gv‡bi Av‡cw¶K Ae¯vb wbY‡qi †kô c_vU n‡jv mswkô webvm¸‡jvi cwiwgZ e¨eavb‡K we‡ePbvq G‡b cwZwU A‡kwaZ gv‡bi Rb¨ cwiwgZ gvb wbYq Kiv|

cwiwgZ gv‡bi ˆewkó¨ (Characteristics of Standard Scores)

cwiwgZ gv‡bi wKQymwbw`ó ˆewkó¨ i‡q‡Q| cwiwgZ gvb¸‡jv n‡jv †gŠwjK msL¨v| KviY, A‡kwaZ msL¨v¸‡jv cwiwgZ msL¨vq i¡cvš‡ii mgq Zv‡`i cwigv‡ci GKK¸‡jv ewZj n‡q hvq| A_vr, cwiwgZ gvb n‡jv GKK wenxb ¯vaxb msL¨v| hw` webv‡mi cwZwU gv‡bi Rb¨ c„Kfv‡e cwiwgZ gvb wbYq Kiv nq Ges †m¸‡jvi GKwU MYmsL¨v webvm †jLwP‡Î Dc¯vcb Kiv nq, Z‡e †`Lv hv‡e †h, cwiwgZ gvb¸‡jvi †jLwPÎwqZ webvm A‡kwaZ gv‡bi wPÎwqZ webv‡mi Abyiæc n‡e| †jLwP‡Îi Abyfwg K A‡¶ Dc¯wcZ A‡kwaZ gv‡bi msL¨v gvb¸‡jvi cwie‡Z ïaygvÎ cwiwgZ gv‡bi gvb¸‡jv Dc¯wcZ n‡e| wb‡gi †jLwPÎ `ÿZvi gva¨‡g A‡kwaZ gv‡bi msL¨v gvb I cwiwgZ msL¨v gv‡bi GB m¤úK© n‡q D‡V|

cwiwgZ gv‡bi wKQymwbw`ó ˆewkó¨ i‡q‡Q| cwiwgZ gvb¸‡jv n‡jv †gŠwjK msL¨v| KviY, A‡kwaZ msL¨v¸‡jv cwiwgZ msL¨vq i¡cvš‡ii mgq Zv‡`i cwigv‡ci GKK¸‡jv ewZj n‡q hvq| A_vr, cwiwgZ gvb n‡jv GKK wenxb ¯vaxb msL¨v|



(K) A‡kwaZ gj‡- gv‡bi ‡eb¨vm (L) i¦cvš—‡iZ cwigZ gv‡bi ‡eb¨vm

wPÎ 5.6.2: A‡kwaZ gj‡- gv‡bi ‡eb¨vm (K) Ges i¦cvš‡iZ cwigZ gv‡bi ‡eb¨vm (L)

Dc‡iv³ wPÎ `ÿvU †_‡K Avgiv cwigZ gv‡bi Av‡iv wKQy¸‡i ZcY¨ ˆe‡ók¨ †`L‡Z cvB| †hgb, ‡h †Kvb ‡eb¨v‡mi Rb¨ cwigZ gvbmg‡ni Mo ($\bar{z}$) memgqB ÔOÕ n‡e Ges cwigZ e¨eavb n‡e 1| hLb ‡eb¨vm‡U A‡c¶vKZ.gmwY ev mylg (smooth or symmetrical) nq, ZLb A‡aKvsk cwigZ msL¨v gvbB –3 †_‡K +3 cwigZ e¨eav‡bi g‡a¨B ‡eivR K‡i| Ab¨ K_vq, mylg ‡eb¨v‡mi A‡aKvsk gvbB Mo †_‡K 3 cwigZ e¨eavb Dc‡i ev bx‡P Ae¯vb K‡i| Z‡e G‡U g‡b ivLv c‡qvRb †h, GB ‰e‡ók¨‡U †Kvb †jŠn K‡Vb ‡bqg bq| KviY, A‡bK mgq –3-Gi †P‡q A‡bK bx‡P Ges +3-Gi A‡bK Dc‡i I cwigZ gvb Ae¯vb K‡i| cwigZ gv‡bi Av‡iK‡U AvKlYxq ˆe‡ók¨ n‡jv, eMKZ.cwigZ gv‡bi mg‡ó ‡eb¨v‡mi †gvU msL¨v gv‡bi mgvb| A_vr, $\sum z^2 = N$ |

