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Abykí cix¶v
Hypothesis Testing

wm×všgj-K cwimsL¨v‡bi kvLvq Abykí cix¶v GKwU ¸i‡Zc¥ welq| ZË MV‡bi Rb¨ AwfÁZvjä m½wZ¸‡jv‡K GKwÎZ K‡i KZ¸‡jv cvi¯úwiK m¤úKhy³ †hŠw³K c¯vebv wbgvY Kiv hvq| GB c¯vebv¸‡jv cKZc‡¶ M‡elKi Ávbw`ß Abygvb, hv Abykí bv‡g AwfwnZ nq| wegZZvi wewfb gvÎvq Abykí wbwgZ nq Ges wewfb Drm †_‡K D™e N‡U| hZ¶Y bv G¸‡jv cixw¶Z n‡"Q ev G¸‡jvi h_v_Zv cgwYZ n‡"Q, ZZ¶Y Avgiv Rwb bv †h GB Abygvb¸‡jv mZ¨ wKbv| GB †hŠw³K c¯vebvi g‡a¨Kvi m¤úK¸‡jv cix¶vi gva¨‡g cgwYZ n‡j Zv ZwËK KvVv‡gvi As‡k cwiYZ nq| GKwU Z‡Ëi wfZ‡i wewfb cZ¨q I c¯vebvi g‡a¨Kvi m¤úK¸‡jv ev¯‡ei mv‡_ m½wZc¥ wKbv Zvl cix¶v K‡i †`Lv nq| Abykí cix¶v n‡jv bgybv chŠe¶‡Yi Dci wfwË K‡i mgMK m¤ú‡K Dcmsnvi Uvbvi GKwU wm×všgj-K c×wZ| Abykí cix¶vi wewfb c×wZ i‡q‡Q| GB c×wZ¸‡jvi c‡qvM wbfi K‡i Pj‡Ki cKwZ, bgybvi AvKvi Ges bgybv web¨v‡mi Dci| GB BDwb‡U Abykí cix¶vi wewfb welq wb‡q we¯wiZ Av‡jvPbv Kiv n‡e|

GB BDwb‡U Avgiv †h cvV¸‡jv Aa¨qb Ki‡ev †m¸‡jv n‡jv:

- cvV - 1 : Abykí - msÁv, cKvi‡f`, Drm Ges MV‡bi mgm¨v
- cvV - 2 : cwimsL¨vbMZ cix¶vi wewfb avc
- cvV - 3 : cwimsL¨vbMZ cix¶v 1: z-cix¶v
- cvV - 4 : cwimsL¨vbMZ cix¶v 2: t-cix¶v
- cvV - 5 : cwimsL¨vbMZ cix¶v 3: χ^2 (KvB-eM) cix¶v

cvV - 1

Ab¶wí – msÁv, cKvi‡f`, Drm Ges MV‡bi mgm¨v
Hypothesis – Definition, Types, Sources and Problems in Formulations

GB cvV †k‡l hv Rvbv hv‡e —

- Ab¶wí wK
- Ab¶‡íi Drm
- Ab¶‡íi cKvi‡f`
- Ab¶wí wbgv‡Y mgm¨v
- Ab¶wí cix¶v wK
- Ab¶wí cix¶vi cKvi‡f`

Ab¶wí wK (What is Hypothesis)?

> GKRb M‡elK we¤fb Pj‡Ki g‡a¨ m¤ú‡Ki wfwË‡Z †h ÔÁvbw`ß AbygvbÕ K‡ib †mB Abygvb¸‡jv n‡jv Ab¶wí|

Z_¨gvjv‡K A_c~Y© n‡Z n‡j ZwËK KvVv‡gvi Dci wbfikxj n‡Z nq| KviY, ZË Z_¨iwk‡K k¦„LwjZ K‡i Ges A_© c`vb K‡i| ZË Z_¨ AbymÜv‡bi w`K wb‡`kbv I w`‡q _v‡K| hLb Z_¨iwk‡K GKwÎZ K‡i k¦„LwjZ Kivi ci GKwU m¤ú‡Ki g‡a¨ †`Lv nq ZLb Zv Z‡Ë i|cvšwiZ nq| AZGe, ZË †Kvb Abygvb bq, eis Zv Z‡_¨i Dci wbwgZ wKQy†hŠw³K m¤ú‡Ki mw¤§jb| Z‡Ëi Ašfy³ wewfb Z_¨iwk‡K †hŠw³Kfv‡e we‡kl Y Kiv hvq Ges Z‡_¨i Ašfy³ bq Ggb m¤ú‡Ki Abygvb Kiv hvq| GKRb M‡elK wewfb Pj‡Ki g‡a¨ m¤ú‡Ki wfwË‡Z †h ÔÁvbw`ß AbygvbÕ (enlightened guess) K‡ib †mB Abygvb¸‡jv n‡jv Ab¶wí| GB Abygvb¸‡jv mZ¨ wKbv Zv G chv‡q Avgiv wbwðZfv‡e ej‡Z cwi bv| wKš GB m¤f~e× Abygvb¸‡jvB Ab¶wí MVb K‡i Ges G¸‡jvi h_v_Zv cgwYZ n‡j †m¸‡jv fwel¨r ZË MV‡bi As‡k cwiYZ nq| Ab¨ K_vq, Ab¶wí ev¯‘e I Z_¨wfwËK| A_vr, ev¯‘e AwfÁZvjä Abygvbg~jK c¯—vebv¸‡jv n‡jv Ab¶wí Ges G¸‡jvi h_v_Zv wbavi‡Yi Rb¨ cix¶v Kiv hvq|

Ab¶wí MV‡bi Abygvb¸‡jv c‡e cwiPwjZ M‡elYv ev ZË †_‡K AvnwiZ nq| Avgiv hv ZË †_‡K BwZg‡a¨ †R‡bwQ, hw` Zv w`‡q ïiæ Kwi Z‡e `ª‚Z ev Z‡ZwaK PjK wKfv‡e m¤úwKZ ‡m m¤ú‡K ‡hŠw³K Abygvb Ki‡Z cwi| †hgb, ˆRweK Pvc ZË ( biological stress theory) Avgv‡`i e‡j ‡h, `xN-¯’vqx kvixwiK Pvc Ae‡k‡l D"P i³ Pvc ˆZix K‡i Ges mvgvwRK AwZ¦KiY ZË (social assimilation theory) e‡j †h, GKwU wfb ms¯‹wZ‡Z AwZ¦KiY mnRvZfv‡eB gvbwmK Pvc m„ó K‡i| GB `ywU ZË †_‡K Abygvb Kiv hvq †h, †h mKj Awfevmx (immigrants) bZzb ms¯‹wZ‡Z Lvc LvB‡q wb‡Z Ly‡e †ekx Amyeavi m¤§yxb nb Zviv hviv Amyeavi m¤§yxb nb bv Zv‡`i †P‡q †ekx D"P i³ Pv‡c fy‡eb|

GKwU Ab¶wí‡K Dc‡hvMx n‡Z n‡j KZ¸‡jv ˆewkó¨ _vKv AZ¨vek¨K| †hgb, Ab¶wí‡K Aek¨B cZ¨qMZfv‡e m¯úó n‡Z n‡e| Ab¶‡íi AwfÁZvg~jK ¯§viK (empirical referent) _vK‡Z n‡e| Ab¶wí‡K mywbw`ó n‡Z n‡e| Ab¶wí‡K we‡kl‡Yi Rb¨ we`¨gvb †KŠkjmg~n Ges GKwU ZwËK KvVv‡gvi mv‡_ m¤úKhy³ _vK‡Z n‡e|

AbyK‡íi Drm (Sources of Hypothesis)

AbyK‡íi Drcw Ë nq wewfbœ Drm †_‡K| c_gZt, GKwU we‡kl ms¯‹Z‡_‡K| †h ms¯‹Z‡Z GKwU weÁvbi D™e I weKvk N‡U Zv A‡bK †gŠwjK AbyK‡íi Rb¥ †`q| †hgb, BD‡ivc, Av‡gwiKv I Gwkqvi mgv‡R mvgwRK MwZkxjZv I cwZ‡hvwMZvgj¨K AvPi‡Yi Aa¨q‡b †`Lv hvq †h, G welqwU m¤ú‡K gvbyl i Dcjwäi g‡a¨ wfbœv i‡q‡Q| GB wfbœv †_‡K wfbœ Z‡Ëi Rb¥ †bq Ges wfbœ wfbœ AbyK‡íi Drcw Ë nq| Av‡iv D`vniY †`qv †h‡Z cv‡i| †hgb, evsjv‡`‡ki cwievi MV‡bi cwµqv, aiY I D‡Ï‡k¨i mv‡_ Av‡gwiKv I BD‡ivcxq mgv‡Ri cv_K¨ i‡q‡Q| GB cv_K¨ †_‡K wfbœ AbyKíbwgZ nq| wewfbœ mgv‡R e¨w³K mgv‡eva m¤ú‡K avivYvi wfbœv †_‡K wfbœ AbyK‡íi Rb¥ †bq| GgbwK, GKwU mvgv‡Ri wewfbœ AÂ‡ji ms¯‹„ZK ˆewP‡Îi Kvi‡Y I wfbœ AbyKíbwgZ nq|

wØZxqZt, wfbœ wfbœ weÁvbi ga¨ †_‡K wfbœ wfbœ AbyK‡íi Rb¥ nq| †hgb, A_bxwZi †¶‡Î evRvi m¤ú‡Ki cKwZ. GK ai‡Yi AbyK‡íi Rb¥ †`q| mgvRweÁvb K‡l. m¤ú‡Ki ev¯‹evZv wfbœ ai‡Yi AbyKí ˆZix K‡i| RbmsL¨vi MwZcKwZ. we‡kl‡Y RbweÁvbxiv Av‡iK ai‡Yi AbyKí cix¶v K‡i _v‡Kb|

ZZxqZt, Zzjbv A‡bK mgq ¸iæZc¥ AbyK‡íi Rb¥ †`q| †hgb, WviDB‡bi ˆRweK weeZ‡bi ZË‡K nvevU †¯úÝvi mvgwRK weeZ‡b c‡qvM K‡i‡Qb| †igÛ cvj gwQi ce„×i nvi‡K gvbyl i mv‡_ Zzjbv K‡i RbmsL¨v ce„×i nvi‡K we‡kl‡Y K‡i‡Qb| D™¢` Ges cvYxi cwZ‡ek (ecology) m¤úwKZ ZË c‡qvM K‡i gvbyl i cwZ‡e‡ki ZË weKwkZ n‡q‡Q| Gi d‡j A‡bK bZzb AbyK‡íi Rb¥ wb‡q‡Q Ges cix¶Z n‡q‡Q|

PZzZt, e¨w³MZ e¨wZµgagx AwfÁZv †_‡KI A‡bK ¸iæZc¥ AbyK‡íi Rb¥ nq| †hgb, weL¨vZ weÁvbx _gvm †nbix nv·jx WviDB‡bi †jLv 'Origin of Species' MšÍU co‡Z wM‡q we®‹q cKvk K‡i e‡jwQ‡jb †h, wZwb GZ wb‡eva †Kb? †Kb wZwbI WviDB‡bi gZ Ggbfv‡e wPšÍv K‡ib wb| gRvi welqwU n‡jv, WviDB‡bi msMÖnxZ ARm Z_¨iwk ZZ¶Y chš GKwU k„LwjZ i¡c †bqwb hZ¶Y bv GKwU AvKw¯‹K NUbv N‡U| wZwb cRwZmg‡ni cwieZ‡bi KviYwU wKQyZB e‡S DV‡Z cviwQ‡jb bv| g¨vj_v‡mi RbmsL¨vi bxwZgvjvi Dci iPbwU covi ciB wZwb 'Struggle for Survival'-Gi avivYwU weKwkZ Ki‡Z m¶g nb| GKBfv‡e, MvQ †_‡K Av‡cj covi NUbv †_‡K wbDU‡bi gva¨vKIY Z‡Ëi Rb¥ Ges †PŠev"Pv †_‡K cwb Qj‡K covi AwfÁZv †_‡K AwKwgW‡mi †mvbvq Lv` cwigv‡ci Dcvq wbavivY e¨w³MZ AwfÁZv †_‡K AbyK‡íi Drcw Ëi wKQz ¸iæZc¥ D`vniY wnmv‡e D‡jL Kiv †h‡Z cv‡i|

AbyK‡íi cKvi‡f` (Types of Hypothesis)

AbyKí‡K wewfbfv‡e †kYx wef³ Kiv hvq| Z‡e cwÛZMY wegvZZvi gvÎvi (level of abstraction) wfwË‡Z AbyK‡íi cavbZt wZb fv‡M fvM K‡i‡Qb| †m¸‡jv n‡jv:

1. AwfÁZvjä m½wZmg‡ni eYbvgj¨K AbyKí (Hypothesis Concerned with the Description of Empirical Uniformities)
2. RwUj Av`k i¡‡ci mv‡_ m¤úwKZ AbyKí (Hypothesis Concerned with Complex Ideal Types)
3. we‡kl‡Yi wfbœvi mv‡_ m¤úwKZ AbyKí (Hypothesis Concerned with the Relations of Analytic Variations)

AwfÁZvjä m½wZmg‚‡ni eYbvg~jK Abygvb (Hypothesis Concerned with the Description of Empirical Uniformities)

> GK ai‡Yi Abygvb i‡q‡Q hv †Kej AwfÁZvjä m½wZ¸‡jvi Aw¯Z‡K eYbv K‡i| G ai‡Yi Abygvb¸‡jv AwaKvsk †¶‡Î mvaviY Ávb‡bi c¯vebv¸‡jvi ˆeÁvwbK cix¶v‡K cwZwbwaZ K‡i|

GK ai‡Yi Abygvb i‡q‡Q hv †Kej AwfÁZvjä m½wZ¸‡jvi Aw¯Z‡K eYbv K‡i| G ai‡Yi Abygvb¸‡jv AwaKvsk †¶‡Î mvaviY Ávb‡bi c¯vebv¸‡jvi ˆeÁvwbK cix¶v‡K cwZwbwaZ K‡i| mvgwRK weÁv‡b G iKg eû Abygvb wb‡q M‡elYv cwiPwjZ n‡q _v‡K| †hgb, GKwU Rb‡Mvôxi Av_-mvgwRK cUf‚wg, GKwU kn‡ii ewmYwR¨K cwZôvbmg~n, GKwU †`‡ki agxq Rb‡Mvôxi web¨vm, BZ¨vw`| G ai‡Yi Abygvb¸‡jv A‡bK mgq GKwU we‡kl Rb‡Mvôxi AvPi‡Yi i¦c‡K eYbv K‡i| †hgb, c_g e‡li Qv‡Î-Qv‡Îx‡`i †kYx K‡¶i AvPiY, Mv‡g›Um kwgK‡`i e¨‡qi cKwZ, BZ¨vw`|

GB me Abygv‡bi wei¦‡× Av‡cwË †Zvjv †h‡Z cv‡i †h, G ai‡Yi Abyg×vb KL‡bv Abygvb cix¶v K‡i bv| G¸‡jv wKQyZ_¨i ms‡hvRb Qvov Avi wKQyb q| Av‡iv Av‡cwË †Zvjv †h‡Z cv‡i †h, G¸‡jv †Kvb Dc‡hvMx Abygvb bq| KviY, G¸‡jv †m mKj wel q‡K cwZwbwaZ K‡i hv mevi Rvbv| GB Av‡cwË¸‡jvi Rev‡e ejv hvq †h, Z‡_¨i mvaviY ms‡hvRb n‡jI GB Abygvb¸‡jv gvby‡li mvaviY Ávb‡K M‡elYvi ga¨ w`‡q GKwU ˆeÁvwbK wfwË c`v‡bi gva¨‡g weÁv‡bi Kv‡Vv‡gvi g‡a¨ mgwšZ K‡i|

mevB hv Rv‡bb Zv mZ¨ bvI n‡Z cv‡i| †h †Kvb weÁv‡bi KvR n‡”Q mvaviY Ávb¸‡jv‡K cix¶v wbix¶v K‡i m~²fv‡e msÁwqZ cZ¨‡qi Aax‡b myse× Kiv| †hgb, mgv‡R gvbyl A‡bK †jvKR a¨vb aviYv wb‡q cwZw`‡bi KgKvÛ cwiPvjbv K‡i| mvgwRK weÁvbxi KvR n‡jv GB †jvKR a¨vb aviYv¸‡jvi †Kvb AwfÁZvjä m½wZ i‡q‡Q wKbv Zv M‡elYvi gva¨‡g chw‡jvPbv K‡i †m¸‡jv‡K Dc‡hvMx Áv‡b cwiYZ Kiv| GwU Ki‡Z wM‡q M‡elK‡K wZbwU KvR Ki‡Z nq| c_gZt, a¨vb-aviYv¸‡jv †_‡K gy~¨‡eva m¤úwKZ c¶cvZ‡K AcmviY Ki‡Z nq| wØZxqZt, a¨vb-aviYvi Aš‘f³ c`¸‡jv‡K †kYxe× Ki‡Z nq| ZZxqZt, h_v_Zvi cix¶v c‡qvM Ki‡Z nq| GKwU D`vniY †`qv hvK, gvbyl i g‡a¨ GKwU mvaviY aviYv i‡q‡Q †h, Ôm¤ú`kvjx gvbyli mshgnxb Rxeb hvcb K‡ib e‡j Zv‡`i g‡a¨ D”P nv‡i weevn we‡”Q` N‡U|Õ GB e³e¨wU †hfv‡e i‡q‡Q ûeû †mfv‡e cix¶v Kiv hv‡e bv| KviY, GB e³e¨ hZUv bv Z‡_¨i Dci cwZwôZ Zvi †P‡q †ekx cwZwôZ fve-ceYZvh³ Abygv‡Zi Dci| GB e³‡e¨i g‡a¨ e¨eüZ c`¸‡jv my¯úófv‡e msÁwqZ bq| GB e³e¨wU‡K GKwU Dc‡hvMx Abygv‡b cwiYZ Kiv hv‡e ZLbB hLb Gi ga¨ †_‡K ˆbwZKZvi welqwU AcmviY Kiv n‡e Ges c`¸‡jv‡K myb¨w`ófv‡e msÁwqZ Kiv n‡e|

ïaymvgwRK weÁv‡b bq, Ab¨vb¨ mKj weÁv‡bB G ai‡Yi mvaviY Ávb A‡bK ¸iæZc~Y M‡elYvi Rb¨ w`‡q‡Q| †hgb, GK mgq Avgv‡`i aviYv wQj †h, mh© c„w_exi Pviw`‡K N‡i| wKš ˆeÁvwb‡Kiv M‡elYvi gva¨‡g cgvY K‡i‡Qb †h, mh© c„w_exi Pviw`‡K N‡i bv, eis c„w_ex m‡h©i Pviw`‡K N‡i| Avgv‡`i GI aviYv wQj †h, fe‚ô mgZj| wKš ˆeÁvwb‡Kiv cgvY K‡i‡Qb †h, fe‚ô mgZj bq, eis Zv †Mvjvkvi| hvB †nvK, mevB hv Rv‡bb Zv cKZ.Rvbv nq bv hZ¶Y bv Zv cix¶xZ n‡”Q| AZGe, †`Lv hv‡”Q †h, AwfÁZvjä m½wZi AbymÜv‡bi gva¨‡g mö cv_wgK Abygvb¸‡jv weÁv‡bi weKv‡k GKwU ¸iæZc~Y fwgKv cvjb K‡i|

RwUj Av`k i¦‡ci mv‡_ m¤úwKZ Abygvb (Hypothesis Concerned with Complex Ideal Types)

> RwUj Av`k i¦‡ci mv‡_ m¤úwKZ Abygvb¸‡jvi j¶¨ n‡jv AwfÁZvjä m½wZ¸‡jvi g‡a¨ †hŠw³Kfv‡e AvšwiZ m¤úK¸‡jvi Aw¯Z‡K cix¶v Kiv|

GB Abygvb¸‡jvi j¶¨ n‡jv AwfÁZvjä m½wZ¸‡jvi g‡a¨ †hŠw³Kfv‡e AvšwiZ m¤úK¸‡jvi Aw¯Z‡K cix¶v Kiv| †hgb, AwfÁZvjä m½wZi mv‡_ m¤úwKZ Abygv‡bi †¶‡Î †`Lv hvq †h, fwg-m¤úK, ewmYwR¨K cwZôv‡bi aiY, ¶z RwZmËv, †ckvMZ web¨vm Ges G iKg Av‡iv

Ab¨vb¨ cc‡Âi g‡a¨ m‡›`nvZxZ fv‡e wKQyAwfÁZvjä m½wZ i‡q‡Q| GB m½wZ¸‡jvi cieZx †hŠw³K we‡kl bZzb wKQyAbyK‡íi Rb¨ †`q| ‡mB AbyKí¸‡jv‡K Av`k iƒc AbyKí e‡j|

GB wegZ© AbyKíMyjv mgv‡R RwUj ¯^viK (complex referent) ˆZix ga¨ w`‡q mij AwfÁZvjä m½wZi cZ¨vkv‡K AwZµg K‡i hvq| †hgb, me kniB wbLyZ GK‡Kw›`K eË©wewkó bq, mgv‡Ri me gvbylB cwšK bq, BZ¨vw` | G ai‡Yi cwiw¯wZ‡Z Av`k iƒc AbyKí wbgvY K‡i cix¶v Kiv LyeB Dc‡hvMx n‡q _v‡K| †h‡nZzG¸‡jv AwfÁZvjä m¤úK‡K AwZµg K‡i, †m‡nZzGB wbgvY¸‡jv‡K Av`k iƒc ejv n‡q _v‡K| RwUj AbygvÜv‡bi †¶‡Î G ai‡Yi AbyK‡íi cavb KvRwU n‡jv fwel¨r M‡elYvi Rb¨ wbZ¨ bZzb †KŠkj ˆZix Kiv|

we‡kl‡Yi wfbZvi mv‡_ m¤úKhy³ AbyKí (Hypothesis Concerned with the Relations of Analytic Variations)

Av`k iƒc AbyK‡íi wbgvY‡K AwZµg K‡i wegZ©Ki‡Yi Av‡iK avc Dc‡i D‡V GB AbyKí¸‡jv‡K wbgvY Kiv nq| †hLv‡b AwfÁZvjä m½wZm¤úb AbyKí¸‡jv ccÂmg‡ni g‡a¨ mvaviY cv_K¨‡K ch‡e¶Y K‡i Ges Av`k iƒc AbyKí¸‡jv we‡kl we‡kl mgRvZxq NUbvi g‡a¨ wfbZv‡K ch‡e¶Y K‡i, †mLv‡b we‡klYgj–K PjK¸‡jvi M‡elYvi Rb¨ GKwU Pj‡Ki GKwU ˆewk‡ó¨i g‡a¨ cwieZ‡bi mv‡_ Ab¨ Pj‡Ki GKwU ˆewk‡ó¨i g‡a¨ cwieZ‡bi m¤úK‡K ch‡e¶‡Yi c‡qvRb c‡o|