Avgiv Avil R‡b †h, GK‡U mylg ‡eb¨v‡mi g‡a¨ M‡oi Dc‡i Ges bx‡P 3‡U K‡i cwigZ e¨eavb i‡q‡Q| cwZ‡U cwigZ e¨eav‡bi Af¨š‡i wK cwigvY NUbmsL¨v i‡q‡Q cwigZ gv‡bi gva¨‡g Zvi AvqZb †ei Kiv hvq| †hgb, ±1 cwigZ e¨eav‡bi Aax‡b 68.27 kZvsk, ±2 cwigZ e¨eav‡bi Aax‡b 95.45 kZvsk, Ges ±3 cwigZ e¨eav‡bi Aax‡b 99.74 kZvsk NUbmsL¨v i‡q‡Q| Av‡iv mybw`ófv‡e ejv hvq †h, GK‡U wbLyZ mylg †iLvi Aax‡b †gvU AvqZb n‡jv 1 (wPÎ 5.4.1 `óe¨)|

cwigZ gvb wbYq (Computing Standard Scores)

GK‡U Dcv˧iwki †h †Kvb GK‡U A‡kwaZ gv‡bi (x) cwigZ gvb n‡jv D³ A‡kwaZ gvb †_‡K ‡eb¨v‡mi M‡oi cv_K¨‡K $(x_i - \bar{x})$ Dcv˧iwki cwigZ e¨eavb (s) w`‡q fvM Ki‡j †h msL¨v gvbwU cvIqv hvq Zv| GK‡U ‡eb¨v‡mi cwZ‡U msL¨v gv‡biB GK‡U cwigZ gvb i‡q‡Q, hv †mB ‡eb¨v‡mi Mo I cwigZ e¨eav‡bi gv‡bi Dci wbfi K‡i| GLb Avgv‡`i D`vniY‡U m‡Î c‡qvM K‡i wgZv I id‡Ki Av‡c¶K Ae¯vb‡U wbYq K‡i †`L‡ev| cwigZ gvb wbY‡qi exRMwYZK m‡Î n‡jv,

$$z = \frac{x_i - \bar{x}}{s}$$

†hLv‡b, z = cwigZ gvb
x_i = cwZ‡U A‡kwaZ gvb
$\bar{x}$ = MwYZK Mo
s = cwigZ e¨eavb

Avgiv †R‡bwQ †h, DcvËiwki GKwU wbw`ó msL¨v gvb Mo †_‡K KZ cwiwgZ e¨eavb Dc‡i ev bx‡P Ae¯vb Ki‡Q Zv cwiwgZ gv‡bi gva¨‡g Rvbv hvq| hw` cix¶vi b¤‡ii Mo 60 Ges cwiwgZ e¨eavb 20 nq, Z‡e cix¶vq cvß 80 b¤i Mo †_‡K GK cwiwgZ e¨eavb Dc‡i Ae¯vb Ki‡e| Aci w`‡K, hw` cix¶vq cvß b¤‡ii Mo 60 Ges cwiwgZ e¨eavb 5 nq, Z‡e cix¶vq cvß 80 b¤i Mo †_‡K Pvi cwiwgZ e¨eavb Dc‡i Ae¯vb Ki‡e| mviwY 5.6.1-G Avgv‡`i D`nvi‡Yi msL¨v¸‡jv e¨envi K‡i wgZv I iwd‡Ki `ÿZv cix¶vq cvß b¤‡ii cwiwgZ gvb wbYq K‡i †`Lv‡bv n‡jv|

mviwY 5.6.1: wgZv I iwd‡Ki cix¶vq cvß A‡kwaZ gv‡bi cwiwgZ gvb wbYq

	ga¨eZx cix¶v $\bar{x}$ = 60, s = 20	Pevš cix¶v $\bar{x}$ = 60, s = 5
wgZv	A‡kwaZ gvb = 80 $z = \frac{80 - 60}{20} = +1.0$	A‡kwaZ gvb = 55 $z = \frac{55 - 60}{5} = -1.0$
iwdK	A‡kwaZ gvb = 55 $z = \frac{55 - 60}{20} = -2.5$	A‡kwaZ gvb = 80 $z = \frac{80 - 60}{5} = +4.0$

Pevš cix¶vq iwd‡Ki cwiwgZ gvb +4.0 Øviv †evSv hvq †h, †m 80 †c‡q Mo †_‡K 4 cwiwgZ e¨eavb Dc‡i Ae¯vb Ki‡Q| Aciw`‡K, ga¨eZx cix¶vq wgZvi cwiwgZ gvb +1.0 Øviv †evSv hvq †h, †m 80 †c‡q Mo †_‡K 1 cwiwgZ e¨eavb Dc‡i Ae¯vb Ki‡Q| myZivs, iwd‡Ki cvß 80 b¤i wgZvi cvß 80 b¤‡ii †P‡q AwaK fv‡jv djvdj| cwiwgZ gvb abvZ¥K Ges FYvZ¥K DfqB n‡Z cv‡i| FYvZ¥K cwiwgZ gvb A_ n‡jv †h, GKwU msL¨v gv‡bi Mo †_‡K bx‡P Ae¯vb Kiv‡K wb‡`k K‡i|

cwiwgZ gv‡bi myweavi (Advantages of Standard Scores)

A‡kwaZ gv‡bi Zzjbvq cwiwgZ gv‡bi myweavi A‡bK †ekx| GKwU D`vni‡Yi gva¨‡g welqwU †evSvi †Póv Kiv hvK| aiv hvK, †Kvb GKwU cwZôv‡b KgPvix wb‡qv‡Mi Rb¨ cv_x‡`i wZb ai‡Yi cix¶vq Ask wb‡Z nq — c_gwU n‡jv 500 b¤‡ii wjwLZ cix¶v, wØZxqwU n‡jv 100 bš‡ii ¯§iYkw³ cix¶v, Ges ZZxqwU n‡jv 3 c‡q‡›Ui fvlv `¶Zvi cix¶v| cix¶v Abyôz nevi ci ‡`Lv †M‡jv †h, GKRb cv_x wjwLZ cix¶vq 370, ¯§iYkw³ cix¶vq 60 Ges fvlv `¶Zv cix¶vq 1.6 †c‡q‡Q| GLb ck n‡jv, mweKfv‡e mswkó cv_x ‡Kgb K‡i‡Q? wewfb gvÎvq cwigvc Kiv GB A‡kwaZ gvb¸‡jv‡K Avgiv wKfv‡e Zzjbv Ki‡ev? Ab¨vb¨ cv_xi cvß b¤‡ii †P‡q GB cv_x KZ fv‡jv ev Lvivc K‡i‡Q?

DwjwLZ b¤i¸‡jv †`‡LB Avgiv Gme c‡ki Reve †c‡Z cwi bv| KviY, wZbwU cix¶vq e¨eüZ gvÎv wfb — 500, 100, Ges 3| myZivs, G †¶‡Î cwiwgZ gv‡bi gva¨‡g Avgiv G mKj c‡ki Reve †c‡Z cwi| Avgiv Rwb †h, †Kvb web¨v‡mi Mo I cwiwgZ e¨eavb Rvbv _vK‡j Avgiv †mB web¨v‡mi †Kvb GKwU wbw`ó msL¨vgvb‡K cwiwgZ gv‡b i¡cvšwiZ Ki‡Z cwi| G ‡¶‡Î I mKj cix¶v_xi b¤‡ii Mo I cwiwgZ e¨eavb †R‡b Avgiv Av‡jvP¨ cix¶v_xi cwiwgZ gvb wbYq Ki‡Z cwi|

aiv hvK, we wfb ci x ¶vq Av‡jvP¨ ci x ¶v_x i cvß b¤‡i i cwiwgZ gvb¸‡jv wbgi¡c,

z = –0.05 (wjwLZ ci x ¶v)

z = 0.1 (¯§iYkw³ ci x ¶v)

z = –2.0 (fvlv `¶Zvi ci x ¶v)

cwiwgZ gvb¸‡jv chv‡jvPbv Ki‡j †`Lv hvq †h, Ab¨vb¨ ci x ¶v_x i Zzjbvq Avgv‡`i Av‡jvP¨ ci x ¶v_x `ÿvU ci x ¶vq M‡oi †P‡q bx‡P Ae¯vb Ki‡Q Ges GKwU ci x ¶vq Mo †_‡K 1 cwiwgZ e¨eavb Dc‡i i‡q‡Q| A_vr, D³ ci x ¶v_x mweKfv‡e †Zgb KZKvhZv †`Lv‡Z cv‡i wb|

mvivsk

wfb cwigvc GKK m¤wjZ `ÿvU DcvËiwki g‡a¨ wfbZvi Zzjbvi Rb¨ wbi¼k e¨eavb¸‡jv‡K Av‡cw¶K c‡` i¡cvšwiZ Kivi c‡qvRb nq| Av‡cw¶K we¯vi cwigv‡ci GKwaK c×wZ i‡q‡Q| †hgb, PZzK e¨eavbv¼, Mo e¨eavbv¼, e¨eavbv¼, cwiwgZ gvb, BZ¨`| †Kvb DcvËiwki Mo Ges cwiwgZ e¨eav‡bi kZKiv Abyqvz‡K †mB DcvËiwki e¨eavbv¼ ejv nq| we¯wZi Av‡cw¶K cwigvc wbY‡qi Rb¨ we¯wZi †h †Kvb wbi¼k cwigvc e¨eüZ n‡j Av‡cw¶K cwigv‡ci †¶‡Î cvq memgqB Avgiv cwiwgZ e¨eavb e¨envi K‡i _wK| GK ¸”Q ch‡e¶‡Yi g‡a¨ GKwU we‡klZ ch‡e¶Y wKfv‡e Ges KZUkzwfb Zv wb‡`‡ki Rb¨ cwiwgZ gvb GKwU we‡klfv‡e Dc‡hvMx c×wZ wnmv‡e KvR K‡i| cwiwgZ gvb‡K z-gvbl ejv n‡q _v‡K|

cv‡VvËi gj¨vqb

^be¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| †Kvb DcvËiwki Mo I cwiwgZ e¨eav‡bi kZKiv Abyôv‡Z‡K:
   K. †f`v¼ e‡j
   L. e¨eavbv¼ e‡j
   M. cwiwgZ e¨eavb e‡j
   N. cwimi e‡j