†hLv‡b AwfAZvjä m½wZm¤úb AbyKí¸‡jv ccÂmg‡ni g‡a¨ mvaviY cv_K¨‡K ch‡e¶Y K‡i Ges Av`k iƒc AbyKí¸‡jv we‡kl we‡kl mgRvZxq NUbvi g‡a¨ wfbZv‡K ch‡e¶Y K‡i, †mLv‡b we‡klYgj–K PjK¸‡jvi M‡elYvi Rb¨ GKwU Pj‡Ki GKwU ˆewk‡ó¨i g‡a¨ cwieZ‡bi mv‡_ Ab¨ Pj‡Ki GKwU ˆewk‡ó¨i g‡a¨ cwieZ‡bi m¤úK‡K ch‡e¶‡Yi c‡qvRb c‡o|

†hgb, Avgiv Rwb †h, gvbylI Rb¨kxjZvi (human fertility) mv‡_ m¤ú`, ag, wk¶v, evm¯vb Ges m¤ú`v‡qi AvKv‡ii AwfÁZvjä m½wZ i‡q‡Q| evsjv‡`‡ki `ªyU AÂ‡ji D”P Rb¨kxjZvi cwi‡cw¶‡Z GwU we‡ePbv Kiv †h‡Z cv‡i| GKwU n‡jv PUMvg wefvM Ges Ab¨wU n‡jv wm‡jU wefvM| GB `ªyU wefv‡Mi wewfb ˆewkó¨ †hgb, wk¶vi nvi, A_©‰bwZK Ae¯v, agxq cfve, i¶YkxjZvi gvÎv, BZ¨vw` we‡ePbv K‡i Rb¨kxjZvi Dci G‡`i cfve‡K we‡klY Kiv †h‡Z cv‡i| G ai‡Yi AbyK‡íi wbgvY I cix¶v Rb¨kxjZv Ges DwjwLZ wefvM `ªyUi wewfb ˆewk‡ó¨i we‡klY Avgv‡`i Rb¨kxjZvi MwZkxjZv m¤ú‡K Av‡iv DbZZi aviYv I cwigvc †`‡e|

GB AbyKí¸‡jv ïay‡h Ab¨ AbyKí¸‡jvi †P‡q AwaK wegZ© ZvB bq, G¸‡jv n‡jv me‡P‡q cwikwjZ wbgvY| AbyK‡íi wegZ©Ki‡Yi GB gvÎvq †h PjK¸‡jv wbevPb Kiv nq Ges we‡klY Kiv nq †m¸‡jv ïaygvÎ GKwU ZË¡ Øviv wbawiZ nq| †h‡nZzZË¡ wb‡RB GKwU weKv‡ki chv‡q _v‡K, †m‡nZzZ‡Ëi weKv‡k bZzb bZzb we‡klYgj–K M‡elYvi †¶‡Î ˆZix n‡Z _v‡K Ges weÁvb‡K mg„× K‡i|

AbyKí wbgv‡Y mgm¨v (Problems in Formulating Hypothesis)

mgvRZË¡Ges mvgvwRK M‡elYv cwiPvjbvi Rb¨ AbyK‡íi c‡qvRbxqZvi welqwU Abyaveb Kiv we‡kl c‡qvRb| KviY, AbyKí QvovI M‡elYv‡K j¶¨nxb jvMvgQvov G‡jv‡g‡jv AwfÁZvgj–K wePiY ejv hvq| AbyKí wbgv‡Y †hgb mZKZv Aej¤b Kiv c‡qvRb †Zgwb Dc‡hvMx AbyKí wbgv‡Y Amyeav¸‡jv‡KI g‡b ivLv c‡qvRb| AbyKí wbgv‡Y M‡elKMY cavbZt wZbwU Amyeavi m¤§yLxb nb| c_gZt, GKwU my¯úó ZvwËK KvVv‡gvi Afve ev ZvwËK KvVv‡gvi Ávv‡bi Afve| Avgiv Av‡MB e‡jwQ †h, AbyKí ZvwËK KvVv‡gvi mv‡_ we‡klfv‡e m¤úKhy³| GKw`‡K, Avgiv Avgv‡`i AwfÁZvjä m½wZ¸‡jv‡K cix¶v wbix¶vi gva¨‡g fwel¨r Z‡Ëi Ask wnmv‡e M‡o Zzwj| Ab¨w`‡K, we`¨gvb Ávb KvVv‡gvi ga¨ †_‡K wewfb c¯vebv‡K AbyKí Abygvb K‡i cix¶v wbix¶vi gva¨‡g †m¸‡jvi h_v_Zv cgvY Kwi| Kv‡RB, hw` ‡Kvb ZvwËK KvVv‡gvi Abycw¯wZ _v‡K ev we`¨gvb ZvwËK KvVv‡gv m¤ú‡K Ávv‡bi Afve _v‡K, †m †¶‡Î h_vh_

AbyKí wbgv‡Y M‡elKMY cavbZt wZbwU Amyeavi m¤§yLxb nb| c_gZt, GKwU my¯úó ZvwËK KvVv‡gvi Afve ev ZvwËK KvVv‡gvi Ávv‡bi Afve| wØZxqZt, ZvwËK KvVv‡gv‡K e¨envi‡ii `¶Zvi Afve| ZZxqZt, ZvwËK KvVv‡gv e¨envi‡ii Rb¨ we`¨gvb †KŠkj¸‡jv m¤ú‡K AewnZ bv _vKv|

AbjKí wbgvY KwVb n‡q c‡o| wØZxqZt, ZwiËK KvVv‡gvi Dcw¯wZ ev ZwiËK KvVv‡gv m¤ú‡K Ávb _vK‡jB ïaywPj‡e bv, GKRb M‡elYv †mB ZwiËK KvVv‡gv‡K e¨envi‡Yi `¶Zv _vK‡Z n‡e| GB `¶Zvi Afve A‡bK mgq Dc‡hvMx AbjKí wbgv‡Y mgm¨v ˆZix K‡i| ZZxqZt, aiv hvK, GKRb M‡elK Zviv M‡elYv m¤úwKZ ZwiËK KvVv‡gv m¤ú‡K cÖvavcwi lqwKenvj Ges †mB ZwiËK KvVv‡gv e¨envi‡Yi `¶ZvI Zviv i‡q‡Q| wKš‘wZb hw` †mB ZwiËK KvVv‡gv e¨envi‡Yi Rb¨ we`¨gvb †KŠkj ¸‡jv m¤ú‡K AewnZ bv _v‡Kb Z‡e Zviv c‡¶ Dc‡hvMx AbjKí wbgvY Kiv KwVb n‡q c‡o|

hw` QvÎ-QvÎx‡`i ck Kiv nq †h, GKwU ¸iæZc¥ I cix¶v‡hvM¨ ck ˆZix Kiv KZUKzKwVb? AwaKvsk QvÎ-QvÎxB GB c‡ki mwVK Reve w`‡Z e¨_ n‡e| hw` I ev †KD †KD wKQyc¯vebvi D‡jL Ki‡Z cvi‡e, wKš cKZ.AbymÜv‡b †`Lv hv‡e †h, †m me c¯vebvi AwaKvskB AbjKí bq| hviv AbjKí wbgv‡Yi KvQvKwQ †h‡Z cvi‡e, †`Lv hv‡e †h, wbwgZ AbjKí ¸wj AwfÁZvjä m½wZi D‡a DV‡Z cv‡iwb| hw` Avgiv GKwU myjbw`ó ZwiËK KvVv‡gv Ges h_vh_ M‡elYv †KŠkj wbaviY K‡i ïiæ Kwi, Zvn‡j Avgiv `ÿvU ev Zvi AwaK Pj‡Ki g‡a¨ m¤ú‡Ki m½wZ ch‡e¶Y K‡i GKwU Dc‡hvMx AbjKí wbevPb Ki‡Z cvi‡ev|

AbjKí cix¶v wK (What is Hypothesis Testing)?

AbjKí cix¶v Kvh-KviY m¤úK© welqK c‡ki mvgvavb †c‡Z mvnvh¨ K‡i| AbjKí cix¶v M‡elK‡K cwimsL¨vbMZ wm×vš Mn‡Yi Dcvq e‡j †`q|

bgybv ch‡e¶‡Yi Dci wfwË K‡i mgMK m¤ú‡K Dcmsnvi Uvbvi Rb¨ †h me wm×všg¡‡K c×wZi i‡q‡Q †m me c×wZi †¶‡Î c‡hvR¨ GKwU c` n‡jv ÔAbjKí cix¶vÕ| Av‡iv mnR K‡i ejv hvq †h, AbjKí cix¶v Kvh-KviY m¤úK© welqK c‡ki mvgvavb †c‡Z mvnvh¨ K‡i| †hgb, aygcvb wK K¨vÝvi Kviv? AbjKí cix¶v cv_K¨ wbavi‡YI mvnvh¨ K‡i| †hgb, wewfb ˆewk‡ó¨i wfwË‡Z wK ejv hvq †h, wewfb †Mvôxi g‡a¨ we`¨gvb cv_K¨ mwZ¨Kvi? mbvZb c×wZi cwie‡Z Avayb wK D™vebxg¡‡K c×wZ‡Z wk¶v`vb wK mwZ¨B QvÎ-QvÎx‡`i djvd‡ji Dci cfve †d‡j? cwimsL¨vbMZ AbjKí cix¶vi gva¨‡g G mKj Kvh-KviY m¤ú‡Ki h_v_Zv cgvY Kiv †h‡Z cv‡i| wewfb ai‡Yi AbjKí cix¶v c×wZ mgM‡Ki ev¯eZv m¤ú‡K M‡elK‡K GKwU cwimsL¨vbMZ wm×v‡š †cŠQ‡Z mvnvh¨ Kivi D‡Ï‡k¨B wbgvY Kiv n‡q‡Q| GB me c×wZ wbi¼kxfv‡e wKQyB cgvYZ K‡i bv, Z‡e hLb m¤vebvi AvKv‡i Dc¯wcZ nq ZLb G¸‡jv M‡elK‡K fvel¨ØvYx Ki‡Z mvnvh¨ K‡i| Ab¨ K_vq, AbjKí cix¶v M‡elK‡K cwimsL¨vbMZ wm×vš Mn‡Yi Dcvq e‡j †`q|

AbjKí cix¶vi cKvi‡f` (Types of Hypoethesis Testing)

Z_¨ I ZË †_‡K Avnwiz †hŠw³K Abyqvb¸‡jv‡K AwfÁZvg¡‡K cix¶v K‡i †`Lv c‡qvRb| †h ‡Kvb weÁv‡b AbjKí cix¶v Kiv nq wVK D‡ëv w`K †_‡K| A_vr, `ÿ ev Z‡ZwaK Pj‡Ki g‡a¨ Ôt‡Kvb m¤úK© †bBÕ ev Ôt‡Kvb cv_K¨ †bBÕ GB Abyqvb †_‡KB weÁvbm¤úZ AbjKí cix¶v ïiæ nq| †Kvb cv_K¨ †bB ev †Kvb m¤úK© †bB Ggb Abyqv‡bi cix¶v‡K Ôbw¯ AbjKíÕ (null hypothesis) e‡j| bw¯ AbjKí cix¶vi hy³wU A‡bKUv Av`vj‡Z Acivax‡`i wePviK¨v‡hi gZ| GKRb e¨w³ hZ¶Y chš m‡›`nvZxZfv‡e Acivax cgvwYZ bv n‡"Qb ZZ¶Y chš †mB e¨w³ Acivax e‡j we‡ewPZ n‡eb bv| Z‡e AvBb c‡qvMKvix ms¯vi m`m¨iv †hgb Zv‡`i Abyqvb‡K cgvY Ki‡Z Pvb †h Awfhy³ e¨w³ mwZ¨Kvi A‡_ Acivax, M‡elKI †Zgb Zviv Ávbw`ß Abyqvb‡K cgvY Ki‡Z Pvb bw¯ AbjKí‡K cZ¨vL¨vb K‡i| M‡el‡Ki GB Abyqvb‡K ÔM‡elYv AbjKíÕ ev ÔwKí AbjKíÕ (research hypothesis or alternative hypothesis) e‡j|

mvs‡KwZK wP‡ýi gva¨‡g GB m¤úK©¸‡jv‡K wbgœwjwLZ fv‡e †`Lv‡bv †h‡Z cv‡i:

bvw¯— Abygvb $H_0 : \mu_0 = k$

weKí Abygvb $H_1 : \mu_0 \neq k$

‡hLv‡b, H_0 = bvw¯— Abygvb

H_1 = weKí Abygvb

μ_0 = †h mgMK †_‡K bgybv msMÖn Kiv n‡q‡Q †mB mgM‡Ki Mo

k = aªæeK

Abygvb cixÿv‡K cÖavbZt `yÕfv‡M fvM Kiv hvq| GKwU cviwgwZK (parametric) Ges Ab¨wU n‡jv AcviwgwZK (non-parametric) Abygvb cixÿv| cviwgwZK cixÿvi gva¨‡g aviYvwU n‡jv †h, bgybvwU †h mgMK †_‡K Pqb Kiv nq †mB mgMKwU ¯^vfvweKfv‡e web¨¯— Ges mgM‡Ki cwiwgZ e¨eavbwU Rvbv _v‡K| Abygvb¸‡jv cÖgv‡bi †ÿ‡Î eûj e¨eüZ _‡K| cviwgwZK cixÿvi mv‡_ msweró Abygvb¸‡jvi Rb¨ †h cwimsL¨vb wbY©q nq †m¸‡jv‡K e¨wßgvb ev AbycvZgvcK gvÎvq cwigvcKZ.n‡Z nq| wKš' mvgwRK ev Av¯—eZvq Avgv‡`i nv‡Z cviwgwZK Abygvb cixÿvi Rb¨ kZ© c~iY K‡i Ggb DcvË memgq _v‡K bv| G iKg cwiw¯'wZ †gvKv‡ejvi Rb¨ cwimsL¨vbwe`MY †ek wKQz Abygvb cixÿv wbgvY K‡i‡Qb †h¸‡jv AcviwgwZK cixÿv e‡j cwiwPZ|

AcviwgwZK kãwUi A_© GwU bq †h, †h mgMK †_‡K bgybvqb Kiv nq †mB mgM‡Ki †Kvb cvigvb †bB| cviwgwZK cixÿvi mv‡_ AcviwgwZK cixÿvi cv_©K¨wU n‡jv †h, cviwgwZK cixÿvq e¨eüZ cwimsL¨vb¸‡jv wbY©‡qi Rb¨ mgMK m¤ú‡K© †Kvb KwVb kZ© Av‡ivc Kiv nq bv| A_©vr, Abygvb¸‡jv cÖgv‡bi †ÿ‡Î wbwgZ nq bv| Ab¨ K_vq, AcviwgwZK Abygvb cixÿv‡K web¨vmgy³ cixÿv e‡j| KviY, G¸‡jv cviwgwZK cixÿvi Zzjbvq Kg kZ©hy³| Aek¨ GB wkw_jZvi Kvi‡Y cviwgwZK cixÿvi Zzjbvq AcviwgwZK cixÿv¸‡jv Kg kw³kvjx nq Ges Gi d‡j A‡bK bvw¯— Abygvb cÖZ¨vL¨vb Kiv hvq bv hv cviwgwZK cixÿvq Kiv †hZ| AcviwgwZK cixÿv¸‡jv bvgmPK I µgmPK gvÎvq cwigvcKZ.Pj‡Ki †ÿ‡Î c‡qvM Kiv hvq|

cviwgwZK cixÿv¸‡jvi g‡a¨ †h¸‡jv me‡P‡q †ekx e¨eüZ nq †m¸‡jv n‡jv z-cixÿv, t-cixÿv Ges F-cixÿv| AcviwgwZK cixÿv¸‡jvi g‡a¨ †hwU me‡P‡q †ekx e¨eüZ nq †mwU n‡jv KvB-eM (Chi-Square) cixÿv, hv χ^2 -cixÿv bv‡g cwiwPZ| GQvovI GB cwie‡i i‡q‡Q Wilcoxon Matched-Pairs, Single Ranked Test Ges Wilcoxon Rank-Sum Test| Abygvb cixÿvi †h mKj c×wZ I †Kškj Aej¤^b Kiv nq Zv cieZx© Aa¨v‡q we¯—vwiZfv‡e Av‡jvPbv Kiv n‡e|

mvivsk

M‡elYv ïiæi cv_wgK c`‡¶c n‡”Q GKwU Abygvb cYqb| Abygvb‡K Aek¨B cZ¨qMZfv‡e m¯úó n‡Z n‡e| GKwU Abygv‡bi Ab¨Zg ˆewkó¨ n‡”Q G‡Z Kgc‡¶ `yÕwU PjK _vK‡Z n‡e hv‡K cixÿv Kiv m¤¢e Ges hv mvaviYZt AwfÁZv †_‡K Rb¥ jvf K‡i| Abygv‡bi Drmmgn n‡”Q AwfÁZv, ˆeÁwbK Ávb Ges Zzjbvg~jK Av‡jvPbv| m¤úó ZwËK KvVv‡gv Qvov Abygvb MVb j¶¨nxb n‡q co‡e|

cv‡VwËi gj¨vqb

^be¨w³K ck

mwVK DË‡i i cv‡k wUK (√) wPý w`b –

1| GKwU fv‡jv Abyjf‡í i Ab¨Zg ^ewkó¨ n‡"Q:

K. cZ¨qMZ fv‡e mv¯úó n‡Z n‡e
L. mybw`ó n‡Z n‡e
M. ZwËK KvVv‡gv i mv‡_ m¤úKhy³ n‡Z n‡e
N. Dc‡i i me

2| Abyjf‡í ‡K cavbZt ..... fv‡M fvM Ki v hvq|

K. GK
L. `ß
M. wZb
N. Pvi

3| ‡Kvb m¤úK ‡bB Ggb Abygv‡bi ci x¶v‡K ..... e‡j|

K. bw¯ Abyjf‡í
L. weKí Abyjf‡í
M. M‡elbv Abyjf‡í
N. ci x¶vgj¨K Abyjf‡í

msw¶ß ck

1. Abyjf‡í wK?
2. Abyjf‡í wbgv‡Yi mgm¨v¸‡jv wK wK?

iPbvgj¨K ck

1| mgvR M‡elYvq Abyjf‡í i c‡qvRbxZv Av‡jvPbv Ki‡b|

2| Abyjf‡í i cKvi‡f`¸‡jv Av‡jvPbv Ki‡b|

cvV - 2

cwimsL¨vbMZ cix¶vi wewfbœ avc
Steps in Hypothesis Testing

GB cvV †k‡l Avcwb Rvb‡Z cvi‡eb —

- cwimsL¨vbMZ cix¶vi wewfbœ avc
- Abygvb wbgvY
- bgybv web¨vm wbYq
- Zvrch gvÎv Ges msKU GjvKv wbYq
- cix¶v cwimsL¨vb wbYq
- wm×vš— MnY

cwimsL¨vbMZ cix¶vi wewfbœ avc (Steps in Statistical Test)

AbyK í cix¶vi Rb¨ cwimsL¨vbwe`‡K avc avc wKQz mywbw`ó c×wZ AbymiY Ki‡Z nq| DcvË we‡kl‡Yi c‡eB GB chvqµwgK c×wZ¸‡jv m¤ú‡K wm×vš— MnY Ki‡Z nq| cwimsL¨vbMZ cix¶vi wewfbœ avc¸‡jv n‡jv:

1. Abygvb wbgvY (Making Assumptions)
2. bgybv web¨vm wbYq (Obtaining the Sampling Distribution)
3. Zvrch gvÎv Ges msKU GjvKv wbevPb (Selecting Significance Level and Critical Region)
4. cix¶v cwimsL¨vb wbYq (Computing the Test Statistics)
5. wm×vš— MnY (Making a Decision)

Abygvb wbgvY (Making Assumptions)

AbyK í cix¶vi Aš—wbwnZ c×wZwU gƒ-Zt m¤¢vebv Z‡Ëi Dci wfwË K‡i `uvwo‡q i‡q‡Q| GKwU h_vh_ m¤¢vebv web¨vm cvevi Rb¨ m¤¢vebv Z‡Ëi e¨envi Ki‡Z n‡j Avgv‡`i Aek¨B mgMK Ges bgybvqb c×wZ m¤ú‡K KZ¸‡jv Abygvb wbgvY Ki‡Z nq| GB Abygvb¸‡jv `yÕai‡Yi n‡q _v‡K| GK ai‡Yi Abygvb i‡q‡Q †h¸‡jv m¤ú‡K Avgiv Av‡cw¶Kfv‡e wbwðZ ev †h¸‡jv‡K Avgiv MnY Ki‡Z PvB| Av‡iK ai‡Yi Abygvb i‡q‡Q †h¸‡jv m¤ú‡K Avgv‡`i m‡›`n i‡q‡Q Ges ‡m¸‡jv wb‡qB Avgv‡`i AvMn †ekx| c_g ai‡Yi Abygvb¸‡jv‡K Avgiv ÔAv`kÕ ev Ôg‡WjÕ ewj Ges wØZxq ai‡Yi Abygvb¸‡jv‡K Avgiv ewj ÔAbyK íÕ hv Avgiv cix¶v Ki‡Z PvB|

GKwU D`vniY †`qv hvK| aiv hvK, GKRb M‡elK wewfbœ mvgvwRK †MvôxI (G †¶‡Î mvgvwRK †kYx) EaŸgyLx MwZkxjZvi AvKv•Lvi Dci AbyK í cix¶v Ki‡Z Pvb| M‡elK `yÕwU Abygvb wbgvY Ki‡jb ‡h,

1. †h mgMK †_‡K bgybv Pqb Kiv n‡q‡Q †mB mgM‡Ki g‡a¨ EaŸgyLx mPjZvi AvKv•Lv m¤¢wjZ ga¨weË Ges wbgweË Rb‡Mvôxi cwigvY mgvb; Ges
2. bgybvq Aš—fy³ mKj Dcv`vb‡K ˆØePwqZfv‡e wbevPb Kiv n‡q‡Q|