2| ‡Kvb DcvËiwki †Kvb we‡kl GKwU gv‡bi Ae¯vb Rvbvi Rb¨:
   K. cwiwgZ e¨eavb Rvbv c‡qvRb
   L. cwiwgZ Mo Rvbv c‡qvRb
   M. cwiwgZ †f`v¼ Rvbv c‡qvRb
   N. cwiwgZ gvb Rvbv c‡qvRb

3| cwiwgZ gvb wbY‡qi exRMwYZK m¤úwU n‡jv:

   K. $z = \frac{x_i - \overline{x}}{s}$

   L. $z = \frac{\overline{x} - x_i}{s}$

   M. $z = \frac{x_i^{\,2} - \overline{x}}{s}$

   N. $z = \frac{x_i - (\overline{x})^2}{s}$

msw¶ß ck

1| cwiwgZ gv‡bi myeav Av‡jvPbv Kiæb|
2| cwiwgZ gv‡bi ^ewkó¨ Av‡jvPbv Kiæb|

iPbvgj¨K ck

1| cwiwgZ gv‡bi e¨envi D`vniYmn Av‡jvPbv Kiæb|
2| e¨eavbv¼ wbY‡qi c‡qvRbxqZv wK? D`vniYmn e¨eavbv¼ wbY‡qi c×wZ Av‡jvPbv Kiæb|

cvV - 8

we¯wZi cwigv‡ci wewfb c×wZi g‡a¨ Zzjbv Ges wbevPb
Comparison and Choice between Measures of Dispersion

GB cvV †k‡l hv Rvbv hv‡e —

- we¯wZi cwigvcmg‡ni g‡a¨ Zzjbv
- we¯wZi h_vh_ cwigvc wbevPb

we¯wZi cwigvcmg‡ni g‡a¨ Zzjbv (Comparison between Measures of Dispersion)

we¯wZi cwZwU cwigv‡ci wbR¯ wKQy^ewkó¨ i‡q‡Q Ges cwZwUiB wKQymyjeav Ges Amyjeav i‡q‡Q| G¸‡jv c‡qv‡Mi wm×vš wbfi K‡i Dcv‡Ëi aiY Ges M‡elYvi D‡Ï‡k¨i Dci| cwZwU c×wZi ^ewkó¨ Ges Dc‡hwMZv we‡ePbv K‡i Avgiv Zv‡`i g‡a¨ GKwU Zzjbvgjv‡K we‡klY Ki‡Z cwi Ges †Kvb cwigvcwU AwaK MnY‡hvM¨ †m wel‡q l GKwU wm×v‡š †cÖQ‡Z cwi|

cwigvgZ e¨eavb Ges †f`v¼‡KB e¨wßgjv‡K gvÎvq †ekx e¨envi Kiv nq|

cwigv‡ci gvÎv: Avgiv Rwb †h, wewfb gvÎvq cwigvcKZ.Dcv‡Ëi Rb¨ we¯vi cwigv‡ci wewfb c×wZ i‡q‡Q| bvgmPK Dcv‡Ëi †¶‡Î we¯vi cwigv‡ci Rb¨ Dchy³ c×wZ wbevPb Kiv m¤¢e bq| hw` µgmPK gvÎvq cwigvcKZ.DcvË _v‡K, Z‡e Avgiv cwimi, AvšPZzK cwimi ev PZzK e¨eavb e¨envi K‡i _vK| e¨wßgjv‡K gvÎvq cwigvcKZ.Dcv‡Ëi Rb¨ Avgiv Mo e¨eavb, cwigvgZ e¨eavb I †f`v¼ e¨envi K‡i _vK| Z‡e Mo e¨eavb wbY‡qi †¶‡Î FYvZ‡K wPý D‡c¶v Kivi Rb¨ †h‡nZzAb‡c¶Zv‡K e¨envi Kiv nq, †m‡nZzMo e¨eavb‡K exRMwYwZK cwiMYbvq e¨envi Kiv hvq bv Ges ZËMZfv‡e e¨vL¨v Kiv I hvq bv| ZvB cwigvgZ e¨eavb Ges †f`v¼‡KB e¨wßgjv‡K gvÎvq †ekx e¨envi Kiv nq|

Mo e¨eavb cieZx exRMwYwZK cwiMYbvq e¨envi‡hvM¨ bq Ges GwUi ZËMZ e¨vL¨v m¤¢e bq e‡j G †¶‡Î cwigvgZ e¨eavb I †f`v¼i e¨envi‡B ‡ekx n‡q _v‡K|

**‡K›`xq ceYZvi cwigv‡ci mv‡_ m¤úK:** KL‡bv KL‡bv we¯wZi cwigvcmgn †K›`xq ceYZvi cwigvcmg‡ni mv‡_ m½wZ †i‡L c‡qvM Kiv nq| hw` ‡Kvb DcvËiwki eYbvq ga¨gv e¨envi Kiv nq, Z‡e Zvi we¯vi cwigv‡ci †¶‡Î PZzK e¨eavb e¨eüZ nq| KviY, Dfq c×wZB Ae¯vb wfwËK cwigvc Ges kZnwi‡Ki Dci wbfi K‡i| M‡oi gva¨‡g Dcv‡Ëi we¯wZi cwigv‡ci †¶‡Î Mo e¨eavb, cwigvgZ e¨eavb I †f`v¼ c‡hvR¨| wKš Mo e¨eavb cieZx exRMwYwZK cwiMYbvq e¨envi‡hvM¨ bq Ges GwUi ZËMZ e¨vL¨v m¤¢e bq e‡j G †¶‡Î cwigvgZ e¨eavb I †f`v¼i e¨envi‡B ‡ekx n‡q _v‡K|

cwigvgZ e¨eav‡bi gvb DcvËiwki Amvaviy gvb Øviv mn‡RB cfweZ nq| ZvB ew¼g webv‡mi †¶‡Î cwimi ev cwigvgZ e¨eav‡bi c‡qvM h_v_ bq| G †¶‡Î PZzK e¨eavbB AwaK wbivc` cwigvc|

ew¼gZv: Avgiv Rwb †h, †Kvb DcvËiwk‡Z Pig gvb ev Amvaviy gvb _vK‡j Zv AwaK ew¼gZv c`kb K‡i| Avgiv GI Rwb †h, cwigvgZ e¨eav‡bi gvb DcvËiwki Amvaviy gvb Øviv mn‡RB cfweZ nq| ZvB ew¼g webv‡mi †¶‡Î cwimi ev cwigvgZ e¨eav‡bi c‡qvM h_v_ bq| eis, G †¶‡Î PZzK e¨eavbB AwaK wbivc` cwigvc| KviY, PZzK e¨eavb c_g I ZZxq PZz‡Ki g‡a¨ Zzjbv K‡i e‡j DcvËiwki g‡a¨ we`¨gvb Pig gvb¸‡jv Gi we‡ePbvi ev‡B‡i †_‡K hvq|

‡kYx e¨wßi cKwZ: cwimi DcvËiwki m‡ev”P I mewbg gv‡bi e¨eavb c`kb K‡i| wKš Dbœ³ †kYx e¨wßi †¶‡Î m‡ev”P ev mewbg gv‡bi †h †Kvb GKwU ev Dfq gvbB Avgv‡`i ARvbv †_‡K hvq| myZivs, Dbœ³ †kYx e¨wßi †¶‡Î cwimi c‡qvM Kiv hvq bv| Mo e¨eavb ev

cwiwgZ e¨eavb wbY‡qi †¶‡Î DcvËiwki cwZwU gvb‡K we‡ePbvq G‡b MYbv Kiv nq Ges †m ‡¶‡Î †kYx e¨eavb GKwU ¸iæZc¥~welq| †h‡nZzDb¥³ †kYx e¨wßi †¶‡Î †Kvb †Kvb †kYxi e¨eavb Avgv‡`i ARvbv †_‡K hvq, †m‡nZz†mLv‡b Mo e¨eavb, cwiwgZ e¨eavb ev †f`v¼ wbYq Kiv hvq bv| mÿZiv‡s, Db¥³ †kYx e¨wßi †¶‡Î mewaK Dchy³ c×wZ n‡jv PZzK e¨eavb| G †¶‡Î, Db¥³ †kYx mxgv‡K c_g I ZZxq PZz‡Ki evB‡i †i‡L mn‡RB PZzK e¨eavb wbYq Kiv hvq| wKš‘ hw` Db¥³ †kYx e¨wßwU c_g ev ZZxq PZz‡Ki g‡a¨ Ae¯vb K‡i, †m ‡¶‡Î PZzK e¨eavbI c‡hvR¨ bq|