DcvË msM‡ni ci †`Lv †Mj †h, wbgœe‡Ëi Zzjbvq ga¨we‡Ëi g‡a¨ EaŸgyLx mPjZvi AvKv•Lv m¤wjZ gvbyli msL¨v A‡bK †ekx| GB chv‡q M‡elK wbwðZfv‡e a‡i wb‡Z cv‡ib †h, DcwiwjwLZ `yÕU Abyœv‡bi g‡a¨ †h †Kvb GKwU fÿ| GLb ck n‡jv †h, †Kvb AbyœvbwU‡K M‡elK cZ¨vL¨vb Ki‡eb? wZwb c_g AbyœvbwU‡K cZ¨vL¨vb Ki‡Z cv‡ib| KviY, c_g Abyœv‡b `yÕU ‡kYxi g‡a¨ mgvb cwigv‡Y AvKv•Lvi Dcw¯'wZ‡K Abyœvb Kiv n‡q‡Q| wKš‘ GB wm×vš— Mn‡Yi c‡¶ M‡elK‡K wbwðZ n‡q wb‡Z n‡e †h, wZwb †Kvb c¶cvZgy³K bgybvqb c×wZ e¨envi K‡ib wb| hw` wZwb bgybv wbevPb ^ØePqbK cÖwµqv wbwðZ K‡i _v‡Kb Z‡e wZwb wØZxq AbyœvbwU‡K Av`k© wnmv‡e MnY K‡i Dcmsnvi Uvb‡Z cv‡ib †h, c_g AbyœvbwU fÿ| wØZxq AbyœvbwU‡K Av`k© wnmv‡e MnY Kivi †cQ‡b gy³ wfwËwU n‡jv e¨eüZ bgybvqb c×wZ m¤ú‡K h_vh_ Áv‡bi e¨envi|

G Qvov I M‡elK GKwU wbw`ó Abyœvb‡K MnY K‡ib c‡¶ cwiPwjZ M‡elYvi djvdj‡ji wfwË‡Z| GLv‡b †h welqwU ¸iæ‡Zi mv‡_ g‡b ivLv c‡qvRb ‡mwU n‡jv M‡elK KZK.cwiPwjZ cix¶v †Kvbfv‡eB åvš— Abyœvb‡K wPwýZ Ki‡Z cv‡i bv| †h‡nZzAv`kwU M‡elYv c×wZi wbwðZ c‡qv‡Mi wfwË‡Z MnY Kiv n‡q _v‡K Ges †h‡nZzAbyKíwU‡KB cix¶v Kiv nq, †m‡nZz MnY ev cZ¨vLv‡bi welqwU mvaviYZt AbyK‡íi Abyœv‡bi DciB †K›`ªxfZ— _v‡K| Z‡e †¶‡Î we‡kl, Av`k‡KI cZ¨L¨vb Kiv †h‡Z cv‡i, hw` Zv åvš— aviYv I c×wZi Dci cwZwôZ nq|

AbyKí cix¶vi †¶‡Î Avgv‡`i cvqkt GKwU c‡ki m¤§yLxb n‡Z nq †h, ‡Kvb gvb`‡Ûi wfwË‡Z GKwU we‡kl cwimsL¨vbMZ cix¶v‡K wbevPb Ki‡ev? cix¶vi Rb¨ MÖnxZ AbyqgZ Av`kwU‡K GKwU gvb`Û wnmv‡e we‡ePbv Kiv †h‡Z cv‡i| wKš‘ G Qvov I Av‡iv wKQzgvb`Û we‡ePbvq ivL‡Z nq| ‡hgb, DcvË‡Ëi aiY I cKwZ m¤ú‡K AbyKí cix¶vi ïiæ‡ZB Abyœvb wbgvY Ki‡Z nq| KviY, DcvË‡Ëi aiY †f‡` cKí cix¶vi c×wZ e`‡j hvq| hw` M‡elYvq e¨eüZ PjKmgn bvgmPK ev µgmPK gvÎvq cwigvcKZ.nq, †mLv‡b †h cwimsL¨vbMZ c×wZ Avgiv c‡qvM Ki‡ev Zv e¨wߧgy³K gvÎvq cwigvcKZ.Pj‡Ki †¶‡Î c‡hvR¨ n‡e bv| `yÕU bgybv M‡oi g‡a¨ cv_©K¨i Zvrch cix¶v Ki‡Z Avgiv †h cwimsL¨vbK m~Î I AbyKí c‡qvM Kwi Zv GKwU bgybv M‡oi mv‡_ mgM‡Ki AbyqgZ M‡oi cv_©K¨i Zvrch cix¶vq c‡hvR¨ m~Î I AbyK‡íi Abyiƒc bq|

AbyKí cix¶vi †¶‡Î mgM‡Ki cwiwgZ e¨eavb Rvbv Ges bv Rvbvi welqwU ¸iæZc~Y©|

AbyKí cix¶vi †¶‡Î mgM‡Ki cwiwgZ e¨eavb Rvbv Ges bv Rvbvi welqwU ¸iæZc~Y©| mgM‡Ki cwiwgZ e¨eavb hw` Avgv‡`i Rvbv _v‡K Z‡e Avgiv †h c×wZ c‡qvM Kwi, Zv mgM‡Ki cwiwgZ e¨eavb ARvbv _vK‡j c‡hvR¨ n‡e bv| mgM‡Ki cwiwgZ e¨eavb Rvbv _vK‡j Avgiv mivmwi z-cix¶v e¨envi Kwi| †m ‡¶‡Î bgybvi AvKvi‡K Avgiv we‡ePbvq Awb bv| Z‡e mgM‡Ki cwiwgZ e¨eavb Avgv‡`i ARvbv _vK‡j cix¶v c×wZ c‡qv‡Mi ‡¶‡Î bgybvi AvKvi GKwU ¸iæZc~Y© we‡eP¨ welq n‡q `vovq| †hgb, mgM‡Ki cwiwgZ e¨eavb Rvbv bv _vK‡j Ges bgybvi AvKvi †QvU n‡j Avgiv t-cix¶v Ges bgybvi AvKvi eo n‡j z-cix¶v c‡qvM K‡i _wK| cwimsL¨vbMZ cix¶vi †¶‡Î h_vh_ c×wZ cQ›`'i Rb¨ bgybvi AvKv‡ii ¸iæZc~Y© fwgKv i‡q‡Q| myZivs, †Kvb cwimsL¨vb c‡qvM Ki‡ev Zv wbwðZ nevi Rb¨ Abyœvb wbgv‡Yi chv‡q Avgv‡`i bgybvi AvKvi m¤ú‡K ¯úó aviYv _vK‡Z n‡e| mgM‡Ki web¨vmwU Av‡`Š ¯^vfweKfv‡e web¨¯— wKbv Zv we‡ePbvq Avb‡Z n‡e| mvaviYZt bgybvi AvKvi hw` eo nq, Z‡e GwU a‡i †bqv nq †h, mgMKwU ¯^vfweKfv‡e web¨¯— n‡e| myZivs, Avgv‡`i wbwðZ n‡Z n‡e †h, M‡elYvq Aš—f©³ bgybvi AvKviwU h‡_ó eo wKbv A_ev mgMKwU ¯^vfweKfv‡e web¨¯— wKbv|

bgybv web¨vm wbYq (Obtaining the Sampling Distribution)

AbykÍ cix¶vi c_g av‡c c‡qvRbxq Abygvb wbgv‡Yi ci Avgv‡`i MwYwZK hy³ e¨envi‡i gva¨‡g bgybv web¨vm wbYq Ki‡Z n‡e| GB bgybv web¨vm¸‡jv mybw`ó m¤vebv web¨v‡mi Dci wfwË K‡i iwPZ| †hgb, wØc`x web¨vm, ¯'vfweK web¨vm, KvB-eM web¨vm, BZ¨vw`| GwU g‡b ivLv c‡qvRb †h, wfb civgvb m‡Ë I GKB m¤vebv web¨v‡mi wewfb bgybv gvb wbYq Kiv †h‡Z cv‡i| hLb Avgiv ewj †h, c`Ë GKwU bgybv gv‡bi GKwU bgybv web¨vm i‡q‡Q hv ¯'vfweK, ZLb Avgiv †evSv‡Z PvB †h, Gi GKwU mybw`ó i‡ci m¤vebv web¨vm i‡q‡Q| A_vr, hw` bw¯ AbykÍwU Ges Ab¨vb¨ Abygvb¸‡jv mZ¨ nq, Z‡e †kl chš bgybv gvbwU GKwU wbw`ó cwigvY mg‡q GKwU wbw`ó gvb ev GKwU wbw`ó cwim‡i i gvb MnY Ki‡e|

GKwU D`vniY w`‡q welqwU cwi¯'vi Kiv hvK| aiv hvK, GKwU wØc`x cix¶vq Abygvb Kiv n‡jv †h, GKwU gyv 10 evi Qwo gvivi cix¶vwU m¤ve¨ mKj Qwo gviv‡i NUbv¸‡jv ˆØe bgybvqb Ges GKwU Qwo gviv‡i NUbv Ab¨wU †_‡K ¯'vaxb| GI Abygvb Kiv n‡jv †h, gywU c¶vcZ`ý bq| wØZxq Abygvb‡U n‡jv Avgv‡`i AbykÍ Ges c_gwU Avgv‡`i Av`k| hw` Abygvb `ywU mZ¨ nq Z‡e Avgiv ‡`L‡ev †h, gv_v I †jR cvevi m¤vebv mgvb n‡e| A_vr, gv_v = †jR = 0.5 n‡e| Ab¨ K_vq, p = q = 0.5| GKwU mr gyv‡K 10evi Qwo gvi‡j gv_v I †jR cvevi 1024 ai‡Yi m¤vebvi wgkY (combination) ˆZix n‡e Ges 1024 ev‡i g‡a¨ gvÎ 1 evi me gv_v Ges 1024 ev‡i g‡a¨ 10 evi 9wU gv_v cvevi m¤vebv cZ¨vkv Kiv †h‡Z cv‡i| hw` Abygvb¸‡jv mZ¨ nq Z‡e †Kvb NUbvi ˆØevr NUevi m¤vebvi Ávb ‡hŠw³K wm×vš Mn‡Yi †¶‡Î e¨envi Kiv †h‡Z cv‡i| aiv hvK, Avgiv 10 eviB me gv_v †c‡Z PvB| G †¶‡Î `ywU m¤vebv i‡q‡Q:

K. nq Abygvb¸‡jv mZ¨ Ges GwU †Zgb NUbv¸‡jvi GKwU †hwU GKwU AvKw¯'iK NUbv wnmv‡e N‡U‡Q; A_ev

L. Kgc‡¶ GKwU Abygvb wg_¨v (G †¶‡Î bw¯ AbykÍwU)|

GB `ywU m¤vebvi †KvbwU mZ¨ Avgiv Rwb bv| Rvbv _vK‡j Avgv‡`i cix¶wU cwiPvjbvi †Kvb c‡qvRb n‡Zv bv| GB mgm¨v mvgvav‡bi Rb¨ Avgiv GKwU wbqg cwZwôZ Ki‡Z cwi| ‡hgb, GKwU gyv‡K 10 evi Qwa‡j hw` 10 eviB gv_v Av‡m Z‡e Avgiv Dcmsnvi Uvb‡Z cwi †h, AšZt GKwU Abygvb wg_¨v Ges †mwU‡K cZ¨L¨vb Ki‡Z n‡e| †h‡nZzAvgiv Rwb †h, GKwU mr gyv w`‡q ˆØeµ‡g Avgiv 1024 ev‡i g‡a¨ 10 eviB me gv_v cvevi cZ¨vkv Ki‡Z cwi, †m‡nZzGB wbqg Abymi‡Yi d‡j †kl chš KL‡bv KL‡bv Avgiv åvš Dcmsnvi Uvb‡Z cwi| AZGe, ‡Kvb we‡kl cix¶vi Rb¨ Avgv‡`i wm×v‡ši ï×Zv wbavi‡Y GB wbqg Avgv‡`i mvnvh¨ K‡i bv| Z‡e m¤vebvi wbqg¸‡jvi gva¨‡g †kl chš wK cwigv‡Y KZevi Avgiv mwVK wm×vš cZ¨vkv Ki‡Z cwi Avgv‡`i Zv mwVKfv‡e wb‡`k K‡i| A_vr, m¤vebv c×wZi DciB Avgv‡`i wbfi Ki‡Z nq, †Kvb we‡kl NUbv NUevi Dci Mp¸‡jv wm×v‡ši Dci bq|

Zvrch gvÎv I msKU GjvKv wbevPb (Selecting Significance Level and Critical Region)

Av`kMZfv‡e, c‡e Mp¸‡jv Abygvb¸‡jv mwVK wKbv †m m¤ú‡K wm×vš MnY Ki‡Z nq cix¶v cwiPvjbv ev DcvË we‡kl‡Yi Av‡MB| ZvB bgybv web¨v‡mi Áv‡bi Dci wfwË K‡i Avgiv m¤ve¨ djvd‡ji KZ¸‡jv weKí wbevPb Kwi hvi gva¨‡g c‡qvRb‡ev‡a Abygvb¸‡jv‡K cZ¨vL¨vb Ki‡Z cwi| GB m¤ve¨ djvdj¸‡jv `ywa‡Yi n‡q _v‡K| GK ai‡Yi djvdj hvi wfwË‡Z Avgiv Avgv‡`i Abygvb‡K cZ¨vL¨vb Ki‡ev; Ges Av‡iK ai‡Yi djvdj hv Avgv‡`i

Abygvb‡K cZ¨vL¨vb Ki‡Z †`‡e bv| GB djvdj¸‡jv MnY ev cZ¨vL¨v‡bi Rb¨ Avgv‡`i `ÿZv wel‡q wm×vš— MnY Ki‡Z nq| GKwU n‡jv Zvrch gvÎv wbevPb Ges Av‡iKwU n‡jv msKU GjvKv wbevPb|

msKU GjvKv wbevP‡bi Rb¨ Avgv‡`i Av`k I Abyk×í m¤ú‡K Abygvb wbgv‡Yi wm×v‡šÍi AwZwi³ Avi I `ÿZv wm×vš— MnY Ki‡Z nq| c_gZt, Avgv‡`i wbaviY Ki‡Z n‡e †h, Avgiv fyj‡v‡e GKwU mZ¨ bw¯— Abyk×í‡K cZ¨vLvb Kivi ev GKwU wg_¨v bw¯— Abyk×í‡K cZ¨vL¨vb Ki‡Z e¨_ nevi Sz‡K KZUkzt be? wØZxqZt, Avgv‡`i wm×vš— wb‡Z n‡e †h, Avgv‡`i msKU GjvKv bgybv webv‡mi `ßi cvš‡K (two tails) Aš—f³ Ki‡e, bwK GK cv‡š— (one tail) mxgve× _vK‡e?

GKwU bw¯— Abyk×í‡K cZ¨vL¨vb Kivi c‡¶ hZ kw³kvjx cgvYB _vKz‡k bv †Kb bw¯— Abyk×í‡K cZ¨vL¨vb Kivi wm×vš—wU fyj n‡Z cv‡i| GKRb wb‡`vl e¨w³ †hgb A‡bK mgq kw¯— †c‡q hvq, †Zgwb mZ¨ bw¯— Abyk×í I A‡bK mgq cZ¨vL¨vZ nq| GKwU bw¯— Abyk×í‡K fyj‡v‡e cZ¨vL¨vb Kivi wm×vš—‡K UvBc-1 åvwš— ev Avjdv (α) åvwš— e‡j| UvBc-1 åvwš— Kievi m¤vebwU n‡jv GKwU cix¶vi Zvrch gvÎv Ges bgybv webv¨vmwU hw` Awe”Qb nq Z‡e Zv †h ‡Kvb Kv•LZ gvÎvq wbaviY Kiv hvq| cKZc‡¶, Avgiv KL‡bv Rvb‡Z cvwi bv †h, Avgiv UvBc-1 åvwš— K‡ivwQ| Z‡e cgv‡Yi kZ wnmv‡e Avgiv K‡Vvi wbqg Av‡ivc K‡i G ai‡Yi åvwš— Gov‡bvi Rb¨ mZKZv Aej¤b Ki‡Z cvwi| Avjdv gvÎv‡K Av‡iv wb‡g wbaviY K‡i Zv Kiv m¤e| Gi d‡j msKU GjvKv‡K Mo †_‡K Av‡iv `ªæi mvi‡q †`qv nq| †hgb, Avjdv gvÎv 0.05-G wbaviY Ki‡j Avgiv hZ mn‡R GKwU bw¯— Abyk×í‡K cZ¨vL¨vb Ki‡Z cvi‡ev Avjdv gvÎv 0.01-G wbaviY Ki‡j ZZ mn‡R Zv cvi‡ev bv|

wKš— mgm¨v n‡jv †h, hLb Avjdv gvÎv‡K wbg gvÎvq wbaviY Kiv nq ZLb msKUgy³ GjvKv †e‡o hvq| Gi A_ n‡jv †h, hLb Avgiv GKwU mZ¨ bw¯— Abyk×í‡K cZ¨vL¨vb Ki‡Z e¨_ nB ZLb Avgiv UvBc-2 åvwš— ev weUv (β) åvwš— Kwi| GwU GKRb †`vlx e¨w³‡K wb‡`vl e‡j gyy³ †`evi gZ| hw` I ejv n‡q _v‡K †h, †h‡nZz`ßai‡Yi åvwš— wecixZfv‡e m¤úwKZ, †m‡nZz GKB bgybvi g‡a¨ `ßai‡Yi åvwši Sz‡K hyMcrfv‡e Kgv‡bv m¤e bq| wKš— cKZc‡¶, GKB cix¶vq `ßai‡Yi åvwš— GK mv‡_ Kiv hvq bv| KviY, hw` bw¯— Abyk×íwU mZ¨ nq Z‡e Avgiv GKgvÎ ‡h åvwš— Ki‡Z cvwi, Zv n‡jv bw¯— Abyk×íwU‡K cZ¨vLvb Ki‡Z cvwi| wecixZfv‡e, hw` bw¯— Abyk×íwU wg_¨v nq †m †¶‡Î Avgiv GKgvÎ †h fyjwU Ki‡Z cvwi, Zv n‡jv bw¯— Abyk×íwU‡K cZ¨vL¨vb Ki‡Z e¨_ nB| †h‡nZzAvgiv Rwb bv †h, bw¯— Abyk×íwU cKZc‡¶B mZ¨ ev wg_¨v wKbv, †m‡nZzAvgiv wX‡jXvjvfv‡e ewj †h, UvBc-1 I UvBc-2 åvwši Sz‡K wecixZfv‡e m¤úKhy³| Ab¨ K_vq, UvBc-1 åvwši Sz‡K hZ Kg n‡e UvBc-2 åvwši m¤vebv ZZ †e‡o hv‡e|

mvgwRK weAvbxiv UvBc-2 åvwš— bv ewo‡q UvBc-1 åvwš—i m¤vebv‡K Kgv‡Z †Póv K‡ib| Avi GwU Zviv Kivi †Póv K‡ib Ab¨ GKwU bgybv wb‡q M‡elYwU‡K cyivq cwiPvjbvi gva¨‡g|

Zvn‡j GB mgm¨v t_‡K D‡Ëvi‡Yi Dcvq wK? mvgwRK weÁvbxiv UvBc-2 åvwš— bv ewo‡q UvBc-1 åvwši m¤vebv‡K Kgv‡Z †Póv K‡ib| Avi GwU Zviv Kivi †Póv K‡ib Ab¨ GKwU bgybv wb‡q M‡elYwU‡K cyivq cwiPvjbvi gva¨‡g| hw` PviwU wfb wfb M‡elYvq bw¯— Abyk×íwU cZ¨vL¨vZ nq Z‡e M‡elK wbwðZ n‡Z cv‡ib †h, wZwb mwVK wm×vš— wb‡q‡Qb| bgybvi AvKvi e„× K‡i I åvwš— Kgv‡bv †h‡Z cv‡i| wfb cwimsL¨vbMZ cix¶vi e¨envi‡K åvwš— Kgv‡bvi Av‡iKwU Dcvq wnmv‡e we‡ePbv Kiv n‡q _v‡K| Ggb wK M‡elYwU‡K cÖ‡qvRbxq cwiKwíZ K‡i I `ÿZv åvwš—‡K wbqš¿Y ivLvi †Póv Kiv nq|

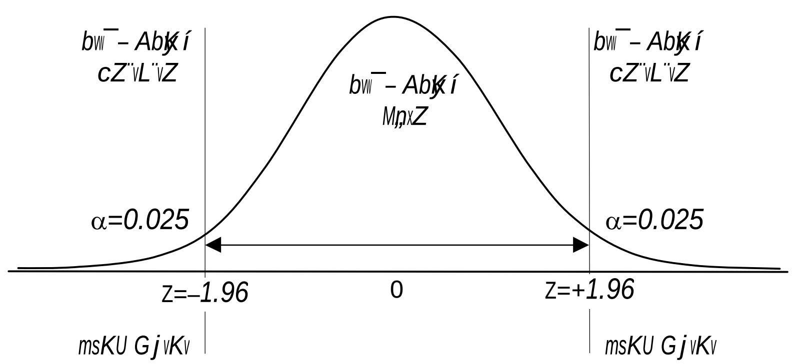
msKU GjvKv I Zvrch gvÎv wbevP‡bi wm×vš—wU wbfi K‡i `ßai‡Yi åvwši Rb¨ M‡elK KZUkzgj¨ †`‡eb Zvi Dci| cwimsL¨vbgj¨K M‡elYvq mvaviYfv‡e †h Zvrch gvÎv¸‡jv

e¨eüZ nq Zv n‡jv 0.05, 0.01, Ges 0.001 gvÎv| M‡elK hw` bvw¯— Abygvb cZ¨vL¨vb Ki‡Z bv Pvb Z‡e wZwb wbivc‡` _vK‡eb hw` Zvrch gvÎv 0.10, 0.20 ev 0.30 gvÎvq wbavi©Y Kiv nq| Gfv‡e wZwb UvBc-2 åvwš—i SzuwK Kgv‡Z cv‡ib|

Zvrch wbavi‡Yi Rb¨ wbw`©ó †Q` we›`yK (cutoff point) Avjdv (α) gvÎv ev Zvrch gvÎv e‡j| GKwU wbw`©ó Avjdv gvÎvi A_© n‡jv †h, GKRb M‡elK cwZ 100 bgybvq wK cwigvY fyj‡K MnY K‡i †b‡eb| †hgb, α = 0.05-Gi A_© n‡jv wZwb 100wU bgybv gv‡bi g‡a¨ 5wU ev Zvi Kg fyj gvb‡K MnY Ki‡eb| GKBfv‡e, α = 0.01-Gi A_© n‡jv 100wU bgybv gv‡bi g‡a¨ 1wU fyj‡K MnY Ki‡eb| Ab¨ K_vq, M‡elK a‡i †bb †h, kZKiv 5wU ev 1wU NUbv ˆØeµ‡g N‡U‡Q| GB djvdj¸‡jv mvavi‡Yfv‡e NUevi m¤¢vebv _v‡K bv| hw` bvw¯— Abygv‡bi GKwU cwimsL¨vbgZ fzj‡K cix¶v ce© wbavwiZ Avjdv gvÎv AR©‡b e¨_© nq Z‡e Avgiv bvw¯— Abygvb‡K cZ¨vL¨vb Kwi bv| Avjdv gvÎv hZ †QvU wbavi©Y Kiv n‡e bvw¯— Abygvb cZ¨vL¨vb Kiv ZZ KwVb n‡q co‡e| KviY, Avgiv †QvU Avjdv gvÎv wbavi‡Yi d‡j weiƒc djvdj hv‡Z cv‡`i Kv‡Q AbvKvw•¶Z †bwZevPK Ae¯’vb ˆZwi Ki‡Z bv cv‡i †mw`‡K †Lqvj ivwL|

wbavwiZ Avjdv gvÎvi Dc‡ii ev bx‡Pi GjvKv‡K msKU GjvKv e‡j| Ab¨ K_vq, msKU GjvKv n‡jv bgybv weZi‡Yi †mB GjvKv, hv bgybvi Am¤¢e¨ djvdj¸‡jv‡K Aš—fy³ K‡i| msKU GjvKv‡K eRb GjvKvI ejv nq| wb‡gi wPÎwU w`‡q welqwU †evSvi †Póv Kiv hvK| wPÎ 9.2.1-G ‡`Lv hv‡"Q †h, Avjdv gvÎv 0.05-†K `yÕcv‡š— we¯— Ki‡j cÖwZwU cv‡ki Rb¨ Avjdv gvÎv n‡e 0.025| GB gvÎvwU n‡jv Avgv‡`i MnY I eRb GjvKvi g‡a¨ †Q`we›`y hvi Dc‡i ev bx‡P cvß gvb¸‡jv‡K Avgiv AcZ¨vwkZ Ges Am¤¢vebvgq ˆØe NUbv wnmv‡e wPwýZ Ki‡ev| hw` †Kvb cix¶vq cvß gvbwU †h †Kvb cv‡š— eRb GjvKvi g‡a¨ c‡o hvq Z‡e Avgiv bvw¯— Abygvb‡K cZ¨vL¨vb Kwi| hw` ch©‡e¶YKZ.gvbwU FYvZ¥K nq Z‡e Zv evg cv‡š— Ae¯’vb

> msKU GjvKv n‡jv bgybv weZi‡Yi †mB GjvKv, hv bgybvi Am¤¢e¨ djvdj¸‡jv‡K Aš—fy³ K‡i|



Ki‡e Ges hw` abvZ¥K nq Z‡e Zv Wvb cv‡š— Ae¯’vb Ki‡e|

wPÎ 9.2.1: Zvrch gvÎv 0.05 gvÎvq z-gvb I msKU GjvKv

wKš‘ Avgiv wbi¼z‡k msL¨v w`‡q GwU wbavi©Y Ki‡Z cvi‡ev bv| ‡m Rb¨ Avgv‡`i ¯^vfvweK gv‡bi mvi‡Yi mvnv‡h¨ wb‡Z nq| Avgiv Rvwb †h, Avjdv gvÎv (α) = 0.05-Gi ¯^vfvweK gvb n‡jv 1.96| hw` cix¶vq wbYx©Z ch©‡e¶YKZ.gvbwU ±1.96-Gi mgvb ev Zvi †P‡q eo nq †m †¶‡Î Avgiv Avgv‡`i bvw¯— Abygvb cZ¨vL¨vb Kwi| GLv‡b GKwU welq D‡jL Kiv cÖ‡qvRb †h, Avgiv bvw¯— Abygvb‡K †KejgvÎ cZ¨vL¨vb ev MnY Ki‡Z cvwi, cgvY Ki‡Z cvwi bv| Ab¨ K_vq, m¤¢vebvi

> m¤¢vebvi ¯^fveRvZ †KejgvÎ GKwU M‡elYv ck‡K cix¶v Kiv hvq, wKš‘ KL‡bv fyj ev ï× cgvY Kiv hvq bv|

wfwË‡Z †Kejgv Î GKwU M‡elYv ck‡K cix¶v Kiv hvq, wKš KL‡bv fyj ev ï× cgvY Kiv hvq bv|

cix¶v cwimsL¨vb wbYq (Computing Test Statistics)

Abygvb wbgvY, bgybv web¨vm wbYq Ges msKU GjvKv I Zvrch gvÎv wbav‡iY ci Avgv‡`i cix¶v cwimsL¨vb wbYq Ki‡Z nq| cix¶v cwimsL¨vb wbY‡qi ch©v‡qB Avgiv Avgv‡`i wbYxZ bgybv web¨vmwUi cKZ. c‡qvM K‡i _wK| GKwU cix¶v cwimsL¨vb n‡jv Ggb GKwU cwimsL¨vb, hvi eYbvg‡j Kfv‡e ‡Kvb mnRvZ AvKlY †bB Z‡e Zv Abyôvb cix¶vq e¨envi Kiv nq| GwU †mB cwimsL¨vb hvi bgybv web¨vm i‡q‡Q Ges hv mivmwi cix¶vq e¨eüZ nq| Ab¨ K_vq, Avgiv bgybv DcvË †_‡K GKwU msL¨v wbYq Kwi hv m¤¢ebv ZË Abyôvqx ÁvZfv‡e cwiewZZ nq| Zvici Avgiv †mB msL¨wU‡K bgybv web¨vm †_‡K cvß ZwËK gv‡bi mv‡_ Zzjbv Kwi Ges GwU NUevi m¤¢ebvi Øviv gj¨vqb Kwi|

GKwU D`vniY †`qv hvK| aiv hvK, Avgv‡`i nv‡Z wbgwjwLZ Z_¨ i‡q‡Q hvi wfwË‡Z Avgiv GKwU cix¶v cwimsL¨vb wbYq Ki‡ev| G †¶‡Î z-cix¶vi K_v we‡ePbv Kiv †h‡Z cv‡i|

$$\mu = 18,\ \sigma = 4,\ \bar{x} = 19,\ \text{Ges } N = 100$$

GLb ck Kiv †h‡Z cv‡i †h, 100 bgybv t_‡K cvß bgybv Mo 19 cvevi m¤¢ebv KZUzKz hLb mgM‡Ki Mo 18 Ges cwigvZ e¨eavb 4? Avgiv Avjdv gvÎv wbaviY K‡i wQ 0.05 gvÎvq hvi z-gvb n‡jv 1.96| Avgv‡`i nv‡Z †h mKj Z_¨ i‡q‡Q Zvi wfwË‡Z z-gvbwU wbYq Ki‡Z n‡e| z-gvb wbY‡qi m¤ú~wU n‡jv:

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

$$= \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

‡hLv‡b, z = ch‡e¶YKZ..z -gvb

$\bar{x}$ = bgybv Mo

μ = mgMK Mo

$\sigma_{\bar{x}}$ ev $\frac{\sigma}{\sqrt{n}}$ = M‡oi web¨v‡mi cwigvZ e¨eavb

m¤ú~Î gvb ewm‡q,

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$= \frac{19 - 18}{\frac{4}{\sqrt{100}}}$$

$$= \frac{1}{\frac{4}{10}}$$

$$= \frac{1}{0.4}$$

$$= 2.50$$

$\therefore$ ch‡e¶YKZ.z-gvbwU n‡jv 2.50

D`vniY wnmv‡e Avgiv GLv‡b ¯'vfwweK web¨v‡mi GKwU ci¤¶v cwimsL¨vb (z) wbYq K‡iwQ| wKš'wewfb ai‡Yi bgybv web¨v‡mi Rb¨ wewfb cKvi ci¤¶v cwimsL¨vb i‡q‡Q| †m¸‡jv cieZx Aa¨vq¸‡jv‡Z wbYq K‡i †`Lv‡bv n‡e| Gevi Avgv‡`i wm×vš Mn‡Yi cvjv|

wm×vš MnY (Making a Decision)

ci¤¶v cwimsL¨vb wbY‡qi ci Avgv‡`i wm×vš MnY Ki‡Z n‡e| ci¤¶vq cvß djvd‡ji wfwË‡Z nq Avgiv bw¯ Abyk×í‡K cZ¨vL¨vb Ki‡ev, bZzev cZ¨vL¨vb Ki‡Z e¨_ n‡ev| hw` djvdj¸‡jv msKU GjvKvi g‡a¨ c‡o, Z‡e UvBc-1 åwš'i GKwU ÁvZ m¤vebvi gva¨‡g Avgiv Avgv‡`i bw¯ Abyk×í‡i Abygvb‡K cZ¨vLvb Ki‡ev| hw` djvdj¸‡jv msKU GjvKvi g‡a¨ bv c‡o, Z‡e Avgiv bw¯ Abyk×í‡i Abygvb‡K cZ¨vL¨vb Ki‡ev bv Ges UvBc-2 åwši SzuK †be| Avgv‡`i Zvrch gvÎv wbawiZ n‡q‡Q 0.05 gvÎvq, hvi z-gvb n‡jv 1.96| wm×vš Mn‡Yi mÂwU n‡jv ‡h, hw` Ab‡c¶ z-gvb Zvrch gvÎvi gv‡bi mgvb ev Zvi †P‡q eo nq Z‡e Avgiv bw¯ Abyk×í‡K cZ¨vL¨vb Ki‡ev| A_vr,

hw` | ch‡e¶YKZ.z-gvb| ≥ 1.96 nq, Z‡e bw¯ Abyk×íwU cZ¨vL¨vZ n‡e|

Avgv‡`i D`vni‡Yi ‡¶‡Î, ch‡e¶YKZ.z-gvbwU ZwËK z-gvb †_‡K eo e‡j Avgiv bw¯ Abyk×í cZ¨vL¨vb Ki‡ev| A_vr,

†h‡nZz |2.50| > 1.96, †m‡nZzbw¯ Abyk×íwU cZ¨vL¨vZ n‡jv|

wKš hw` Avgv‡`i Avjdv gvÎv 0.01 n‡Zv, hvi ZwËK z-gvb n‡jv 2.58, Z‡e ch‡e¶YKZ.z-gvbwU ZwËK z-gv‡bi †P‡q †QvU e‡j Avgiv bw¯ Abyk×íwU‡K cZ¨vL¨vb Ki‡Z cviZvg bv| wKš eo bgybv AvKvi m¤úb M‡elYvi †¶‡Î †h †Kvb Abyk×í cZ¨vL¨vb Kiv m¤e| †hgb, hw` bgybvi AvKvi 10000 nq, Z‡e 0.001 Zvrch gvÎvq bw¯ Abyk×í cZ¨vL¨vZ n‡Z cv‡i|

mvivsk

cwimsL¨vbMZ wm×v‡š †cÖQv‡bvi Rb¨ M‡elK Abyk×í ci¤¶v K‡i _v‡Kb| Abyk×í ci¤¶vi Rb¨ M‡elK‡K wKQzmybw`ó avc AwZµg Ki‡Z nq| avc¸‡jv n‡"Q Abygvb wbgvY, bgybv web¨vm wbYq, Zvrch gvÎv Ges msKU GjvKv wbevPb, ci¤¶v cwimsL¨vb wbYq Ges wm×vš MnY| `ÿai‡Yi Abygvb wbgvY i‡q‡Qt Av‡cw¶Kfv‡e wbwðZ Abygvb Ges m‡›`nRbK Abygvb| M‡elKiv wØZxq ai‡Yi Abygvb wb‡qB KvR K‡i _v‡Kb| bgybv web¨vmi †¶‡Î MwYwZK hy³ gj-Zt wfwË wnmv‡e KvR K‡i| Ab¨w`‡K, msKU GjvKv wbevP‡bi Rb¨ GK cwšK/wØcwšK Aš'fy³ Ki‡Yi wm×všwU ¸iæZc¥|

cv‡VwËi gj¨vqb

^be¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| Pj‡Ki Kvh-KviY m¤úK© welqK c‡ki mgvavb †c‡Z mvnvh¨ K‡i:

K. Mo ci¤¶v
L. e¨eavb ci¤¶v
M. Abyjvf ci¤¶v
N. bgybv ci¤¶v

2| Abyjvf ci¤¶vi Aš—bwnZ c×wZwU gj¨Zt wbfi K‡i:

K. m¤vebv Z‡Ëi Dci
L. Am¤vebv Z‡Ëi Dci
M. Pj‡Ki ^ewk‡ó¨i Dci
N. Pj‡Ki e¨eav‡bi gvÎvi Dci

3| bw_ Abyjvf‡K fy³fv‡e cZ¨Lvb Kivi wm×vš‡K ..... e‡j|

K. wmMgv åvwš
L. Avjdv åvwš
M. Mo åvwš
N. cwiwgZ e¨eavb åvwš

msw¶ß ck

UxKv wjLby:

K. ci¤¶v cwimsL¨vb wbYq
L. wm×vš MnY

iPbvgj¨K ck

1| ms‡¶‡c cwimsL¨vbMZ ciw¶vi wewfb avc Av‡jvPbv Ki‡b|

2| UvBc-1 I UvBc-2 åvwš wK? Zvrch gvÎv I msKU GjvKv wbevP‡b UvBc-1 I UvBc-2 åvwši Syjvf KZUzKz Zv D`vniYmn Av‡jvPbv Ki‡b|

cvV - 3

cwimsL¨vbMZ cix¶v 1: z-cix¶v
Statistical Test 1: z-Test

GB cvV †k‡l hv Rvbv hv‡e —

- eo bgybvi AbyK‚í cix¶v: z-cix¶v
- GKwU bgybvi Rb¨ AbyK‚í cix¶v
- `ywU bgybvi g‡a¨ cv_©‡K¨i Rb¨ AbyK‚í cix¶v
- GK-cvwšK ebvg wØ-cvwšK cix¶v

eo bgybvi AbyK‚í cix¶v: z-cix¶v (Hypothesis Testing for Large Samples: z-test)

GKwU bgybvi AvKvi hZ eo n‡e †mB bgybv †_‡K cvß gvbwU mgMK gv‡bi ZZ wbKUeZx n‡e|

‡K›`ªxq mxgv Dccv`¨ (central limit theorem)-Gi gva¨‡g Avgiv Rvwb †h, GKwU bgybvi AvKvi hZ eo n‡e †mB bgybv †_‡K cvß gvbwU mgMK gv‡bi ZZ wbKUeZx n‡e| d‡j, cwigvY e¨eav‡bi gvbwUI ZZ †QvU n‡e| wKš' bgybvi AvKvi KZ eo n‡j eo bgybv ejv hv‡e GB m¤ú‡K †Kvb ¯úó †f`‡iLv UvbvmnR bq| Z‡e cwimsL¨vbwe`MY m¤vebv web¨v‡mi gva¨‡g GKwU mvavirY wbqg cwZwôZ K‡i‡Qb †h, hw` GKwU bgybvi AvKvi 60 AwZµg K‡i Z‡e Zv ¯^vfvweKZvi (normality) w`‡K †h‡Z _v‡K Ges hw` Zv 120 AwZµg K‡i Z‡e †mB bgybv ¯^vfvweKZvi ˆewkó¨ ARb K‡i|

hw` bgybvi AvKviwU 30 ev Zvi †P‡q †QvU nq Z‡e †mwU‡K †QvU bgybv ejv nq| eo bgybvi AbyK‚í cix¶v †QvU bgybvi AbyK‚í cix¶v †_‡K wfbœ n‡q _v‡K| KviY, eo bgybvi †¶‡Î †h Abygvb¸‡jv †bqv nq †mB Abygvb¸‡jv †QvU bgybvi †¶‡Î c‡hvR¨ bq| eo bgybvi ‡¶‡Î †h Abygvb¸‡jv MÖnY Kiv nq †m¸‡jv n‡jv:

1. bgybv wbevPb ˆØePwqZ n‡Z n‡e|
2. bgybv web¨vmwU ¯^vfvweK n‡Z n‡e|
3. bgybv gvbwU mgMK‡K GZUvB cwZwbwaZ Ki‡e ‡h mgM‡Ki ¯^vfvweKZvi AbygvbwU wkw_j Kiv †h‡Z cv‡i|
4. cwigv‡ci gvÎwU n‡Z n‡e e¨wßgyjK gvÎvq|

GB Abygvb¸‡jvi wfwË‡Z eo bgybvi Rb¨ z-cix¶v cwiPvjbv Kiv n‡q _v‡K|

GKwU bgybvi Rb¨ AbyK‚í cix¶v (Hypothesis Testing for a Single Sample)

bgybv †_‡K cvß ŷ hw` y-Gi GKwU c¶cvZgy³ cv°jb nq Z‡e Avgv‡`i wbgœwjwLZ bvw¯Í AbyK‚íwU cix¶v Ki‡Z n‡e:

$$H_0 : \hat{y} = y$$

AbyK‚íwU‡K mgMK gv‡bi †cwi¶‡Z Dc¯'vcb Ki‡j Zvi Awfe¨w³wU `vovq wbgœiƒc,

$$H_0 : \mu_0 = \mu$$

GKwU bgybvi AvKvi hZ eo n‡e †mB bgybv †_‡K cvß gvbwU mgMK gv‡bi ZZ wbKUeZx n‡e|

†hLv‡b, H_0 = bvw¯— AbyK í

μ = mgMK Mo

μ_0 = mgMK M‡oi AbygvZ gvb

Z‡e †h‡nZz bgybv Mo ($\bar{x}$) n‡j v mgMK Mo (μ)-Gi †kô c¶cvZnxb cv°jb, †m‡nZz Avgiv bgybv Mo ($\bar{x}$)-Gi bgybv webïv‡mi Dci Avgv‡`i `„wó †K›`ªxfz— Ki‡ev| hw` bgybv webïvmwU ¯^vfvweK nq Z‡e,

$$z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}}$$

‡hLv‡b, z = ¯^vfvweK gvb

$\bar{x}$ = bgybv Mo

μ = mgMK Mo

$\sigma_{\bar{x}}$ = M‡oi cwiwgZ åvš—

hw` mgM‡Ki cwiwgZ e¨eavb Rvbv _v‡K Z‡e M‡oi cwiwgZ åvš—i cwigvcwU n‡e,

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

wKš‘ hw` mgM‡Ki cwiwgZ e¨eavb Rvbv bv _v‡K Z‡e M‡oi cwiwgZ åvš—i cwigvcwU n‡e,

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n-1}} = \frac{\hat{s}}{\sqrt{n}}$$

GKwU M‡oi Rb¨ AbyK í cix¶vi GKwU D`vniY †`qv hvK| aiv hvK, GKwU M‡elYvq 376 Rb QvÎ-QvÎxi Mo eqmbv¼ cvIqv †M‡Q 93.56, hvi cwiwgZ e¨eavb n‡jv 12.18| wKš‘ cÖ_vi mgM‡Ki Mo eqmbv¼ n‡jv 100, hvi cwiwgZ e¨eavb 15| A_©vr, bgybv †_‡K cvß MowU mgMK Mo †_‡K 6.44 Kg| GLb cÖkœ n‡jv †h, GB cv_©K¨wU wK ˆØeµ‡g cvß? †h‡nZz M‡elK bgybv wb‡q KvR Ki‡Qb, †m‡nZz Zvi wbwðZ bb †h, cv_©K¨wU wK cKÖZ, bvwK ˆØeµ‡g cvß| GwU wbwðZ nevi Rb¨ M‡elK AbygvY Ki‡eb †h cv_©K¨wU cKZ.bq| M‡elK GKwU weKí AbygvbI MnY Ki‡eb †h, cv_©K¨wU cKZB ev¯—‡e we`¨gvb| GB Abygvb `ywU‡K mvs‡KwZK wP‡ýi gva¨‡g wbgœi‡c cKvk Kiv hvq:

bvw¯— AbyK í, $H_0 : \mu_0 = \mu$

weKí AbyK í, $H_1 : \mu_0 \neq \mu$

GB AbyK í cix¶vi Rb¨ bgybv webïvmwU wbYq Ki‡Z Ly e †ekx †eM †c‡Z nq bv| KviY, †h‡nZz Avgiv Rwb †h, bgybv M‡oi bgybv webïvmwU ¯^vfvweK A_ev cvq ¯^vfvweK, †m‡nZz M‡elK bgybv webïvm wnmv‡e miivmwi ¯^vfvweK webïvm‡K e¨envi Ki‡Z cv‡ib hv z-webïvm bv‡g cwiwPZ| ¯^vfvweK webïvmwU GKwU mviwY AvKv‡i Dc¯'wcZ nq hv Avgiv BDwbU 6-G we¯—wiZfv‡e Av‡jvPbv K‡iwQ| Gi cieZx av‡c M‡elK Zvrch gvÎv I msKU GjvKv wPwýZ Ki‡eb| GB KvRwU Ly e mnR bq| KviY, h_vh_ Zvrch gvÎv I msKU GjvKv wPwýZKi‡Yi wm×vš—wU wbfi K‡i UvBc-1 I UvBc-2 åvš—i g‡a¨ ‡KvbwUi Rb¨ M‡elK KZUKz gyj¨ †`‡eb Zvi Dci| hvB †nvK, M‡elK wVK Ki‡jb †h, `ywU M‡oi g‡a¨Kvi cv_©K¨wU kZKiv 5 ev Zvi ‡P‡q Kgevi

NU‡j wZwb a‡i ‡b‡eb †h, cv_K¨wU cKZ.bq| Avi hw` Zvi †P‡q †ekxevi N‡U Z‡e wZwb a‡i †b‡eb †h, cv_K¨wU mwZ¨Kvi fv‡e ev¯‡e we`¨gvb| A_vr, α gvÎv n‡jv 0.05, hvi z-gvb 1.96|

Gevi M‡elK cix¶v cwimsL¨vb wbYq Ki‡eb| Avgiv Rwb †h, z-cix¶vi exRMwYZK mÿgZv n‡jv,

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

‡hLv‡b, z = ¯vfvweK gvb
$\bar{x}$ = bgybv Mo
μ = mgMK Mo
$\frac{\sigma}{\sqrt{n}}$ = cv_‡K¨i cwigvZ åwš

m¤ÿÎ gvb ewm‡q,

$$z = \frac{93.56 - 100}{\frac{15}{\sqrt{376}}}$$

$$= \frac{-6.44}{\frac{15}{19.39}}$$

$$= -\frac{6.44}{0.774}$$

$$= -8.32$$

$\therefore z = -8.32$

wm×vš Mn‡Yi †¶‡Î Avgiv Rwb †h, hw` ch‡e¶YKZ gvbwU abvZ¥K nq Z‡e a‡i wb‡Z n‡e †h, cv_K¨wU ¯vfvweK †iLvi Wvb cv‡š Ae¯vb Ki‡Q Ges hw` FYvZ¥K nq Z‡e a‡i wb‡Z n‡e †h, cv_K¨wU evg cv‡š Ae¯vb Ki‡Q| Z‡e Avgiv Rwb †h, wm×vš Mn‡Yi †¶‡Î ZwËK gv‡bi mv‡_ ch‡e¶YKZ gv‡bi Zzjbv Kivi mgq wPý‡K AeÁv Ki‡Z nq| AZGe ‡`Lv hv‡"Q †h, ch‡e¶YKZ.Ab‡c¶ z-gvbwU | 8.32 | ZwËK z-gvb‡K (1.96) A‡bK †ekx AwZµg K‡i wM‡q‡Q Ges Zv FYvZ¥Kfv‡e N‡U‡Q| AZGe, M‡elK wm×vš †b‡eb †h, bgybv Mo I mgMK M‡oi g‡a¨Kvi cv_K¨wU ˆØeµ‡g N‡Uwb, GB cv_K¨wU ev¯‡eB i‡q‡Q| wKš GwU g‡b ivLv c‡qvRb †h, Ab¨ GKwU bgybvi †¶‡Î ev Av‡iv msKxY Zvrch gvÎvi †¶‡Î wm×všwU wfb n‡Z cv‡i| Abyfí cix¶vq wm×všwU wbgwjwLZfv‡e Dc¯vcb Kiv nq,

$$z = -8.32, \qquad p < 0.05$$

Gi A_ n‡jv, ZwËK z-gv‡bi (z = 1.96) m¤vebwU ch‡e¶YKZ.z-gv‡bi †P‡q Kg| AZGe, bw¯ Abyk×íwU cZ¨vL¨vZ n‡q‡Q| hw` ch‡e¶YKZ.gv‡bi †P‡q ZwËK gvbwU eo n‡Zv Z‡e Avgv‡`i p > 0.05 wjL‡Z n‡Zv|