Db¥³ †kYx e¨wßi †¶‡I mewaK Dchy³ c×wZ n‡jv PZzK e¨eavb|

exRMwYwZK cwiMYbv: †h †Kvb cwimsL¨vbK c×wZ wbevP‡bi †¶‡Î exRMwYwZK cwiMYbv GKwU Ab¨Zg ¸iæZc¥~we‡eP¨ welq| we¯vi cwigv‡ci †¶‡ÎI Gi e¨wZµg N‡U bv| Gi me¸‡jv c×wZ we‡ePbv K‡i Avgiv †`L‡Z cvB †h, GKgvÎ ‡f`v¼ I cwiwgZ e¨eavbB cieZx exRMwYwZK cwiMYbvi Rb¨ me‡P‡q †ekx Dchy³| KviY, wewfb gvÎvq cwigvcKZ. DcvË¸‡jvi g‡a¨ GKgvÎ e¨wßg‡jK ev Abyqvwgg‡jK gvÎvq cwigvcKZ.DcvË w`‡q Avgiv AwaK exRMwYwZK cwiMYbvi w`‡K AMmi n‡Z cwi| Avi G iKg Dcv‡Ëi Rb¨ cwiwgZ e¨eavbB me‡P‡q Dchy³ cwigvc| GKwaK DcvËiwki cwiwgZ e¨eavb †_‡K Avgiv GKwU mgw¤Z cwiwgZ e¨eavb wbYq Ki‡Z cwi, hvi gva¨‡g wewfb DcvËiwki g‡a¨ Zzjbv Kiv m¤e nq| we¯wZi me¸‡jv wbi¼k cwigv‡ci g‡a¨ GKgvÎ cwiwgZ e¨eavbB DcvËiwki me¸‡jv gvb‡K cKZfv‡e we‡ePbv K‡i| Mo e¨eavb me¸‡jv gvb‡K we‡ePbv Ki‡jI G‡Z e¨eüZ Ab‡c¶Zv exRMwYwZK cwiMYbvi †¶‡Î Gi Dc‡hvMxZv‡K Kwg‡q †`q|

GKgvI ‡f`v¼ I cwiwgZ e¨eavbB cieZx exRMwYwZK cwiMYbvi Rb¨ me‡P‡q †ekx Dchy³|

mnR‡eva¨Zv: mnR‡eva¨Zvi we‡ePbvq we¯vi cwigv‡ci c×wZ¸‡jvi g‡a¨ cwimiB me‡P‡q †ekx MnY‡hvM¨| KviY, DcvËiwki ïaygvÎ `ÿwU gvb (mewbg I m‡ev”P) Rvbv _vK‡jB cwimi wbYq Kiv m¤e| ZvB, hLb Lye `ªæZZvi mv‡_ we¯vi cwigvc m¤ú‡K avivYv †bevi c‡qvRb nq ZLb cwimiB n‡jv Dchy³ cwigvc|

hLb Lye `ªæZZvi mv‡_ we¯vi cwigvc m¤ú‡K avivYv †bevi c‡qvRb nq ZLb cwimiB n‡jv Dchy³ cwigvc|

we¯wZi h_vh_ cwigvc wbevPb (Selection of Appropriate Measure of Dispersion)

we¯wZi wewfb cwigv‡ci gvb¸‡jv, hw` DcvËiwk Zzjbvg‡jK fv‡e Qwo‡q wQwU‡q _v‡K Z‡e cwigv‡ci gvb¸‡jv eo n‡e, Avi hw` DcvËiwk Zzjbvg‡jK fv‡e KvQvKwQ Ae¯vb K‡i Z‡e †m¸‡jvi gvb †QvU n‡e| GLb ck n‡jv †h, we¯wZi †Kvb cwigvcwU †kô? GK K_vq Gi Reve †`qv m¤e bq| KviY, cwigvc¸‡jvi cZ¨qMZ Abb¨Zv Dcv‡Ëi cKwZ. Ges DcvË eYbvi †¶‡Î †m¸‡jv‡K wKfv‡e e¨envi Kiv n‡e GB mKj wel‡qi Dci mg¨K avivYv †bevi ciB †Kejgv‡Î Gi Reve †`qv m¤e| cwimi GKwU web¨v‡mi m‡ev”P I mewbg gv‡bi g‡a¨ `iZ‡K wb‡`k K‡i| µgmwPK I e¨wßg‡jK gvÎvq cwigvcKZ.Dcv‡Ëi we¯wZ. eYbvi Rb¨ GwU‡K e¨envi Kiv nq| cwim‡ii mÿeawU n‡jv, GwU mn‡R wbYq Kiv hvq Ges †evSv hvq| wKš‘ Amÿeawu n‡jv, GwU GKwU A¯wZkxj cwigvc| KviY, GwU m¤úK©fv‡e `ÿwU ch‡e¶Y gv‡bi Dci wbfikxj|

ew¼g web¨v‡mi †¶‡Î AvšPZzK cwimi GKwU Dchy³ cwigvc wnmv‡e e¨eüZ nq| AvšPZzK cwimi ZZxq Ges c_g PZzK A_ev 75Zg Ges 25Zg kZnwi‡Ki ga¨Kvi cv_K¨‡K c`kb K‡i| hw` I cwim‡ii Zzjbvq AwaKZi w¯wZkxj Ges µgmwPK ev Zvi †P‡q D”P gvÎvq cwigvcKZ. Dcv‡Ëi †¶‡Î c‡qvM Kiv hvq, AvšPZzK cwimi web¨v‡mi Aš—f³ mKj DcvË‡K cwZdwjZ K‡i bv|