`ÿwU bgybvi g‡a¨ cv_‡K¨i Rb¨ Abyk×íci x¶v (Hypothesis Testing for the Difference between Two Samples)

aiv hvK, Avgv‡`i KwíZ M‡elK Qv‡Î Ges Qv‡Îx‡`i g‡a¨Kvi ey×bv‡¼i cv_K¨wU‡K ci x¶v K‡i †`L‡Z Pvb| Gi Rb¨ wZwb ˆØePqb c×wZ Abyôvb Y K‡i 150 Rb Qv‡Î Ges 150 Rb Qv‡Îxi GKwU bgybv wbe©vPb Ki‡jb| Qv‡Î‡`i bgybvq Mo ey×bv¼ n‡jv 97.5 I cwigvZ e¨eavb 10.4, Ges Qv‡Îx‡`i Mo ey×bv¼ n‡jv 94.4 I cwigvZ e¨eavb 11.2| `ÿwU bgybvi g‡a¨ cvß Mo ey×bv‡¼i cv_K¨wU cwimsL¨vbMZfv‡e Zvrchc©wKbv Zv ci x¶vi Rb¨ bw¯ I weKí Abyk×í `ÿwU n‡jv h_vµ‡g:

bw¯ Abyk×í $H_0 : \mu_1 = \mu_2$

weKí Abyk×í $H_1 : \mu_1 \neq \mu_2$

bgybv web¨vm wnmv‡e M‡elK z-web¨vm‡K wbe©vPb K‡i‡Qb| Zvrch gv‡Î wnmv‡e MnY K‡i‡Qb $\alpha = 0.05$| ci x¶v cwimsL¨vb wbYq Ki‡Z wM‡q wZwb wbgwjwLZ m¤ú~e¨envi Ki‡eb|

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\hat{\sigma}_{\bar{x}_1 - \bar{x}_2}}$$

‡hLv‡b, z = ¯vfwweK gvb

$\bar{x}_1$ = Qv‡Î‡`i ey×bv‡¼i Mo

$\bar{x}_2$ = Qv‡Îx‡`i ey×bv‡¼i Mo

$\hat{\sigma}_{\bar{x}_1 - \bar{x}_2}$ = cv_‡K¨i cv°wjZ cwigvZ åwš

cv_‡K¨i cwigvZ åwš‡K wbgwjwLZfv‡e msÁwqZ Kiv hvq:

$$\hat{\sigma}_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

†hLv‡b, s_1^2 = Qv‡Î‡`i ey×bv‡¼i †f`v¼

s_2^2 = Qv‡Îx‡`i ey×bv‡¼i †f`v¼

n_1 = Qv‡Î‡`i bgybv msL¨v

n_2 = Qv‡Îx‡`i bgybv msL¨v

bgybv †_‡K cvß djvdj ¸‡jv n‡jv,

Qv‡Î‡`i bgybv: $\bar{x}_1$ = 97.5, s_1 = 10.4, n_1 = 150

Qv‡Îx‡`i bgybv: $\bar{x}_2$ = 94.4, s_2 = 11.2, n_2 = 150

gvb¸‡jv m‡š^Î ewm‡q,

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$= \frac{97.5 - 94.4}{\sqrt{\frac{(10.4)^2}{(150-1)} + \frac{(11.2)^2}{(150-1)}}}$$

$$= \frac{3.1}{\sqrt{\frac{108.16}{149} + \frac{125.44}{149}}}$$

$$= \frac{3.1}{\sqrt{0.7259 + 0.8419}}$$

$$= \frac{3.1}{\sqrt{1.252}}$$

$$= 2.476$$

$$\therefore \quad z = 2.476, \quad p < 0.05$$

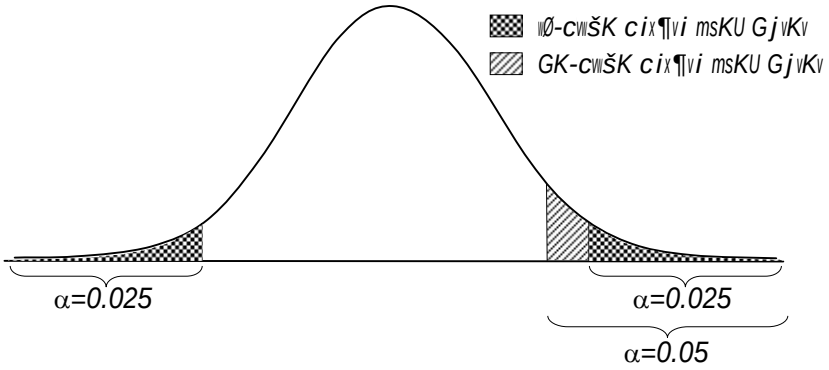
ch‡e¶YKZ.z-gvb Abycvqx †`Lv hv‡"Q †h, GwU ZwËK gvb‡K abvZ¥Kfv‡e Qvwo‡q wM‡q‡Q| Kv‡RB, bwj AbyK í wU‡K cZ¨vL¨vb Kiv n‡jv| A_vr, Qv Î-QvÎx‡`i g‡a¨ Mo eyw×bv‡¼i Zdvr wU 0.05 gvÎvq cwimsL¨vbMZfv‡e Zvrch©c~Y©| Z‡e hw` M‡elK Zvrch gvÎv wnmv‡e $\alpha = 0.01$ wbevPb Ki‡Zb Zvn‡j wZwb bwj AbyK íwU‡K cZ¨vL¨vb Ki‡Z cvi‡Zb bv| KviY, z-mviwY Abyhvqx, $\alpha = 0.01$-Gi z-gvb n‡jv 2.58, hv ch‡e¶YKZ.z-gv‡bi †P‡q eo|

GK-cwšK ebvg wØ-cwšK cix¶v (One-Tailed Versus Two-Tailed Test)

‡h `yÕai‡Yi AbyK í cix¶vi D`vniY Avgiv G chš Av‡jvPbv K‡iwQ †m me †¶‡Î bwj Ges weKí `yÕwU AbyK í Dc¯vc‡bi †¶‡Î M‡elK c¯ve Ki‡Qb †h, bgybv M‡oi mv‡_ mgMK M‡oi †Kvb cv_K¨ †bB, ev i‡q‡Q Ges `yÕwU bgybv M‡oi g‡a¨ †Kvb cv_K¨ †bB, ev i‡q‡Q| G ai‡Yi c¯vebvi gva¨‡g cwimsL¨vbMZfv‡e cgwYZ cv_K¨wU Kg ev †ekx wKbv Zv †evSv hvq bv | wKš Avgiv GKwU bgybvi Rb¨ cix¶vi †¶‡Î †`‡LwQ †h, ch‡e¶YKZ.gvbwU FYvZ¥K Ges `yÕwU bgybvi Rb¨ cix¶vi †¶‡Î gvbwU abvZ¥K n‡q‡Q| A_vr, `yÕwU cix¶vi †¶‡Î ch‡e¶YKZ. cv_K¨wU `yÕwU wfbœ w`K wb‡`k Ki‡Q| hLb AbyK í cix¶vi c¯vebvq †Kvb w`K wb‡`k Kiv _v‡K bv ZLb †mB cix¶v‡K w`Kwb‡`kbvnxb (non-directional) ev wØ-cwšK (two-tailed) cix¶v e‡j|

wKš A‡bK mgq wеKí AbyK í c¯vebvi †¶‡Î w`K wb‡`k Kivi c‡qvRb n‡q c‡o| hLb wеKí AbyK í c¯vebvi †¶‡Î msKU Gjv‡K GKwU wbw`ó w`‡K (Wv‡b ev ev‡g) wb‡`k Kiv nq ZLb Zv‡K w`K wb‡`kbvgyK (directional) ev GK-cwšK (one-tailed) cix¶v e‡j| hLb GKwU AbyK‡íi g‡a¨ w`K wb‡`kbv‡K wPwýZ Kiv nq, ZLb GKB Zvrch gvÎvq wØ-cwšK cix¶vi cwie‡Z GK-cwšK cix¶v DËg djvdj c`vb K‡i| KviY, Gi d‡j cÖ‡qvRbxq msKU

GjvKv‡K bgybv web¨v‡mi h_vh_ cv‡š †K›`xfzZ K‡i GKwU e„Ëi msKU GjvKv cvIqv m¤e nq| wPÎ 9.3.1-G 0.05 Zvrch gvÎvq GK-cwšK I wØ-cwšK ci x¶vi msKU GjvKvi GKwU Zzjbv t`Lv th‡Z cv‡i|



wPÎ 9.3.1: 0.05 Zvrch gvÎvq GK-cwšK I wØ-cwšK ci x¶vi msKU GjvKvi Zzjbv

GLv‡b `ÿvU t¶‡ÎB UvBc-1 åwši m¤vebv GKB iKg| wKš djvdj hw` cKZc‡¶ cZ¨wkZ w`‡K N‡U Z‡e GK-cwšK ci x¶vi gva¨‡g bw¯ AbykÍ‡K cZ¨vL¨vb Kiv Awa KZi m¤vebv _vK‡e| KviY, cZ¨wkZ w`‡Ki djvdj ¸‡jv e„Ëi msKU GjvKvi g‡a¨ covi m¤vebv e„× cv‡e| wØ-cwšK ci x¶vi Zzjbvq GK-cwšK ci x¶vq UvBc-2 åwš Kivi Sz‡K Kg _v‡K|

mnR K_vq ejv hvq th, hLb bgybv Mo bgybv web¨v‡mi `ÿcv‡šB c‡o Ges mgMK Mo m¤úwKZ AbykÍwU‡K cZ¨vL¨vb Kiv nq ZLb †mwU‡K wØ-cwšK ci x¶v e‡j| hLb bgybv Mo bgybv web¨v‡mi th †Kvb GKwU cv‡š c‡o Ges mgMK Mo m¤úwKZ AbykÍwU‡K cZ¨vL¨vb Kiv nq ZLb †mwU‡K GK-cwšK ci x¶v e‡j| mviwY 9.3.1-G wewfb Zvrch gvÎvq GK-cwšK I wØ-cwšK ci x¶vi ZwËK z-gv‡bi Zzjbv K‡i t`Lv‡bv n‡jv|

mviwY 9.3.1: wewfb Zvrch gvÎvq GK-cwšK I wØ-cwšK ci x¶vi ZwËK z-gv‡bi Zzjbv

Zvrch gvÎv	0.10	0.05	0.01	0.005	0.002
GK-cwšK ci x¶vi Rb¨ ZwËK z-gvb	–1.28 ev 1.28	–1.65 ev 1.65	–2.33 ev 2.33	–2.58 ev 2.58	–2.88 ev 2.88
wØ-cwšK ci x¶vi Rb¨ ZwËK z-gvb	–1.645 ev 1.645	–1.96 ev 1.96	–2.58 ev 2.58	–2.81 ev 2.81	–3.08 ev 3.08

mvivsk

bgybvi AvKvi hw` 30 ev Gi †P‡q †QvU nq Z‡e †mwU‡K †QvU bgybv ejv nq| mvaviYZt, †QvU bgybvi ‡¶‡Î th AbykÍ ci x¶v e¨envi Kiv nq e„r bgybvi †¶‡Î Zv e¨envi Kiv nq bv| e„r bgybvi ‡¶‡Î z-ci x¶v Kiv n‡q _v‡K| z-web¨vm n‡"Q bgybvi ¯vfweK web¨vm| hw` ch‡e¶YKZ.gvbwU abvZ¥K nq, Z‡e a‡i wb‡Z n‡e th cv_K¨wU ¯vfweK †iLvi Wvb cv‡š Ae¯vb Ki‡Q Ges hw` FYvZ¥K nq, Z‡e a‡i wb‡Z n‡e th cv_K¨wU evg cv‡š Ae¯vb Ki‡Q| wm×vš Mn‡Yi †¶‡Î ZwËK gv‡bi mv‡_ ch‡e¶YKZ.gv‡bi Zzjbvi mgq wPý‡K AeÁv K‡i cvß gvbwU‡K we‡ePbvq wb‡Z nq|

cv‡VvšÍi g~j¨vqb

ˆbe©¨w³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| hw` GKwU bgybvi AvKvi ............... AwZµg K‡i Z‡e Zv ¯^vfvweKZvi w`‡K †h‡Z _v‡K Ges hw` Zv .... AwZµg K‡i Z‡e †mB bgybv ¯^vfvweKZvi ˆewkó¨ ARb K‡i|

K. 40/80
L. 50/100
M. 55/110
N. 60/120

2| wm×vš Mn‡Yi †¶‡Î ZwËK gv‡bi mv‡_ ch‡e¶YKZ.gv‡bi Zzjbv Kivi mgq wPý‡K .... Ki‡Z nq|

K. Ašf©y³
L. ewnfz©Z
M. AeÁv
N. Aegy³

3| z-cix¶vi exRMwYwZK m~ÎwU n‡jv:

K. $z = \dfrac{\bar{x} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$

L. $z = \dfrac{\sigma - \bar{x}}{\mu}$

M. $z = \dfrac{(n - \bar{x})^2}{\sqrt{\sigma}}$

N. $z = \dfrac{\sigma - \bar{x}}{\sqrt{\mu}}$

msw¶ß ck

UxKv wjLyb:

K. ‡QvU bgybvi Rb¨ Abyi‚c cix¶v|
L. GK-cvwšK ebvg wØ-cvwšK cix¶v|

iPbvg~jK ck

1| D`vniYmn z-cix¶vi e¨envi Av‡jvPbv Ki“b|

2| `yÕwU bgybvi gv‡a¨ cv_©‡K¨i Rb¨ Abyi‚c cix¶v c×wZwU D`vniYmn Av‡jvPbv Ki“b|

cvV - 4

cwimsL¨vbMZ cix¶v 2: t-cix¶v
Statistical Test 2: t-Test

GB cvV †k‡l hv Rvbv hv‡e —

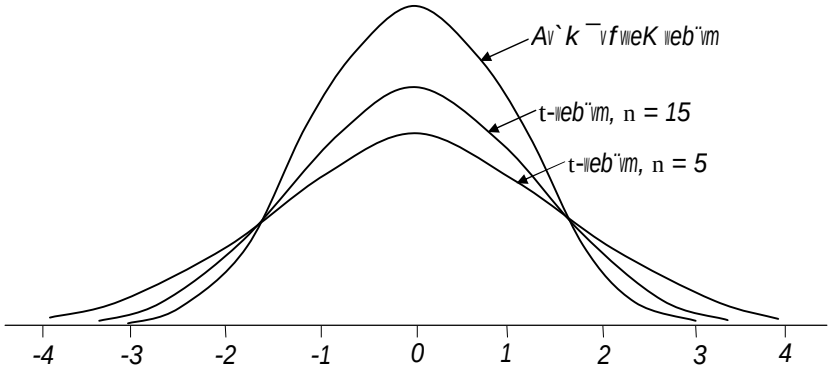
- ‡QvU bgybvi Abyk×í cix¶v: t-cix¶v
- ¯vaxbZvi gvÎv
- t-web¨v‡mi ^ewkó¨mgn
- GKwU bgybvi Rb¨ Abyk×í cix¶v
- `ÿvU bgybvi g‡a¨ cv_‡K¨i Rb¨ Abyk×í cix¶v
- mn-m¤úwKZ M‡oi Rb¨ t-cix¶v

‡QvU bgybvi Abyk×í cix¶v: t-cix¶v (Hypothesis Testing for Small Sample: t-test)

cvV 3-G Avgiv z-web¨vm ev ¯vfweK web¨v‡mi gva¨‡g enr bgybvi Abyk×í cix¶v wb‡q Av‡jvPbv K‡i wQ| †mB Av‡jvPbvq Avgiv †R‡bwQ †h, hw` mgMKwU ¯vfweKfv‡e web¨¯— nq Z‡e bgybvi web¨vm¸‡jvI ¯vfweKfv‡e web¨¯— n‡e| d‡j, bgybv gvb mgMK gvb‡K cwZwbwaZ Ki‡e| G Qvovl enr bgybvi Abyk×í cix¶vi †¶‡Î Avgiv †`‡LwQ †h, mgM‡Ki cwigZ e¨eav‡bi gvbwU Rvbv _v‡K| wKš mvgwRK M‡elYvq Lye Kg †¶‡ÎB mgM‡Ki cwigZ e¨eav‡bi ev †f`v‡¼i gvbwU Rvbv _v‡K| hLb mgM‡Ki cwigZ e¨eav‡bi gvbwU Rvbv _v‡K bv A_ev bgybvi AvKvi wU ‡QvU nq, ZLb Avgiv bgybvi cwigZ e¨eavb‡K mgM‡Ki cwigZ e¨eav‡bi mwbKUeZx Abygvb Ki‡Z cwi bv|

AZGe, Avgv‡`i ïayth bgybv †_‡K mgM‡Ki Mo‡K cv°jb Ki‡Z nq, ZvB bq Avgv‡`i cwigZ åwš‘I cv°jb Ki‡Z nq| G ai‡Yi cwi¯‹Z‡Z bgybv web¨vm wnmv‡e ¯vfweK web¨vm ev z-web¨vm c‡hvR¨ nq bv| G †¶‡Î Ggb GKwU bgybv web¨vm e¨envi Ki‡Z n‡e hvi gva¨‡g G ai‡Yi mgm¨v¸‡jv‡K †gvKv‡ejv Kiv hvq| GB bgybv web¨vmwU t-web¨vm bv‡g cwiwPZ| bgybvi AvKv‡ii welqwU we‡ePbv QvovB t-web¨vm GKwU Rbwcq cwimsL¨v‡b cwiYZ n‡q‡Q, we‡kl K‡i `ÿvU bgybvi M‡oi g‡a¨ Zzjbvi Rb¨| Gi Rbwcq Zvi KviY m¤eZt `ÿvU| c_gZt, †QvU bgybvi Rb¨ GB cwimsL¨vbwU Lye Dc‡hvMx, we‡kl K‡i hLb bgybvi AvKvi 30 ev Zvi †P‡q †QvU nq| wØZxqZt, bgybvi AvKvi hZ eo n‡Z _v‡K t-web¨vm ax‡i ax‡i ¯vfweK web¨v‡mi i¡c wb‡Z _v‡K| bgybvi AvKv‡ii Dci wfwË K‡i cwZwU t-web¨v‡mi AvKwZ. wfb n‡q _v‡K| †hgb, hw` n = 5 nq, Z‡e t-web¨v‡mi D”PZv ¯vfweK web¨v‡mi Zzjbvq Kg nq Ges cvš¸‡jv ck¯ nq, (wPÎ 9.4.1 `óe¨)|

t-web¨v‡mi gva¨‡g mgM‡Ki Mo m¤úwKZ Abyk×í cix¶v, `ÿvU M‡oi g‡a¨ cv_K¨ m¤úwKZ Abyk×í cix¶v, wbfikxj PjK m¤wjZ `ÿvU M‡oi g‡a¨ cv_K¨ m¤úwKZ Abyk×í cix¶v Ges mn-m¤ú‡Ki mnM m¤úwKZ Abyk×í cix¶v cwiPvjbv Kiv n‡q _v‡K| t-web¨vm m¤ú‡K we¯—wiZ Av‡jvPbvi c‡e t-web¨v‡mi mv‡_ m¤úwKZ GKwU ¸iZc¥cZ¨q Av‡jvPbv Kiv c‡qvRb| †mwU n‡j ¯vaxbZvi gvÎv (degrees of freedom)|



wPÎ 9.4.1: ¯vfweK web¨v‡mi mv‡_ t-web¨v‡mi Zzjbv

¯vaxbZvi gvÎv (Degrees of Freedom)

¯vaxbZvi gvÎv gj¨vZt Dcv‡Ëi Dci wbqšY‡K cwZwbwaZ K‡i| †hgb, cvPwU msL¨vi †hvMdj hw` 30 nq, Z‡e †Kejgv Î c_g Pvi wU msL¨v ¯vaxbfv‡e cQ›` Kiv hvq Ges cÂg msL¨wUi cQ›` wbqwšZ n‡q c‡o| aiv hvK, 9, 6, 5 Ges 4 n‡jv †mB Pvi wU msL¨v| cvPwU msL¨vi †hvMdj 30 n‡Z n‡j cÂg msL¨wU‡K Aek¨B 6 n‡Z n‡e, Ab¨ †Kvb msL¨v cQ›` Kivi ¯vaxbZv Avgv‡`i †bB| Avgv‡`i cQ›`wU c_g Pvi wU msL¨vi wbevP‡bi gva¨‡g wbqwšZ n‡q c‡o‡Q|

t-web¨v‡mi †¶‡Î ¯vaxbZvi gvÎv n‡jv bgybvi AvKvi †_‡K †h KqwU civgvb cv°jb Ki‡Z n‡e Zvi we‡qvMdj| hw` n=bgybvi AvKvi nq Ges k=cv°wjZ civgvb ev ¯vaxb agybK nq, Z‡e n−k n‡jv ¯vaxbZvi gvÎvi msL¨v| Avgv‡`i D`vni‡Yi †¶‡Î cvPwU msL¨vi GKwU bgybvq (n = 5) †h‡nZz †kl msL¨wUi wbevPb mxgve× n‡q c‡o‡Q, †m‡nZz k = 1| AZGe, ¯vaxbZvi gvÎv n‡jv (n−k)=(5−1)=4 | hw` I mvavi YZt ¯vaxbZvi gvÎv‡K ms¶ß fv‡e df e‡j cKvk Kiv nq, GwU gj¨vZt MvK eYgvjvi ν (nu) A¶ iwU w`‡q mvs‡KwZKfv‡e wPwýZ Kiv nq|

t-web¨v‡mi ˆewkó¨mgn- (Properties of t-Distribution)

t-web¨v‡mi KZ„‡jv „ i¶Zc¥ˆewkó¨ i‡q‡Q| ‡m„‡jv n‡jv:

1. t-web¨vm ¯vfweK web¨v‡mi gZB –α †_‡K +α chš we¯Z|
2. GwU ¯vfweK web¨v‡mi gZB ¯vfweKfv‡e web¨¯, N›Uv AvKwZi Ges M‡oi Pvicv‡k mgi¡c|
3. ¯vaxbZvi gvÎvi msL¨vi cwieZ‡bi mv‡_ mv‡_ t-web¨v‡mi AvKwZi cwieZb N‡U| hw` ¯vaxbZvi gvÎvi msL¨v 120-Gi Kg nq, Z‡e t-web¨vm cvq ¯vfweK †iLvi i¡c avi Y K‡i| hw` df > 120 nq, Z‡e t-web¨vm Ges ¯vfweK web¨vm GKB i¡c †bq|
4. t-web¨vm ¯vfweK web¨v‡mi gZB GKwU Awe”Qb web¨vm| Z‡e ¯vfweK web¨v‡mi gZ mgM‡Ki Mo (μ) Ges mgM‡Ki cwigvZ e¨ea v‡bi (σ) Dci wbfikxj bq| GwU m¤úY¨fv‡e GKwU gv‡bi Øviv wbawiZ Ges †mwU n‡jv ¯vaxbZvi gvÎv|

5. t-wZwiZ Aš¿fz©³ Z_¨ ... t-web¨vm AwaKZi fv‡e AbwZmPvj| A_vr, ¯^vfweK web¨v‡mi Zzjbvq t-web¨vm †K‡›`ª Kg mPvj Ges `yÑc‡š¿ wKQzUv DPz(wPÎ 9.4.1 `óe¨)|

6. ¯^vfweK web¨v‡mi Zzjbvq t-web¨vm AwaKZi we¯^wZ©Zmµúb|

GKwU bgybvi Rb¨ AbygvKí cix¶v (Hypothesis Testing for a Single Sample)

Avgiv Av‡MB e‡jwQ †h, hLb bgybvi AvKvi †QvU nq Ges mgM‡Ki cwiwgZ e¨eavbwU ARvbv _v‡K ZLb Avgiv t-web¨vm e¨envi K‡i AbygvKí cix¶v K‡i _wK| A‡bK Kvi‡YB mgM‡Ki cwiwgZ e¨eavb ARvbv _v‡K| KviY, cwimsL¨vb eÿ‡iv, †emiKvix cwZôvb Ges msev` gva¨g¸‡jv †h mKj Z_¨ cKvk K‡i †m¸‡jv‡Z cvqkt Mo ev ga¨gvi gZ cwimsL¨v‡bi D‡jL _v‡K Ges AwaKvsk †¶‡Î cwiwgZ e¨eav‡bi D‡jL _v‡K bv| G ai‡Yi cwiw¯wZ‡Z mgM‡Ki cwiwgZ e¨eavb ($\sigma_{\bar{x}}$) cv°j‡bi Rb¨ bgybv †_‡K cvß cwiwgZ e¨eavb (s) e¨envi Kiv n‡q _v‡K| cv°wjZ $\sigma_{\bar{x}}$ wbY‡qi m~ÎwU n‡jv:

$$\hat{\sigma}_{\bar{x}} = \frac{s}{\sqrt{n-1}} = s_{\bar{x}}$$

‡hLv‡b, $\hat{\sigma}_{\bar{x}}$ = mgM‡Ki cv°wjZ cwiwgZ e¨eavb

s = bgybvi cwiwgZ e¨eavb

n = bgybvi AvKvi

$s_{\bar{x}}$ = bgybv †_‡K cv°wjZ cwiwgZ e¨eavb

ejvr bgybvi Rb¨ cwimsL¨vbMZ cix¶vi †¶‡Î †h avc¸‡jv AbymiY Kiv nq †QvU bgybvi Rb¨I GKB avc AbymiY Ki‡Z nq| †hgb, AbygvK wbgvY, bgybv web¨vm wbevPb, Zvrch gvÎv I msKU GjvKv wbaviY, AbygvKí cix¶v wbYq Ges wm×vš MnY| GKwU D`vniY †`qv hvK| aiv hvK, KvhKi wk¶v`vb c×wZi Dci M‡elYvi Ask wnmv‡e ^ØePwqZfv‡e GKwU kn‡ii PZz_© †kYxi 26 Rb QvÎ-QvÎx‡K wbevPb K‡i GKwU AvaybK hš¿ e¨env‡ii gva¨‡g wk¶v`vb Kiv n‡jv| ewlK cix¶vi mgq kn‡ii mKj PZz_© †kYxi QvÎ-QvÎx GKwU mevZ¥K cix¶vq Ask †bevi ci †`Lv †Mj †h, hv‡`i AvaybK hš¿wUi gva¨‡g wk¶v`vb Kiv n‡q‡Q Zv‡`i cvß Mo b¤i n‡jv 81 Ges hv‡`i mbvZb c×wZ‡Z wk¶v`vb Kiv n‡q‡Q Zv‡`i Mo n‡q‡Q 75| djvd‡ji GB Zvdvr wK hš¿wUi KvhKvixZv‡K cgvY K‡i? 26 R‡bi †QvU `jwUi Mo †ekx n‡q‡Q wK hš¿wUi Kvi‡Y, bwK N‡U‡Q †QvU `jwU GKwU fv‡jv `j e‡j, bwK NUbwU N‡U‡Q ^Øeµ‡g? G †¶‡Î mgM‡Ki cwiwgZ e¨eavbwU Avgv‡`i ARvbv Z‡e bgybv mgMKwU n‡jv 16|

mgM‡Ki Mo †_‡K bgybvi M‡oi g‡a¨ †h cv_©K¨ n‡q‡Q Zv wK cKZ, bwK ^Øeµ‡g N‡U‡Q Zv cix¶vi Rb¨ Abygvb wnmv‡e Avgv‡`i AbygvKí `yÕwU n‡e wbgiƒc:

bwj¯ AbygvKí, $H_0 : \mu_0 = \mu$

weKí AbygvKí, $H_1 : \mu_0 \neq \mu$

‡h‡nZzAvgv‡`i bgybvi AvKviwU Lye †QvU Ges †h‡nZzmgM‡Ki cwiwgZ e¨eavb Avgv‡`i ARvbv, †m‡nZzAvgiv Rwb †h, bgybv web¨vm wnmv‡e t-web¨vm‡K e¨envi Ki‡Z n‡e| Avgiv GI Rwb †h, ¯^vaxbZvi gvÎvi Dci wfwË K‡i t-web¨v‡mi AvKwZ cwieZb N‡U| mwVK t-web¨vmwU wbevP‡bi R‡b¨ ZvB Avgv‡`i mswkó ¯^vaxbZvi gvÎvi w`‡K `„wó wbe× Ki‡Z n‡e| Avgv‡`i D`vni‡Yi †¶‡Î ¯^vaxbZvi gvÎvwU n‡jv, $df = (n-1) = (26-1) = 25$ | AZGe,

Zvrch gvÎv I msKU GjvKv wbevP‡bi Rb¨ Avgiv 25 ¯vaxbZvi gvÎvq t-web¨vmwU e¨envi Ki‡ev| cwimsL¨vbMZ Zvrch ci¤¶vi Rb¨ Avgiv 0.05 Zvrch gvÎv wbaviY K‡iwQ| GLb ci¤¶v cwimsL¨vbwU wbYq Ki‡Z n‡e|

GKwU bgybvi Rb¨ Abykí ci¤¶vi †¶‡Î ci¤¶v cwimsL¨vb wbY‡qi myiwU n‡jv:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

†hLv‡b, t = ch‡e¶YKZ.t-Gi gvb

$\bar{x}$ = bgybvi Mo

μ = mgMK Mo

$\frac{s}{\sqrt{n-1}}$ = bgybvi †_‡K cv°wjZ cwigvZ e¨eavb

Gevi cvß gvb¸‡jv m¤Î c‡qvM Ki‡j Avgiv ch‡e¶YKZ.t-cwimsL¨v‡bi gvbwU cvB|

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$

$$= \frac{81 - 75}{\frac{16}{\sqrt{26-1}}}$$

$$= \frac{6}{\frac{16}{\sqrt{25}}}$$

$$= \frac{6}{\frac{16}{5}}$$

$$= \frac{6}{3.2}$$

$$= 1.88$$

$\therefore t = 1.88, \quad p \geq 0.05$

ch‡e¶YKZ.t-gvbwU 0.05 Zvrch gvÎvq cwimsL¨vbMZ fv‡e Zvrchc¥wKbv Zv hvPvB Ki‡Z n‡j Avgv‡`i 0.05 Zvrch gvÎvq t-web¨v‡mi ZwËK gvbwU Rvb‡Z n‡e| t-web¨v‡mi ZwËK gv‡bi mviwY †_‡K Avgiv Zv †c‡Z cwi| t-mviwYi c_g ¯‡¤ ¯vaxbZvi gvÎvi D‡jL _v‡K Ges Ab¨vb¨ ¯‡¤ wewfb Zvrch gvÎvq t-web¨v‡mi ZwËK gv‡bi D‡jL _v‡K| t-web¨v‡mi cÖkvmb mviwYwU GLv‡b Dc¯vcb Kiv m¤e bq, Z‡e GwU‡K †evSvi Rb¨ Gi GKwU msw¶ß fvlv wb‡g Dc¯vcb Kiv n‡jv| m¤úwK mviwYwU cwik‡ói mviwY cwi-3 -G Dc¯wcZ n‡q‡Q|

mviwY 9.4.1: t-webªv‡mi ZwËK gv‡bi mviwYi msw¶ß fvi¨

¯^vaxbZvi gvÎv	GK-cwšK cix¶vi Rb¨ Zvrch gvÎv (α)					
	0.10	**0.05**	**0.025**	**0.01**	**0.005**	**0.0005**
	wØ-cwšK cix¶vi Rb¨ Zvrch gvÎv (α)					
	0.20	**0.10**	**0.05**	**0.02**	**0.01**	**0.001**
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.859
6	1.440	1.943	2.447	3.143	3.707	5.959
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∝	1.282	1.645	1.960	2.326	2.576	3.291

mviwY‡Z †`Lv hv‡"Q †h, wØ-cwšK cix¶vq 0.05 Avjdv gvÎv Ges 25 ¯^vaxbZvi gvÎvq t-ZwËK gvbwU n‡jv 2.060| cix¶v cwimsL¨vb wbYq K‡i Avgiv ch©‡e¶YKZ.gvbwU †c‡qwQ 1.88| wm×v‡šÍi wbqgwU Abymvi Avgiv †`Lv hvq †h, ch©‡e¶YKZ.gvbwU ZwËK gv‡bi †P‡q †QvU nIqvq Avgiv bvw¯Í AbyKí wU‡K cZ¨vL¨vb Ki‡Z e¨_© n‡qwQ| A_©vr, 0.05 Avjdv gvÎvq Avgiv bvw¯Í AbyKí wU‡K MnY K‡iwQ| KviY,

| t-ch©‡e¶YKZ.gvb = 1.88 | < t-ZwËK gvb = 2.060

Avgv‡`i D`vni‡Yi †cw¶‡Z Avgiv ej‡Z cwi †h, 26 Rb PZz_© †kYxi QvÎ-QvÎxi evwIK cix¶vi djvd‡ji Mo (81) Ges kn‡ii mKj PZz_© †kYxi QvÎ-QvÎx‡`i djvd‡ji M‡oi (75) g‡a¨ †h cv_©K¨wU cvIqv †M‡Q Zv cKZ.bq, †mB cv_©K¨wU cvIqv †M‡Q GKwU ˆØe NUbv wnmv‡e| Ab¨ K_vq, cv_©K¨wU wk¶v`v‡bi AvaywbK hš¿ e¨env‡ii dj bq| m¤¢eZt 26 R‡bi †QvU `jwU cKwZMZ fv‡eB GKwU fv‡jv QvÎ-QvÎxi `j wQj|

`yªU bgybvi g‡a¨ cv_©‡K¨i Rb¨ AbyKí cix¶v (Hypothesis Testing for the Difference between Two Samples)

aiv hvK, GKRb M‡elK wewfb †ckvRxex‡`i g‡a¨ Kv‡Ri cwZ Abxnv cwigv‡ci Rb¨ GKwU cwZôvb †_‡K `yªU †QvU †QvU bgybv wbevPb Ki‡jb| GKwU bgybv n‡jv D"Pc`¯^ KgKZv‡`i ga¨ †_‡K Ges Ab¨wU n‡jv wbgc`¯^ KgKZv‡`i ga¨ †_‡K| GB `yªU †ckvRxex †MvôxЇK wbevPb Kivi †cQ‡b M‡el‡Ki ZwËK wfwËU n‡jv †h, †ckvMZ ghv`v, mvgvwRK gh©v`v Ges mš¿wói gvÎv, hv GKRb e¨w³i Kv‡Ri cwZ AvMn ev Abxnvi gvÎvi mv‡_ m¤úwKZ| †h‡nZz gvbyli Pwn`v Ges mš¿wói gvÎvi g‡a¨ mxgv‡iLv Uvbv m¤¢e bq, †m‡nZzcwZwU ‡ckvRxex †Mvôxi g‡a¨ wKQzbv wKQzAmš¿wó we`¨gvb _v‡K hvi d‡j Kv‡Ri cwZ Abxnv †_‡K GK ai‡Yi wew"QbZv‡eva ˆZix nq| Z‡e M‡elK GwUI Abygvb Ki‡jb †h, D"Pc`¯^ KgKZv‡`i Zzjbvq wbgc`¯^ KgKZv‡`i g‡a¨ wew"QbZv‡ev‡ai gvÎvwU †ekx|

Ab¨ K_vq, M‡elK †ckvi wfwË‡Z we‡”QbZv‡eva‡K e¨vL¨v Ki‡Z PvB‡Qb| GB j‡¶¨ wZwb ˆØePwqZfv‡e 12 Rb D”Pc`¯ KgKZv Ges 13 Rb wbgc`¯ KgPvixi bgybv wb‡q †`L‡jb †h, D”Pc`¯ KgKZv‡`i we‡”QbZvi gvÎvi Mo n‡q‡Q 50, hvi cwigvZ e¨eavb 19.54 Ges wbgc`¯ KgPvix‡`i we‡”QbZvi gvÎvi Mo n‡q‡Q 57.69, hvi cwigvZ e¨eavb 14.23| GB DcvË †_‡K M‡elKi GKwU Abygvb AvcvZt`wó‡Z mZ¨ e‡j cgvwYZ n‡q‡Q| A_vr, D”Pc`¯‡`i Zzjbvq wbgc`¯‡`i we‡”QbZvi gvÎv 7.69 †ekx n‡q‡Q| wKš¯vfwweKfv‡e GKwU ck G‡m hvq †h, GB cv_K¨wU wK ev¯‡e we`¨gvb, bwK GwU GKwU ˆØe NUbv? GB c‡ki Reve †c‡Z n‡j Abykxí cix¶v K‡i †`L‡Z n‡e| †h‡nZzAbygvbwU‡Z mybw`ófv‡e ewYZ n‡q‡Q †h, D”Pc`¯‡`i Zzjbvq wbgc`¯‡`i we‡”QbZvi gvÎv †ekx, †m‡nZzAbygv‡bi g‡a¨ GKwU w`K wb‡`kbv i‡q‡Q| Kv‡RB, Abykxí cix¶wU GKwU GK-cwšK cix¶v n‡e| AZGe, cix¶vi Abykxí `ÿwU n‡jv:

bw¯ Abykxí, $H_0 : \mu_1 = \mu_2$

weKí Abykxí, $H_1 : \mu_1 < \mu_2$

‡hLv‡b, μ_1 = D”Pc`¯ KgKZv‡`i Mo we‡”QbZvi gvÎv

μ_2 = wbgc`¯ KgKZv‡`i Mo we‡”QbZvi gvÎv

Abykxí cix¶vi Rb¨ α gvÎv wbaviY Kiv n‡q‡Q 0.05 Zvrch gvÎvq| Gevi cix¶v cwimsL¨vbwU wbYq Kiv hvK| `ÿwU bgybvi g‡a¨ cv_‡K¨i Rb¨ t-cix¶v cimsL¨vb wbY‡qi mÎwU n‡jv:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{n_1 + n_2}{(n_1)(n_2)}}}$$

‡hLv‡b, t = t-weeb¨v‡mi ch‡e¶YKZ.gvb

$\bar{x}_1$ = D”Pc`¯ KgKZv‡`i Mo we‡”QbZvi gvÎv

$\bar{x}_2$ = wbgc`¯ KgKZv‡`i Mo we‡”QbZvi gvÎv

n_1 = D”Pc`¯ KgKZv‡`i bgybvi AvKvi

n_2 = wbgc`¯ KgKZv‡`i bgybvi AvKvi

s_1^2 = D”Pc`¯ KgKZv‡`i bgybvi ‡f`v¼

s_2^2 = wbgc`¯ KgKZv‡`i bgybvi ‡f`v¼

$n_1 + n_2 - 2$ = `ÿwU bgybvi ¯vaxbZvi gvÎv

gvb¸‡jv m‡Î ewm‡q Avgiv ch‡e¶YKZ.t-Gi gvbwU †c‡Z cwi|

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} \sqrt{\frac{n_1 + n_2}{(n_1)(n_2)}}}$$

$$= \frac{50 - 57.69}{\sqrt{\frac{(12)(19.54)^2 + (13)(14.23)^2}{12+13-2}}\sqrt{\frac{12+13}{(12)(13)}}}$$

$$= \frac{-7.69}{\sqrt{\frac{(12)(381.8)+(13)(202.49)}{25-2}}\sqrt{\frac{25}{156}}}$$

$$= \frac{-7.69}{\sqrt{\frac{4581.6+2632.37}{23}}\sqrt{0.1603}}$$

$$= \frac{-7.69}{\sqrt{\frac{7213.97}{23}}(0.40)}$$

$$= \frac{-7.69}{\sqrt{313.65}(0.40)}$$

$$= \frac{-7.69}{(17.71)(0.40)}$$

$$= \frac{-7.69}{7.084} = \frac{\text{`ÿvU M‡oi g‡a¨ cv_K¨}}{\text{cv_‡K¨i cwigZ åwš–}}$$

$$= -1.085$$

$\therefore \quad t = -1.085, \quad p < 0.05$

Avgv‡`i wbYxZ t-gvbwU n‡jv –1.085| GB gvbwU 0.05 gvÎvq cwimsL¨vbMZfv‡e Zvrchc¥K wKbv Zv wbaviY Ki‡Z n‡j Avgv‡`i t-web¨vm mviwYwU ch‡e¶Y Ki‡Z n‡e| †h‡nZzAvgv‡`i mvwYi ÔGK-cwšK cix¶vi Rb¨ Zvrch gvÎvi Õ jvBbwU †`L‡Z n‡e| Avgiv wm×vš wb‡qwQ †h, kZKiv cvÐwU †¶‡Î fyj n‡jI Zv MnY Ki‡ev| A_vr, Avgv‡`i Zvrch gvÎv n‡jv $\alpha = 0.05$| AZGe, 0.05 gvÎvi ¯¤wU a‡i bx‡P bvg‡Z bvg‡Z c_g ¯‡¤ Dwjwl‡Z 23 msL¨wU‡Z wM‡q _vg‡Z n‡e, †Kbbv 23 n‡jv Avgv‡`i ¯vaxbZvi gvÎvi msL¨v| †h RvqMwU‡Z `ÿvU ¯¤ G‡m wg‡j‡Q †mLv‡b Avgiv †`wL †h, msL¨wU n‡jv 1.714| GwUB n‡jv 0.05 Zvrch gvÎvq t-web¨v‡mi ZwËK gvb| Gi A_ n‡jv †h, GKwU GK-cwšK cix¶vq `ÿvU M‡oi g‡a¨ cv_K¨wU 0.05 Avjdv gvÎvq cwimsL¨vbMZfv‡e Zvrchc¥K n‡Z n‡j Mo `ÿvUi cv_K¨‡K cv_‡K¨i cwigZ åwši 1.714 ¸Y ev Zvi †P‡q eo n‡Z n‡e| mnR K‡i ejv hvq †h, ch‡e¶YKZ.t-gvbwU ZwËK t-gv‡bi mgvb ev Zvi †P‡q eo n‡j Avgiv `ÿvU M‡oi cv_K¨‡K Zvrchc¥K ej‡ev Ges †m †¶‡Î Avgiv Avgv‡`i bw¯ AbykÍ‡K cZ¨vL¨vb Ki‡ev| AZGe, †h‡nZzAvgv‡`i ch‡e¶YKZ.gvbwU n‡q‡Q $|1.085|$, hv ZwËK t-gvb 1.714-Gi ‡P‡q †QvU (wPý‡K D‡c¶v K‡i), †m‡nZzAvgiv Dcmsnvi Uvb‡Z cwi †h, Mo `ÿvUi g‡a¨ cv_K¨wU cwimsL¨vbMZfv‡e Zvrchc¥K bq| A_vr, †ckv wew”QbZvi gvÎv‡K e¨vL¨v K‡i bv|

mn-m¤úwKZ M‡oi Rb¨ t-cixÿv (t-Test for Correlated Means)

G chš— Avgiv t-web¨vm e¨enviK‡i †h mKj AbygvbKí cixÿv Av‡jvPbv K‡iwQ †m mKj cixÿvi †¶‡Î Avgiv Abygvb K‡iwQ †h, bgybvqbwU `yÕwU ¯^vaxb mgMK †_‡K ˆ`ePqbZfv‡e wbev©Pb Kiv n‡q‡Q| wKš' A‡bK mgq Avgv‡`i wbfikxj `yÕwU †MvôxiI ga¨ †_‡K bgybv wb‡q M‡oi g‡a¨ cv_©‡K¨i AbygvbKí cixÿv Ki‡Z nq| bgybv `yÕwU wbfikxj nq GB Kvi‡Y †h, †m¸‡jv ‡Rvo-eva Ges GKwU bgybvi cwZwU ch©‡e¶Y wØZxq bgybvi we‡kl †Kvb ch©‡e¶‡Yi mv‡_ m¤úKhy³| G Kvi‡Y t-cixÿv‡K †Rvov evav t-cixÿv ejv nq|

hw` `yÕwU bgybv ci¯úi wbfikxj nq Z‡e Zv‡`i GKK¸‡jv GKB n‡Z n‡e Ges GKwU †Rvovi mv‡_ msw¯ó `yÕwU cwigvc cvi¯úwiKfv‡e m¤úKhy³ n‡Z n‡e| GwU A‡bKUv Ôc‡e Ges c‡iÕ M‡elYvi gZ| †hgb, `yÕwU cwk¶Y c×wZi g‡a¨ Zzjbv Ki‡Z n‡j `yÕwU c×wZi gva¨‡g cvV Mo gv‡bi wfwË‡Z Zzjbv Ki‡Z n‡e| Gi Rb¨ cwk¶Yv_©x‡`i `yÕwU mgvb fv‡M fvM K‡i cÖwZwU `j‡K wfbœ c×wZ‡Z cwk¶Y w`‡Z n‡e| GKwU wbw`©ó mgq c‡i `yÕwU `‡ji KZ©KvhZvi gv‡bi Dci wfwË K‡i Zzjbv Ki‡Z n‡e|

welqwU †evSvi Rb¨ Avgiv wk¶v`v‡b AvaywbK h‡š¿i e¨enva‡ii D`vniYwU e¨envi Ki‡Z cvwi| ˆ`ePqbZfv‡e wbe©vPZ 18 Rb ¯œvZK QvÎ-QvÎx‡K Bs‡iRx fvlvi Dci GKwU mevZ©K cixÿv †bevi ci Zv‡`i cixÿvq cÖvß b¤^‡ii wfwË‡Z 9 †Rvov QvÎ-QvÎxi GKwU `j MVb Kiv nj| Gevi cÖwZ †Rvov †_‡K GKRb‡K `j 1 Ges Ab¨Rb‡K `j 2-G †djv nj| `j 1-†K wbqwgZ cvV¨µ‡gi cvkvcvwk AvaywbK h‡š¿i gva¨‡g GK eQi cwk¶Y †`qv nj Ges `j 2-†K ïaygvÎ wbqwgZ cvV¨µ‡gi gva¨‡g cwk¶Y †`qv n‡jv| cwk¶Y †`evi ci `yÕwU `‡jiB GKwU mevZ©K cixÿv †bqv n‡jv hvi djvdj mviwY 9.4.2-G Dc¯’vcb Kiv n‡q‡Q|

mviwY 9.4.2: `yÕwU `‡ji cixÿvq cÖvß b¤^i I Zv‡`i cv_©‡K¨i wefvRb

‡Rvov	`j 1	`j 2	D	$(D-\overline{D})$	$(D-\overline{D})^2$
1	130	123	7	1.44	2.0736
2	131	125	6	0.44	0.1936
3	120	112	8	2.44	5.9536
4	121	125	-4	-9.56	91.3936
5	111	113	-2	-7.56	57.1536
6	118	115	3	-2.56	6.5536
7	100	91	9	3.44	11.8336
8	102	90	12	6.44	41.4736
9	98	47	11	5.11	29.5936
‡gvU	**1031**	**981**	**50**		**246.2224**

DcvË †_‡K †`Lv hvq †h, c_g `‡ji b¤^‡ii Mo n‡q‡Q 114.55 Ges wØZxq `‡ji Mo n‡q‡Q 109.00| AvaywbK h‡š¿i gva¨‡g cwkwÿZ QvÎ-QvÎxiv wbqwgZ cvV¨µ‡gi gva¨‡g cwkwÿZ QvÎ-QvÎx‡`i †P‡q M‡o †ekx b¤^i †c‡q‡Q| GB cv_©K¨wU wK Zvrchc~Y©, bwK Zv ˆ`eµ‡g N‡U‡Q? `yÕwU `‡ji g‡a¨Kvi cv_©K¨wU cwimsL¨vbMZ cixÿv Kivi ciB †Kejgvθ Avgiv GB c‡ki Reve †c‡Z cvwi|

‡m †¶‡Î Avgv‡`i AbygZ AbyK í `ª̀ÿUi Awfe¨w³wU wKQzUv wfb n‡e| mvaviYfv‡e, ¯^vaxb bgybvwU cix¶vi †¶‡Î AbyK í `ª̀ÿU nq wbgi|c:

bvw¯ AbyK í $H_0 : \mu_1 = \mu_2$

weKí AbyK í $H_1 : \mu_1 \neq \mu_2$

wbfikxj bgybv cix¶vi †¶‡Î AbygZ AbyK í `ª̀ÿU n‡e wbgi|c:

bvw¯ AbyK í $H_0 : \bar{x}_{\bar{D}} = 0$

weKí AbyK í $H_1 : \bar{x}_{\bar{D}} \neq 0$

‡hLv‡b, $\bar{x}_{\bar{D}}$ = cwZ †RvoV ch‡e¶‡Yi g‡a¨ cv_‡K¨i Mo

†h‡nZzAvgv‡`i D`vni‡Y 9 †RvoV ch‡e¶Y i‡q‡Q, †m‡nZzAvgv‡`i ¯^vaxbZvi gvÎv n‡jv (n – 1) = (9 – 1) = 8, Ges Avgiv 0.05 Avjdv gvÎvq cix¶v cwiPvjZ Ki‡ev| GB cix¶v cwimsL¨vb wbY‡qi m~ÎwU n‡jv:

$$t = \frac{\bar{D}}{s_{\bar{D}}} = \frac{\bar{D}}{\frac{\hat{s}_{\bar{D}}}{\sqrt{n-1}}}$$

‡hLv‡b, $\bar{D}$ = cwZ ‡RvoV ch‡e¶‡Yi cv_‡K¨i Mo

$$s_{\bar{D}} = \frac{s_{\bar{D}}}{\sqrt{n}} = \text{cv\_‡K¨i cwigZ åwši cv°jb}$$

$s_{\bar{D}} = \frac{s_{\bar{D}}}{\sqrt{n}}$ ‡K wbgwjwLZ Dcv‡q wbYq Kiv hvq,

$$s_{\bar{D}} = \frac{s_{\bar{D}}}{\sqrt{n}} = \frac{\sqrt{\frac{\sum\left(D-\bar{D}\right)^2}{n-1}}}{\sqrt{n}}$$

cix¶v cwimsL¨vb wbYq Kivi Rb¨ c_‡g c‡qvRbxq gvb¸‡jv wbYq K‡i wb‡Z n‡e| †hgb,

$$\bar{x}_1 = \frac{1031}{9} = 114.55$$

$$\bar{x}_2 = \frac{981}{9} = 109.00$$

$$\bar{x}_1 - \bar{x}_2 = 114.55 - 109.00 = 5.55$$

$$\bar{D} = \frac{50}{9} = 5.56$$

$$s_{\bar{D}} = \frac{\sqrt{\frac{246.2224}{8}}}{\sqrt{9}} = \frac{\sqrt{30.7778}}{3} = \frac{5.55}{3} = 1.85$$

Gevi gvb¸‡jv m‡Î emv‡q Avgiv t-gvbwU cvB,

$$t = \frac{\overline{D}}{s_{\overline{D}}}$$

$$= \frac{5.56}{1.85}$$

$$= 3.01$$

$$\therefore \quad t = 3.01$$

t-mvwiwY ‡_‡K †`Lv hvq †h, 8 ¯^vaxbZvi gvÎv Ges 0.05 Avjdv gvÎvq ZvwË¡K t-gvbwU n‡jv 2.306| †h‡nZzAvgv‡`i ch©‡e¶YKZ.t-gvbwU ZvwË¡K gvb‡K AwZµg K‡i wM‡q‡Q, †m‡nZz Avgiv bvw¯Í Abygvb cZ¨vL¨vb Ki‡Z cvwi Ges Dcmsnv‡i ej‡Z cvwi †h, Avaybv K h‡šÎi gva¨‡g cwk¶Y †h cv_©K¨ m„wó n‡q‡Q Zv 0.05 Zvrch© gvÎv Ges 8 ¯^vaxbZvi gvÎvq cwimsL¨vbMZ fv‡e Zvrch©c~Y©|

mvivsk

`yU M‡oi g‡a¨ Zzjbvi Rb¨ t-cix¶v GKwU Rbwcq cwimsL¨vb c×wZ| bgybvi AvKv‡ii Dci wfwË K‡i cwZwU t-web¨v‡mi AvKvi.wfb n‡q _v‡K| t-web¨v‡mi mv‡_ m¤úwKZ GKwU ¸iæZc~Y© cZ¨q n‡"Q ¯^vaxbZvi gvÎv| ¯^vaxbZvi gvÎv g~j¨Zt Dcv‡Ëi Dci wbqšÿ‡K cwZwbwaZ K‡i| Av‡iv mybxófv‡e ejv hvq †h, t-web¨v‡mi †¶‡Î ¯^vaxbZvi gvÎv n‡jv bgybvi AvKvi †_‡K †h KqwU civgvb cv°jb Ki‡Z nq Zvi we‡qvMdj| t-web¨v‡mi Ab¨Zg ˆewkó¨ n‡"Q ¯^vaxbZvi gvÎvi mv‡_ Gi m¤úK| ¯^vaxbZvi gvÎvi msL¨vi cwieZ‡bi mv‡_ mv‡_ t-wb¨v‡mi AvKwZ.i cwieZb N‡U| hw` ¯^vaxbZvi gvÎvi msL¨v 120-Gi †ekx nq, Z‡e t-wb¨vm Ges ¯^vfvweK wb¨vm GKB i|c avi Y Ki‡e|

cv‡WvËi gj¨vqb

^be¨w³K ck

mwVK DË‡i i cv‡k wUK (√) wPý w`b –

1| t-web¨v‡mi gva¨‡g ..... Abyk×í ci¤¶v cwiPvjbv Kiv n‡q _v‡K|

K. mgMK Mo m¤úwKZ
L. `ÿZv M‡oi g‡a¨ cv_K¨ m¤úwKZ
M. mn-m¤ú‡Ki mnM m¤úwKZ
N. Dc‡i i me KÓwU

2| t-web¨v‡mi †¶‡Î ¯vaxbZvi gvÎv n‡jv bgybvi AvKvi †_‡K †h KqwU civgvb cv°jb Ki‡Z n‡e Zvi:

K. †hvMdj
L. we‡qvMdj
M. ¸Ydj
N. fvMdj

3| t-web¨v‡mi mv‡_ m¤úwKZ ¸i“Zc¥cZ¨qwU n‡jv:

K. Zvrch gvÎv
L. msKU GjvKv
M. ¯vaxbZvi gvÎv
N. Dc‡i i ‡KvbwUB bq

msw¶ß ck

1| t-ci¤¶v wK?

2| ¯vaxbZvi gvÎv wK?

iPbvgj¨K ck

1| t-web¨v‡mi ^ewkó¨¸‡jv Av‡jvPbv Ki“b|

2| mn-m¤úwKZ M‡oi Rb¨ t-ci¤¶v ej‡Z wK †evSvq Zv D`vniYmn Av‡jvPbv Ki“b|

cvV - 5

cwimsL¨vbMZ cix¶v 3: χ^2 (KvB-eM) cix¶v

Statistical Test 3: χ^2 (Chi- square) Test

GB cvV †k‡l hv Rvbv hv‡e —

- χ^2 cix¶v wK
- Dcv‡Ëi mvgÄm¨Zvi χ^2 -cix¶v
- Awew”QbZvi Rb¨ B‡qUm-Gi ms‡kvab
- ¯^vZ‡š¿i χ^2-cix¶v
- χ^2 -cix¶v c‡qv‡Mi cÖwµqv

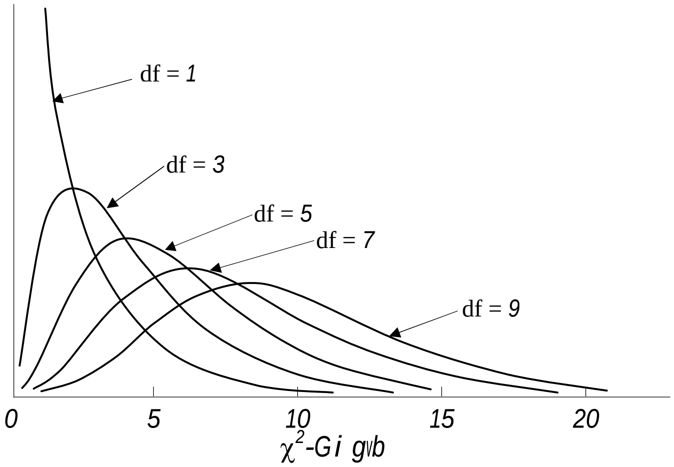
χ^2-cix¶v wK (What is χ^2 Test)?

cvV 2-G Avgiv †R‡bwQ †h, Abygvb cix¶v¸‡jv‡K mvaviYZt `yÕwU fv‡M fvM Kiv hvq: civwgwZK (parametric) Ges AciwgwZK (non-parametric)| civwgwZK cix¶vi D`vniY wnmv‡e cvV 3 Ges cvV 4-G z-cix¶v I t-cix¶v‡K we¯ÍvwiZ fv‡e Av‡jvPbv Kiv n‡q‡Q| Avgiv Rvwb †h, civwgwZK cix¶vi †¶‡Î ‡h mgMK †_‡K bgybv wbevPb Kiv nq †mB mgMK‡K ¯^vfvweKfv‡e web¨¯Í n‡Z nq, Abygvb¸‡jv‡K civgv‡bi †cwi‡¶‡Z ewYZ n‡Z nq Ges PjK¸‡jv‡K e¨wßgvb‡K ev AbycvZgvb‡K gvcvq cwigvcKZ.n‡Z nq| wKš‘ hLb G mKj kZ© c~iY Kiv m¤¢e nq bv ZLb AciwgwZK cix¶v cÖwZ Abymi‡Y Ki‡Z nq| AciwgwZK Abygvb cix¶vq mgMK m¤úwK© kZ©¸‡jv wkw_j Kiv nq| mgM‡Ki web¨vm m¤ú‡K© †Kvb Abygv‡bi c‡qvRb nq bv e‡j GB cix¶v¸‡jv‡K web¨vmgy³ cix¶v ejv nq| AciwgwZK cix¶v¸‡jv bvgmPK I µgmPK gvcvq cwigvcKZ.Pj‡Ki ‡¶‡Î c‡qvM Kiv n‡q _v‡K| AciwgwZK cix¶v¸‡jvi g‡a¨ me‡P‡q ‡ekx RbwcÖq cÖwZwU n‡jv χ^2 (KvBeM) cix¶v, hv χ^2-web¨v‡mi Dci cÖwZwôZ|

χ^2-web¨vm n‡jv GKwU Awew”Qb m¤¢vebv web¨vm hv abvZ¥K w`‡K 0 †_‡K Amxg gvb chš— we¯—Z| Gi †Kvb FYvZ¥K gvb †bB| KviY, $\chi^2$-gvbwU cZ¨wkZ Ges ch©‡e¶YKZ.NUbmsL¨vi cv_K¨‡K eM K‡i wbYxZ nq| ¯^vaxbZvi gvÎvi gv‡bi Dci $\chi^2$-web¨v‡mi AvKwZwU wbfi K‡i| cÖwZwU ¯^vaxbZvi gvÎvi gvb t-web¨v‡mi gZ GK GKwU wfb web¨v‡mi Rb¥ †`q| wPÎ 9.5.1-G wewfb ¯^vaxbZvi gvÎv gv‡bi wfwË‡Z K‡qKwU χ^2-web¨v‡mi D`vniY †`Lv‡bv n‡jv| hLb ¯^vaxbZvi gvÎvi gvb †QvU nq ZLb web¨v‡mi †iLwU Wvb w`‡K mPvj nq| hLb ¯^vaxbZvi gvÎvi gvb eo n‡Z _v‡K, ZLb web¨vmwU ax‡i ax‡i mgiƒc n‡Z _v‡K Ges ¯^vaxbZvi gvÎv 30-Gi AwaK n‡j †kl chš— χ^2-web¨vm ¯^vfvweK web¨v‡mi mwbKUeZx n‡Z _v‡K|

Abygvb cix¶v¸‡jv‡K mvaviYZt `yÕwU fv‡M fvM Kiv nq: civwgwZK Ges AciwgwZK|

χ^2-web¨vm n‡jv GKwU Awew”Qb m¤¢vebv web¨vm hv abvZ¥K w`‡K 0 †_‡K Amxg gvb chš— we¯—Z| Gi †Kvb FYvZ¥K gvb †bB|



wPÎ 9.5.1: wewfb ¯^vaxbZvi gvÎvq χ^2-web¨v‡mi wewfb AvKvZ

χ^2-cix¶v n‡jv me‡P‡q mnR Ges mewaK e¨eüZ Aciv‡gwÎK cix¶v| χ^2-Gi gvbwU cZ¨vkv I ch©‡e¶‡Yi g‡a¨ †h cv_K¨ Zvi cwigvY‡K eYbv K‡i| A_vr, ZË Ges ev¯eZvi g‡a¨ †h e¨eavb %Zix nq †mwU wK ch©‡e¶YKZ.gv‡bi mv‡_ Z‡Ëi mvgÄm¨Zvi Afv‡e n‡q‡Q, bwK ˆØeµ‡g N‡U‡Q, χ^2-cix¶vi gva¨‡g Avgiv Zv Rvb‡Z cwi| hw` χ^2-Gi gvb ÔOÕ nq Z‡e eŷ‡Z n‡e †h, cZ¨wkZ I ch©‡e¶YKZ.NUbmsL¨vi g‡a¨ ûeû wgj n‡q‡Q| χ^2-Gi gvb hZ eo n‡e cZ¨wkZ I ch©‡e¶YKZ.NUbmsL¨vi g‡a¨ e¨eavb ZZ †ekx i‡q‡Q e‡j a‡i wb‡Z n‡e|

Dcv‡Ëi mvgÄm¨Zvi χ^2-cix¶v (χ^2-Test for Goodness-of-Fit)

aiv hvK, 100Rb QvÎ-QvÎx‡K wRÁvmv Kiv n‡jv †h, e¨w³MZ mgm¨v †gvKv‡ejvq civg‡k©i j‡¶¨ mvnv‡h¨i Rb¨ Zviv c_‡g Kvi Kv‡Q hv‡e? QvÎ-QvÎx‡`i KvQ †_‡K cvß DË‡ii wb¤¥wmwU wbgœic:

hv‡`i Kv‡Q mvnvh¨ PvB‡e	NUbmsL¨v
QvÎ-QvÎx civgk© I wb‡`kbv †K›`ª	27
eÜz-evÜe	28
fvB‡evb ev AvZxq¯Rb	24
gvZv-wcZv	11
Kv‡iv Kv‡Q bq	10
‡gvU	**100**

GB Dcv‡Ëi wfwË‡Z Avgiv Rvb‡Z PvB †h, NUbmsL¨v¸‡jv wK wewfb ‡Mvôx‡Z ˆØePwqZfv‡e web¨¯ n‡q‡Q, bwK A‰ØePwqZfv‡e web¨¯ n‡q‡Q| GB †cwÿ‡Z Avgv‡`i AbyKí `yÕwU n‡e wbgœic:

bw¯ AbyKí $H_0 : f_0 = f_e$

weKí AbyKí $H_1 : f_0 \neq f_e$

‡hLv‡b, f_0 = ch©‡e¶YKZ.NUbmsL¨v

f_e = cZ¨wkZ NUbmsL¨v

bw¯ AbykíwUi A_ n‡jv †h, hw` mwZ¨Kvi A‡_ DËi ¸‡jv ˆØePwqZfv‡e wewfb †MvôxZ wef³ n‡q _v‡K Z‡e ch‡e¶YKZ.NUbmsL¨v¸‡jv cZ¨wkZ NUbmsL¨vi mgvb e‡j we‡ewPZ n‡Z cv‡i| Avi weKí AbykíwUi A_ n‡jv †h, ch‡e¶YKZ.I cZ¨wkZ NUbmsL¨v¸‡jvi g‡a¨ ev¯‡eB cv_K¨ i‡q‡Q| $\chi^2$-ci¤¶vq memgq `ªæai‡Yi NUbmsL¨vi c‡qvRb nq — ch‡e¶YKZ.NUbmsL¨v I cZ¨wkZ NUbmsL¨v| cZ¨wkZ NUbmsL¨v wbYxZ nq ch‡e¶‡Yi †gvU msL¨v‡K ch‡e¶Y `dvi †gvUmsL¨v w`‡q fvM K‡i| Avgv‡`i D`vniY Abykvmq †gvU ch‡e¶Y msL¨v n‡jv 100 Ges ch‡e¶Y `dvi (observation category) †gvU msL¨v n‡jv 5| AZGe, cwZwU `dvq cZ¨wkZ NUbmsL¨v n‡e 20| GLb ck n‡jv †h, ch‡e¶YKZ. NUbmsL¨v¸‡jv wK cZ¨wkZ NUbmsL¨v¸‡jv †_‡K Zvrchc¥fv‡e wfb? GwU cgvY Kivi Rb¨ Avgv‡`i χ^2-ci¤¶v cwimsL¨vb wbYq Ki‡Z n‡e| χ^2-ci¤¶v cwimsL¨vb wbY‡qi mvfvwU n‡jv:

$$\chi^2 = \sum \frac{(f_0 - f_e)^2}{f_e}$$

‡hLv‡b, χ^2 = ch‡e¶YKZ.χ^2-Gi gvb

f_0 = ch‡e¶YKZ.NUbmsL¨v

f_e = cZ¨wkZ NUbmsL¨v

$$\chi^2 = \frac{(27-20)^2}{20} + \frac{(28-20)^2}{20} + \frac{(24-20)^2}{20} + \frac{(11-20)^2}{20} + \frac{(10-20)^2}{20}$$

$$= \frac{(7)^2}{20} + \frac{(8)^2}{20} + \frac{(4)^2}{20} + \frac{(-9)^2}{20} + \frac{(-10)^2}{20}$$

$$= \frac{49}{20} + \frac{64}{20} + \frac{16}{20} + \frac{81}{20} + \frac{100}{20}$$

$$= 2.45 + 3.20 + 0.80 + 4.05 + 5.00$$

$$= 15.50$$

$\therefore$ wbYxZ $\chi^2 = 15.50$

cvß χ^2-gvbwUi Zvrch ci¤¶vi Rb¨ Avgv‡`i c_‡g ¯vaxbZvi gvÎv wbYq Ki‡Z n‡e| G †¶‡Î ¯vaxbZvi gvÎvi gvb wbY‡qi mvfvwU n‡jv,

df = k – 1

‡hLv‡b, k = ch‡e¶Y `dvi msL¨v

df = ¯vaxbZvi gvÎv

¯vaxbZvi gvÎvwU wbfi K‡i m¤ve¨ ch‡e¶Y `dvi msL¨vi Dci, ch‡e¶‡Yi †gvU msL¨vi Dci bq| AZGe, ¯vaxbZvi gvÎvi gvbwU n‡jv,

df = 5 – 1

= 4

¯vaxbZvi gvÎvi gvb wbY‡qi ci Avgv‡`i wm×vš wb‡Z n‡e †h, Zvrch gvÎvwU wK n‡e? Avgv‡`i D`vni‡Y cvß χ^2-gv‡bi cwimsL¨vbMZ Zvrch ci¤¶vi Rb¨ $\alpha = 0.05$ wbaviY Kiv

n‡jv| Gevi χ^2- webvm mvivwY‡Z 4 ¯vaxbZvi gvÎv Ges 0.05 Zvrch gvÎvi h_vh_ ZwËK gvbwUi w`‡K j¶¨ Ki‡Z n‡e| mvivwY 9.5.1-G $\chi^2$-webvm mvivwYi GKwU msw¶ß fvl¨ D`vniY wnmv‡e Dc¯vcb Kiv n‡jv| m¤úŠ³ mvivwYwU cwiwk‡ói mvivwY cwi-4 -G Dc¯wcZ n‡q‡Q|

mvivwY 9.5.1: χ^2-webvm mvivwYi msw¶ß fv‡l¨i Dc¯vcb

df	0.20	0.10	0.05	0.02	0.01
3	4.642	6.251	7.815	9.837	11.341
4	5.989	7.779	9.488	11.668	13.277
5	7.289	9.236	11.070	13.388	15.086
13	16.985	16.812	22.362	25.472	27.688
14	18.151	21.064	23.685	26.873	29.141
15	19.311	22.307	24.996	28.259	30.578
25	30.675	34.382	37.652	41.566	44.314
30	36.250	40.256	43.773	47.962	50.892