hw` I M‡elYvi D‡Ik¨ Ges Dcv‡Ëi cKwZ. Abjyvqx †¶Î we‡k‡l Ab¨ c×wZ¸‡jv‡K GKRb M‡elK †ekx Dchy³ g‡b K‡ib, wKš mwnZ we‡Pv‡i, cwigvY e¨eavbB we¯—vi cwigv‡ci me‡P‡q MnY‡hvM¨ c×wZ|

we¯wZ‡K eYbv Kivi Av‡iKwU c×wZ n‡jv, GKwU cwigv‡ci gvÎvq †Kvb wbawiZ we›`y (fixed point) ‡_‡K ch‡e¶Y¸‡jv ev Dcv‡Ëiwki gvb¸‡jv KZUkzwfb Zvi gvÎwU wb‡`k Kiv| MwYwZK hy³ ‡Z GKwU web¨v‡mi Mo‡K GB ¯§viK we›`y (reference point) wnmv‡e wbevPb Kiv nq| Mo †_‡K cwZwU gv‡bi wePvwiZi Dci wbfi K‡i `ÿwU cwigvc Dcv‡Ëi we¯wZ‡K wbYq K‡i – GKwU n‡jv Mo e¨eavb, Ges Ab¨wU n‡jv cwiwgZ e¨eavb| wKš cwigv‡ci AšwbwnZ hy³ Abjyvqx, cwiwgZ e¨eavbB n‡jv me‡P‡q †ekx Dc‡hvMx cwigvc| GB AšwbwnZ hy³ ¸‡jv Avgiv c‡e Av‡jvPbv K‡iwQ| Kv‡RB, G chv‡q G‡m Avgiv Avi †m¸‡jvi cÖvPxwi³ Ki‡ev bv| Z‡e, †hwU msw¶ßfv‡e D‡jL Kiv c‡qvRb Zv n‡jv, GKwU web¨v‡mi we¯vi eYbvi Rb¨ cwiwgZ e¨eavb me‡P‡q ‡ekx KvhKi Ges Dc‡hvMx cwigvc cavbZt Pviwu Kvi‡Y| c_gZt, gj- Dcv‡Ë †h GK‡K cwigvcKZ.nq GwU †mB cwigv‡cB cKwkZ nq; wØZxqZt, GwU web¨v‡mi mKj gvb‡K cwZdwjZ K‡i; ZZxqZt, bgybv wePvwiZi Øviv Kg cfweZ nq e‡j we¯wZi Ab¨vb¨ cwigv‡ci Zzjbvq GwU AwaKZi w¯wZkxj cwigvc; Ges PZzZt, cwiwgZ e¨eav‡bi exRMwYwZK ˆewkó¨¸‡jv GwU‡K RwUj cwimsL¨vbK KgKv‡Û e¨envi‡i Rb¨ mÿhvM K‡i †`q| GK K_vq, hw` I M‡elYvi D‡Ïk¨ Ges Dcv‡Ëi cKwZ. Abjyvqx †¶Î we‡k‡l Ab¨ c×wZ¸‡jv‡K GKRb M‡elK †ekx Dchy³ g‡b K‡ib, wKš mwnZ wePv‡i cwiwgZ e¨eavbB we¯vi cwigv‡ci me‡P‡q MnY‡hvM¨ c×wZ|

kZnwiK (Percentile)

we¯wZ. cwigv‡ci Aci GKwU Ae¯vb wfwËK cwigvc n‡jv kZnwiK| mnR K_vq ejv hvq †h, kZnwiK n‡jv GKwU Pj‡Ki †mB me gvb hv mwie× Z_¨‡K mgvb 100 fv‡M fvM K‡i| A_vr, GKwU mwie× Z_¨wki g‡a¨ Avgiv 99wU kZnwiK cvB, kZnwiK wbY‡q †¶‡Î Avgiv hZZg kZnwiK wbYq Ki‡Z PvB, $\frac{N+1}{100}$-Gi m‡_ ZZ ¸Y K‡iB Avgiv †mB kZnwiKwU cvB| †hgb, GKwU wb‡ek‡b hw` Avgiv 5g kZnwiK eYbv Ki‡Z PvB, Zvn‡j $L_1+\frac{\frac{(N+1)n}{100}-f_c}{f_p}\times c$ Zg msL¨vUB n‡e cÂg kZnwiK| kZnwiK wbY‡qi cwµqv PZzZK wbY‡qi cwµqvi g‡ZvB| A_vr,