χ^2-webvm mvivwY‡Z j¶¨ Kiv hv‡”Q †h, 4 ¯vaxbZvi gvÎv Ges 0.05 Zvrch gvÎvq χ^2-webv‡mi ZwËK gvbwU n‡jv 9.49, hv Avgv‡`i ch‡e¶YKZ.gv‡bi †P‡q A‡bK †QvU| Avgiv Rwb †h, hLb wbawiZ ¯vaxbZvi gvÎv Ges Zvrch gvÎvq ch‡e¶YKZ.$\chi^2$-gvb ZwËK gvb‡K AwZµg K‡i hvq ZLb Avgiv bw¯ Abyký‡K cZ¨vL¨vb Kwi| AZGe, †mB wbqg Abyhvqx Avgv‡`i D`vni‡Y Abyhvq bw¯ Abyký‡K cZ¨vL¨vZ n‡e| Ggb wK †`Lv hvq †h, 0.01 m¤vebvi gvÎvq l bw¯ Abyký‡K cZ¨vL¨vZ nq| DcvË m¤úŠK Avgv‡`i wm×vš‹wU GB Dcmsnvi c`vb K‡i †h, ev¯‡e cwZwU ch‡e¶Y `dv mgvbfv‡e QvÎ-QvÎx‡`i Kv‡Q MnY‡hvM¨ bq| c‡Z¨‡Ki cQ›`i GKwU wfbZv i‡q‡Q|

GLv‡b GKwU welq g‡b ivLv c‡qvRb †h, Ab¨vb¨ cix¶vi gZ wbawiZ Avjdv gvÎv UvBc-1 åvši m¤vebvi gvb‡K cwZwbwaZ K‡i| hLb bw¯ Abyký‡K mZ¨ nq ZLb ch‡e¶YKZ. χ^2-gvbwU ZwËK gv‡bi †P‡q eo nq| ¯vaxbZvi gvÎvi msL¨v e„× Ki‡j χ^2-cix¶vi ¶gZv e„× cvq Ges ch‡e¶Yi `dv e„×i mv‡_ mv‡_ GKwU wg_¨v bw¯ Abyký‡K cZ¨vL¨v‡bi m¤vebv †e‡o hvq|

Awew”QbZvi Rb¨ B‡qUm-Gi ms‡kvab (Yates's Correction for Continuity)

χ^2-we‡klY e¨envi‡i †¶‡Î GKwU ¸i“Zc¥ welq g‡b ivLv c‡qvRb †h, Av`k cwiw¯wZ‡Z Dcwiwjwl‡Z m‡Ši gva¨‡g $\chi^2$-cwimsL¨vb wbY‡q ‡Kvb mgm¨v †bB| wKš hw` GKwU mvivwYi †h †Kvb GKwU N‡i (cell) cZ¨wkZ NUbmsL¨vi gvb 5-Gi †P‡q †QvU nq Zvn‡j χ^2-webv‡mi kZ j•NZ nq| G ai‡Yi cwiw¯wZ‡Z χ^2-gv‡bi ms‡kva‡bi c‡qvRb| KviY, AwfÁZvjä χ^2-gvbwU wew”Qb DcvË †_‡K Ges χ^2-webv‡mi ZwËK bgybv webvmwU GKwU Awew”Qb MwYwZK webvm †_‡K wbYxZ nevi d‡j GB `ÿwU webv‡mi g‡a¨ GKwU Zdvr ^Zix nq| GB Zdvrwu‡K Kgv‡bvi Rb¨ F. Yates GKwU ms‡kvab c¯ve K‡i‡Qb, hv Awew”QbZvi Rb¨ B‡qUm-Gi ms‡kvab e‡j cwiwPZ| msw¶ß fv‡e GwU‡K B‡qUm-Gi ms‡kvab ejv nq|

B‡qUm-Gi ms‡kvab wbY‡qi m¤ÚwU n‡jv:

$$\chi^2\text{- ms‡kwaZ} \quad = \quad \sum \frac{\left(\left|f_0 - f_e\right| - 0.5\right)^2}{f_e}$$

‡hLv‡b, f_0 = ch‡e¶YKZ.NUbmsL¨v

f_e = cZ¨wkZ NUbmsL¨v

$\left|f_0 - f_e\right|$ = ch‡e¶YKZ.Ges cZ¨wkZ NUbmsL¨vi cv_‡K¨i wPý ewRZ gvb

hLb GKwU Pj‡Ki g‡a¨ †Kejgv‡Î `ÿwU ch‡e¶Y `dv _v‡K Ges ¯vaxbZvi gv‡Îvi gvb 1 nq, ZLb B‡qUm-Gi ms‡kvab m¤Ú- c‡qvM K‡i χ^2-web¨v‡mi gvb wbYq Kiv c‡qvRb| wKš wKQz cwimsL¨vbwe` g‡b K‡ib †h, ‡h †Kvb 2×2 mvi wYi Rb¨ B‡qUm-Gi ms‡kvab c‡qvM Kiv c‡qvRb| Ab¨w`‡K, wKQzcwimsL¨vbwe` i‡q‡Qb hvwuw g‡b K‡ib †h, B‡qUm-Gi ms‡kvab $\chi^2$-Gi gvb‡K AwZms‡kvab K‡i _v‡K| Zvn‡j 2×2 mvi wYi †¶‡Î M‡elK wK wm×vš †b‡eb? †mwU wbfi K‡i Abykí cix¶vi †¶‡Î M‡elK KZUkzi ¶Ykxj n‡eb Zvi Dci| hw` M‡elK bw¯ Abykí‡K Lye †ekx cZ¨vL¨vb Ki‡Z bv Pvb Z‡e B‡qUm-Gi ms‡kvab e¨envi Acwinvh| B‡qUm-Gi ms‡kvab AwaKvsk †¶‡Î bw¯ Abykí‡K cZ¨vL¨vb Ki‡Z e¨_ K‡i| Avi hw` wZwb AwaKvsk †¶‡Î bw¯ Abykí‡K mn‡RB cZ¨vL¨vb Ki‡Z Pvb Zvn‡j wZwb B‡qUm-Gi ms‡kvab‡K AeÁv Ki‡Z cv‡ib| Z‡e G cwiw¯wZ‡Z mwVK wm×všwU n‡e Aciva wÎK cix¶vi wfb c×wZi Abyhvqx Kiv ev AwaKZi h_v_ c×wZ e¨envi K‡i we‡klY Kiv|

¯vZ‡š¨i χ^2-cix¶v (χ^2-Test of Independence)

Gi c‡e Avgiv χ^2-cix¶v cwimsL¨vb wbY‡qi gva¨‡g Dcv‡Ëi mvgÄm¨Zvi cix¶v K‡i wQ| †mwU GKgvÎx χ^2-cix¶v e‡j cwiwPZ| wKš A‡bK mgq Avgiv Rvb‡Z PvB †h, `ÿwU PjK ¯Zš (independent) wK bv| A_vr, †h `ÿwU PjK e¨envi K‡i contingency mvi wYwU ˆZix Kiv n‡q‡Q †mB PjK `ÿwU cvi¯úwiKfv‡e m¤úKhy³ wK bv| GwU †evSvi Rb¨ χ^2-cix¶vi †h c×wZwU e¨envi Kiv nq Zv‡K ¯vZ‡ši χ^2-cix¶v e‡j| GwU Avevi wØ-gvÎx χ^2-cix¶v bv‡gI cwiwPZ|

cix¶v cwiPvjbvi Rb¨ ˆØePwqZfv‡e GKwU bgybvi Dci DcvË msMn K‡i Zv GKwU mvi wY‡Z corss-classify K‡i Dc¯vcb Kiv nq| GB corss-classified mvi wY‡K contingency mvi wY e‡j| aiv hvK, `ÿwU PjK‡K h_vµ‡g A I B e‡j wPwýZ Kiv n‡jv Ges A Pj‡K r-wU ch‡e¶Y `dv Ges B Pj‡K c-wU ch‡e¶Y `dv i‡q‡Q| mvi wYi cwZwU N‡ii NUbmsL¨v‡K cell frequencies e‡j| cwZwU mwii †gvU NUbmsL¨v (R_i) Ges cwZwU ¯‡¤i †gvU NUbmsL¨v‡K (C_j) cwšq (marginal) NUbmsL¨v e‡j|

cwZwU N‡ii cZ¨wkZ NUbmsL¨v¸‡jv m¤vebv Z‡Ëi ¸Yb wbq‡gi (multiplicative rule of probability) wfwË‡Z wbYxZ nq| †hgb, hw` †Kvb PjK ev †kYxweb¨v‡mi `ÿwU gvb`Û ¯Zš ev ¯vaxb nq Z‡e Zv‡`i †hŠ_ m¤vebvwU mswkó `ÿwU cwšq m¤vebvi ¸Yd‡jvi mgvb n‡e| AZGe, ‡h †Kvb N‡ii cZ¨wkZ NUbmsL¨v wbY‡qi m¤ÚwU n‡e wbgi¡c:

$$E_{ij} = \frac{R_i C_j}{N}$$

†hLv‡b, E_{ij} = †h †Kvb N‡i i cZ¨wkZ NUbmsL¨v

R_i = ‡h †Kvb mvwi i †gvU NUbmsL¨v

C_j = †h †Kvb ¯‡¤i †gvU NUbmsL¨v

N = ‡gvU NUbmsL¨v

¯vZš¿¨i χ^2-cix¶v cwimsL¨vb wbY©‡qi m~Î wbæiƒc:

$$\chi^2 = \sum\sum \frac{(f_0 - f_e)^2}{f_e}$$

ev $$\chi^2 = \frac{(f_0 - f_e)^2}{f_e}$$

‡hLv‡b, f_0 = ch©‡e¶YKZ NUbmsL¨v

f_e = cZ¨wkZ NUbmsL¨v

¯vaxbZvi gvÎv wbY©‡qi m~Î wbæiƒc:

$$\nu = (r-1)(c-1)$$

†hLv‡b, ν = ¯vaxbZvi gvÎv

r = †gvU mvwi i msL¨v

c = †gvU ¯‡¤i msL¨v

GKwU D`vniY w`‡q welqwU fv‡jv K‡i eySv †bqv hvK| aiv hvK, GKwU M‡elYv cwZôvb niZv‡ji c‡¶/wec‡¶ RbM‡Yi gZvgZ hvPvB-Gi Rb¨ 600 R‡bi GKwU bgybv Rb‡Mvôxi Dci Rwic cwiPvjbv K‡i‡Q| c‡Z¨K‡K GKwU ck wRÁvmv Kiv n‡q‡Q †h, ÔAvcwb niZvj‡K cQ›` K‡ib wK bv?Õ DË‡ii wZbwU `dv i‡q‡Q: Ôn¨vÕ, ÔAwbwðZÕ I ÔbvÕ| †`Lv †Mj †h, 200 Rb DË‡i Ôn¨vÕ e‡j‡Qb, 220 Rb ÔAwbwðZÕ Ges 180 Rb e‡j‡Qb ÔbvÕ| GB ch©‡e¶YKZ. `dv¸‡jv GKwU †_‡K Av‡iKwU ¯Zš¿ wKbv Zv cix¶vi Rb¨ AbygvbÍ `uvwoU n‡jv:

bwj AbygvbÍ $H_0 : f_0 = f_e$

weKí AbygvbÍ $H_1 : f_0 \neq f_e$

AvcvZt`„wó‡Z †`‡L g‡b nq †h, gZvgZ¸‡jv mgvbfv‡e web¨¯ Ges mvgÄm¨Zvi χ^2-cix¶vq †`Lv hvq †h, 2 ¯vaxbZvi gvÎv Ges 0.05 Zvrch gvÎvq wZbwU ch©‡e¶Y `dvq cvß NUbmsL¨v †Kvb Zvrchc~Y© cv_©K¨ c`kb K‡i bv| wKš' hw` ch©‡e¶YKZ.`dv¸‡jv‡K we‡fb eq‡Mvôx‡Z wef³ K‡i cix¶v Kiv hvq Z‡e djvdjwU †ek AvKlYxq n‡q D‡V| wZbwU eq‡Mvôx‡Z wef³ K‡i DcvËmgn‡K mviwY 9.5.2-G Dc¯vcb Kiv n‡jv|

mvi wY 9.5.2: niZv‡ji c‡¶/wec‡¶ gZvg‡Zi Dci eqm wfwËK Dcv‡Ëi web¨vm

ch‡e¶YKZ NUbmsL¨v				
eq‡Mvôx	niZv‡ji c‡¶/wec‡¶ gZvgZ			
	nÖu	Awbw ðZ	bv	‡gvU
30 eQ‡ii †P‡q Kg	110	40	30	180
30-45 eQi	40	100	60	200
45 eQ‡ii †ekx	50	80	90	220
†gvU	**200**	**220**	**180**	**600**

cZ¨wkZ NUbmsL¨v				
eq‡Mvôx	niZv‡ji c‡¶/wec‡¶ gZvgZ			
	nÖu	Awbw ðZ	bv	‡gvU
30 eQ‡ii †P‡q Kg	60	66	54	180
30-45 eQi	66.67	73.33	60	200
45 eQ‡ii †ekx	73.33	80.67	66	220
†gvU	**200**	**220**	**180**	**600**

cZ¨wkZ NUbmsL¨v¸‡jv †hfv‡e wbYq Kiv n‡q‡Q:

$$E_{11} = \frac{R_1 C_1}{N} = \frac{180 \times 200}{600} = \frac{36000}{600} = 60$$

$$E_{21} = \frac{R_2 C_1}{N} = \frac{200 \times 200}{600} = \frac{40000}{600} = 66.67$$

$$E_{31} = \frac{R_3 C_1}{N} = \frac{220 \times 200}{600} = \frac{44000}{600} = 73.33$$

$$E_{12} = \frac{R_1 C_2}{N} = \frac{180 \times 220}{600} = \frac{39600}{600} = 66$$

$$E_{22} = \frac{R_2 C_2}{N} = \frac{200 \times 220}{600} = \frac{44000}{600} = 73.33$$

$$E_{32} = \frac{R_3 C_2}{N} = \frac{220 \times 220}{600} = \frac{48400}{600} = 80.67$$

$$E_{13} = \frac{R_1 C_3}{N} = \frac{180 \times 180}{600} = \frac{32400}{600} = 54$$

$$E_{23} = \frac{R_2 C_3}{N} = \frac{200 \times 180}{600} = \frac{36000}{600} = 60$$

$$E_{33} = \frac{R_3 C_3}{N} = \frac{220 \times 180}{600} = \frac{39600}{600} = 66$$

Gevi gvb¸‡jv m¤Î ewm‡q χ^2-Gi gvbwU wbYq Kiv †h‡Z cv‡i|

$$\chi^2 = \sum\sum \frac{(f_0 - f_e)^2}{f_e}$$

$$= \frac{(110-60)^2}{60} + \frac{(40-66)^2}{66} + \frac{(30-54)^2}{54} + \frac{(40-66.67)^2}{66.67} + \frac{(100-73.33)^2}{73.33} + \frac{(60-60)^2}{60} + \frac{(50-73.33)^2}{73.33} + \frac{(80-80.67)^2}{80.67} + \frac{(90-66)^2}{66}$$

$$= \frac{(50)^2}{60} + \frac{(-26)^2}{66} + \frac{(-24)^2}{54} + \frac{(-26.67)^2}{66.67} + \frac{(26.67)^2}{73.33} + \frac{(0)^2}{60} + \frac{(-23.33)^2}{73.33} + \frac{(-0.67)^2}{80.67} + \frac{(24)^2}{66}$$

$$= \frac{2500}{60} + \frac{676}{66} + \frac{576}{54} + \frac{711.29}{66.67} + \frac{711.29}{73.33} + 0 + \frac{544.29}{73.33} + \frac{0.45}{80.67} + \frac{576}{66}$$

$$= 41.67 + 10.24 + 10.67 + 10.67 + 9.70 + 0 + 7.42 + 0.01 + 8.73$$

$$= 99.11$$

$\therefore$ wbYxZ $\chi^2 = 99.11$

¯vaxbZvi gvÎv n‡jv,

$$\nu = (r-1)(c-1)$$
$$= (3-1)(3-1)$$
$$= 2 \times 2$$
$$= 4$$

AZGe, 4 ¯vaxbZvi gvÎv Ges 0.05 Zvrch gvÎvq χ^2-Gi ZwËK gvbwU n‡jv 9.45 (mviwY 9.5.1 `óe¨)| †h‡nZzAvgv‡`i ch‡e¶YKZ.χ^2-Gi gvb 99.11 hv ZwËK gv‡bi †P‡q A‡bK eo, †m‡nZzbw¯ Abykí wU cZ¨vL¨vZ n‡jv| GgbwK 0.01 gvÎvq I (hvi ZwËK gvb 13.28) bw¯ Abykí wU cZ¨vL¨vZ nq| A_vr, ¯vZ‡š¨i χ^2-cix¶vi gva¨‡g Avgiv †`L‡Z †cjvg †h, eq‡mi mv‡_ niZv‡ji c‡¶/wec‡¶ gZvg‡Zi `dv¸‡jvi g‡a¨ GKwU Nwbó m¤úK i‡q‡Q| PjK `ÿwU ¯Zš ev ¯vaxb bq|

χ^2-cix¶v c‡qv‡Mi ce-kZ (Conditons for the Applicaton of χ^2-Test)

Avgiv Av‡MB e‡jwQ †h, civgwÎK Abykí cix¶v wnmv‡e χ^2-cix¶v GKwU Rbwcq c×wZ| Z‡e Gi Ace¨envi j¶¨ Kiv wM‡q‡Q| ZvB GwU mZKZvi mv‡_ e¨envi Kiv c‡qvRb| †mRb¨ χ^2-cix¶v e¨envi‡i c‡e KZ¸‡jv ce-kZ c‡Y Kiv c‡qvRb| c_gZt, bgybv ch‡e¶Y¸‡jv GKwU †_‡K Ab¨wU ¯vaxb n‡Z n‡e| wØZxqZt, mgMK †_‡K bgybv‡K

ˆØePwqZfv‡e wbevPb Ki‡Z n‡e| ZZxqZt, DcvË‡K cKZ.GK‡K Dc¯wcZ n‡Z n‡e, kZKiv ev Abycv‡Z bq| PZzZt, bgybvq Kgc‡¶ 50wU ch‡e¶Y _vK‡Z n‡e| cÂgZt, mvivYi cwZwU N‡i Kgc‡¶ 5wU bgybv ch‡e¶Y _vK‡Z n‡e| me†k‡l, GwU g‡b ivL‡Z n‡e †h, χ^2-cix¶v AcivgwÎK cix¶v cwiev‡ii GKgvÎ cix¶v bq| c‡qvRb‡ev‡a wfb cix¶v c×wZ e¨envi Ki‡Z n‡e|

mvivsk

AcivgwÎK cix¶v¸‡jvi g‡a¨ me‡P‡q Rbwcq c×wZwU n‡jv KvB-eM (χ^2) cix¶v| KvB-eM cix¶v KvB-eM web¨v‡mi Dci wbfi K‡i| KvB-eM web¨vm GKwU Awew”Qb m¤vebv web¨vm hv abvZ¥K ÔOÕ †_‡K Amxg gvb chš— we¯—Z| Gi †Kvb FYvZ¥K gvb †bB| hw` KvB-eM-Gi gvb ÔOÕ nq Z‡e eySv‡Z n‡e †h, cZ¨wkZ I ch‡e¶YKZ.NUbmsL¨vi g‡a¨ ûeû wgj n‡q‡Q| KvB-e‡Mi gvb hZ eo n‡e cZ¨wkZ I ch‡e¶YKZ.NUbmsL¨vi g‡a¨ e¨eavb ZZ †ekx i‡q‡Q e‡j a‡i wb‡Z n‡e| `yBwU Pj‡Ki g‡a¨ m¤ú‡Ki aiY e¨vL¨vq KvB-eM GKwU Dc‡hvMx cwimsL¨vb cix¶v|

cv‡WvËi gj¨vqb

^be¨vw³K ck

mwVK DË‡ii cv‡k wUK (√) wPý w`b –

1| χ^2-web¨v‡m hLb ¯vaxbZvi gvÎvi gvb †QvU nq ZLb web¨v‡mi †iLwU ..... mPwj nq|
   K. Dfq w`‡K
   L. evg w`‡K
   M. Wvb w`‡K
   N. Dc‡ii me KÓwU

2| χ^2 -ci¤¶v wbY‡qi mÎwU n‡jv:
   K. $\chi^2 = \frac{(f_o - f_e)}{f_e}$
   L. $\chi^2 = \left(\frac{f_e - f_o}{f_e}\right)^2$
   M. $\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e}$
   N. $\chi^2 = \sum \frac{(f_e - f_o)^2}{f_e}$

3| χ^2-ci¤¶v e¨envi‡ii c‡e †h kZ ci¤Y Kiv c‡qvRb:
   K. bgybv ch‡e¶Y¸‡jv GKwU †_‡K Ab¨wU ¯vaxb n‡Z n‡e
   L. mgMK †_‡K bgybv‡K ‰ØePwqZ fv‡e wbe©vPb Ki‡Z n‡e
   M. DcvË‡K cKZ.GK‡K Dc¯wcZ Ki‡Z n‡e
   N. Dc‡ii me KÓwU

msw¶ß ck

1| msÁv w`b I cv_K¨ Av‡jvPbv Ki¤b:
   K. ch‡e¶YKZ© I cZ¨wkZ NUbmsL¨v
   L. bw¯ I weKí Abykí

iPbvgj¨K ck

1| KvB-eM ci¤¶v wK? KvB-eM ci¤¶vi c‡qvRbxqZv D`vniYmn Av‡jvPbv Ki¤b|

2| Dcv‡Ëi mvgÄm¨Zvi χ^2-ci¤¶v I ¯vZ‡š¨i χ^2-ci¤¶vi g‡a¨ Zzjbvgj¨K Av‡jvPbv Ki¤b|