$$n\text{ Zg kZnwiK} = L_1+\frac{\frac{(N+1)n}{100}-f_c}{f_P}\times c$$

GLv‡b, L_1 = n Zg kZnwiK †h †kYx‡Z Ae¯vb Ki‡Q †mB †kYxi wbg mxgv

f_c = kZnwiK †kYxi ceeZx †kYxi µg‡hwRZ MYmsL¨v

f_P = kZnwiK †kYxi MYmsL¨v

c = †kYx e¨eavb

GKwU D`vni‡Yi gva¨‡g welqwU †evSv †h‡Z cv‡i| cieZx cô‡q GKwU Mv‡gi 400 Rb e¨w³i Av‡qi web¨vm wb‡gi mviwY‡Z Dc¯wcZ n‡jv:

mviwY 5.8.1: GKwU Mv‡gi 400 Rb e¨w³i Av‡qi web¨vm

gwmK Avq (UvKv)	Mvgevmxi msL¨v (f_i)	‡hvwRZ MYmsL¨v (f_c)
5000 – 10000	92	92
10000 – 15000	79	171
15000 – 20000	45	216
20000 – 25000	30	146
25000 – 30000	61	307
30000 – 35000	26	343
35000 – 40000	30	373
40000 – 45000	8	381
45000 – 50000	5	386
50000 – 55000	10	396
55000 – 60000	4	400
$\sum f_i = 400$		

Dc‡ii †kYx web¨vm †_‡K hw` Avgiv 30Zg kZnwiK wbYq Ki‡Z PvB, Zvn‡j c_‡g Avgv‡`i Rvb‡Z n‡e 30Zg kZnwiK KZZg msL¨v Ges Zv †Kvb †kYx‡Z Ae¯vb Ki‡Q|

A_vr, $\frac{(N+1)n}{100}$ Zg msL¨v = $\frac{(400+1)30}{100}$ = 120.3Zg msL¨v gvbwUB n‡e 30Zg kZnwiK| GB gvbwU 10000 – 15000 GB †kYx‡Z Ae¯vb Ki‡Q|

mxgvi vs, 30Zg kZnwiK n‡e = $10000 + \frac{120.3 + 92}{171} \times 5000$

= 10000 + 0.165497076 × 5000

= 10000 + 827.48538

= 10827.48538

mxgvi vs 30Zg kZnwiK n‡jv 10827.49 UvKv

ZvB, hLb Lye `ßiZvi mv‡_ we¯vi cwigvc m¤ú‡K avivYv †bevi c‡qvRb nq ZLb cwimiB n‡jv Dchy³ cwigvc|

mvivsk

we¯ÍvZvi cwigv‡ci cwZwUiB wKQzbR¯ ‰ewkó¨ i‡q‡Q| G¸‡jvi h_vh_ c‡qv‡Mi wm×vš wbfi K‡i Dcv‡Ëi ˆewk‡ó¨i Dci| mvaviYZt bvgmPK Dcv‡Ëi †¶‡Î Dchy³ c×wZ wbevPb Kiv m¤e bq| wKš µgmPK cwigv‡ci †¶‡Î cwimi, AvšPZzK cwimi, Ges e¨wßgj¨K cwigv‡ci †¶‡Î Mo e¨eavb, cwigvZ e¨eavb I †f`v¼ e¨envi K‡i _vwK| we¯ÍvZvi Avi GKwU eûj e¨eüZ c×wZ n‡"Q kZnwiK cwigvc| kZnwiK n‡jv GKwU Pj‡Ki †mB me gvb hv mwie× Z_¨‡K mgvb 100 fv‡M fvM K‡i|

cv‡VwËi gj¨vqb

^be¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| ..... GKwU web¨v‡mi m‡ev"P I mewbg gv‡bi g‡a¨ `iZ‡K wb‡`k K‡i|

K. cwigvZ e¨eavb
L. cwigvZ Mo
M. kZnwwiK
N. cwimi

2| ..... n‡jv GKwU Pj‡Ki †mB me gvb hv mwie× Z_¨‡K mgvb 100 fv‡M fvM K‡i|

K. Mo
L. kZnwwiK
M. PZzK
N. Dc‡ii me KÛwU

3| e¨wßgj¨K gvÎvq cwigvcKZ.Dcv‡Ëi Rb¨ Avgiv:

K. Mo e¨eavb e¨envi K‡i _wK
L. cwigvZ e¨eavb e¨envi K‡i _wK
M. ‡f`v¼ e¨envi K‡i _wK
N. Dc‡ii me KÛwU DËiB mwVK

msw¶ß ck

1| msÁv w`b I cv_K¨ Av‡jvPbv Kiæb:

K. ew¼gZv – †kYx e¨wßi cKwZ.
L. kZvnwwiK – c_g PZzK

iPbvgj¨K ck

1| we¯wZi h_vh_ cwigvc wbevP‡b †Kvb †Kvb welq¸wj‡K ¸iæZ t`qv DwPr e‡j Avcwb g‡b K‡ib|

2| Z_¨ e¨vL¨vq kZvnwwi‡Ki ¸iæZ Av‡jvPbv Kiæb|